# 2017 MATHEMATICS

Full marks: 100 Time: 3 hours

### General instructions:

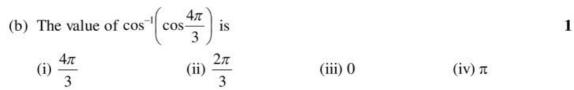
- i) Approximately 15 minutes is allotted to read the question paper and revise the answers.
- ii) The question paper consists of 26 questions. All questions are compulsory.
- iii) Marks are indicated against each question.
- iv) Internal choice has been provided in some questions.
- v) Use of simple calculators (non-scientific and non-programmable) only is permitted.

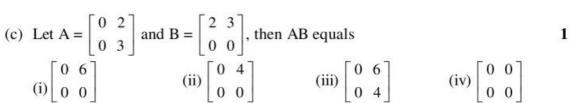
N.B: Check that all pages of the question paper is complete as indicated on the top left side.

### Section - A

## 1. Choose the correct answer from the given alternatives:

(a)	Consider the set $A = \{1, 2, 3, 4\}$ . Which one of the following relations R form	
()	a reflexive relation?	1
	(i) $R = \{(1, 1), (1, 2), (2, 2), (3, 4)\}$	
	(ii) $R = \{(1, 1), (2, 2), (2, 3), (3, 3), (3, 4)\}$	
	(iii) $R = \{(1, 1), (2, 2), (2, 3), (3, 3), (3, 4), (4, 4)\}$	
	(iv) $R = \{(1, 1), (2, 1), (2, 3), (3, 3), (3, 4), (4, 4)\}$	





(d) If 
$$y = \tan^{-1} \frac{x}{2} - \cot^{-1} \frac{x}{2}$$
, then  $\frac{dy}{dx}$  is

(i) 
$$\frac{4}{4+x^2}$$
 (ii)  $\frac{2}{4+x^2}$  (iii)  $\frac{1}{4+x^2}$  (iv)  $\frac{2}{1+x^2}$ 

1

1

1

1

1

2

- (e) If  $y = \log \sqrt{\tan x}$ , then the value of  $\frac{dy}{dx}$  at  $x = \frac{\pi}{4}$  is 1
  - (ii)  $\frac{1}{2}$ (i) 0 (iii) 1
- (f) If the rate of change of area of a circle is equal to the rate of change of its diameter, then its radius is
- (ii)  $\frac{2}{7}$
- (iii)  $\frac{\pi}{2}$
- (iv)  $\pi$

 $(iv) \infty$ 

- (g)  $\int (2x+7)^6 dx$  is equal to
  - (i)  $\frac{(2x+7)^7}{14} + C$

- (ii)  $\frac{(2x+7)^6}{14} + C$
- (iii)  $\frac{(2x+7)^7}{7} + C$
- (iv)  $\frac{-(2x+7)^7}{14} + C$
- (h)  $\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$  is equal to
  - (i)  $\frac{\pi}{2}$  (ii)  $\frac{\pi}{3}$

(i)1

(ii) 2

- (iii)  $\frac{\pi}{4}$
- (i) The degree of the differential equation  $\left(1 + \frac{dy}{dx}\right)^5 = \left(\frac{d^2y}{dx^2}\right)^2$  is
- (j) The direction cosines of the vector  $\vec{a} = \hat{i} \hat{j} 2\hat{k}$  are
  - (i) 1, -1, -2

(ii)  $\frac{1}{\sqrt{6}}$ ,  $-\frac{1}{\sqrt{6}}$ ,  $-\frac{2}{\sqrt{6}}$ 

(iii)  $\frac{1}{4}$ ,  $-\frac{1}{4}$ ,  $-\frac{2}{4}$ 

(iv)  $\sqrt{\frac{1}{6}}$ ,  $-\sqrt{\frac{1}{6}}$ ,  $-\sqrt{\frac{2}{6}}$ 

## Section - B

- Show that the function  $f: \mathbb{N} \to \mathbb{N}$  given by f(1) = f(2) = 1 and f(x) = x 1 for every x > 2 is onto but not one-one.
- 3. Express  $\tan^{-1} \left( \frac{\sin x}{1 + \cos x} \right)$  in the simplest form. 2

- 4. Using determinants, show that the points (a+5, a-4), (a-2, a+3) and (a, a)do not lie on a straight line.
- 5. If  $y = \sin^{-1}(\cos x) + \cos^{-1}(\sin x)$ , prove that  $\frac{dy}{dx} = -2$
- 6. If  $y = \tan x$ , prove that  $\frac{d^2 y}{dx^2} = 2y \frac{dy}{dx}$
- 7. Prove that the tangents to the curve  $y = x^2 5x + 9$  at the points (2, 0) and (3, 0) are at right angles.
- 8. Evaluate  $\int \frac{x^4}{1+x^2} dx$
- 9. Form a differential equation of the curve  $\frac{x}{a} + \frac{y}{b} = 1$ , where a and b are arbitrary constants.
- 10. Find the angle between the vectors  $\vec{a}$  and  $\vec{b}$  if  $\vec{a} = \hat{i} \hat{j}$  and  $\vec{b} = \hat{i} + \hat{j}$
- 11. If A and B are two events such that  $P(A) = \frac{7}{13}$ ,  $P(B) = \frac{9}{13}$  and  $P(A \cap B) = \frac{4}{13}$ , then find (i)  $P(A \cup B)$  (ii)  $P(\overline{B} | \overline{A})$

### Section - C

- 12. Consider the set of integers **Z**. Define a relation R on **Z** as  $R = \{(x, y) : x y = 3k, where k is some integer\}$ . Prove that R is an equivalence relation.
- 13. a. Using properties of determinants, prove that:

$$\begin{vmatrix} x^2 + 2x & 2x + 1 & 1 \\ 2x + 1 & x + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (x - 1)^3$$

**Or b.** For the matrix  $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ , verify that  $A^2 + 8I = 8A$ . Hence, find  $A^{-1}$ .

4

4

14.a. Show that the following function f(x) is continuous at x = -3 but discontinuous at x = 3:

$$f(x) = \begin{cases} |x|+3, & \text{if } x \le -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x+2, & \text{if } x \ge 3 \end{cases}$$

)r

**b.** If  $y = (\tan^{-1} x)^2$ , show that  $(1+x^2)^2 y_2 + 2x(1+x^2)y_1 = 2$ 

15. Evaluate 
$$\int_{0}^{\pi} \frac{x}{1+\sin x} dx$$

16.**a.** Evaluate 
$$\int \left\{ \frac{1}{\log x} - \frac{1}{(\log x)^2} \right\} dx$$

Or 4

**b.** Evaluate 
$$\int \frac{dx}{2\sin x + \cos x + 3}$$

17. Solve the differential equation 
$$2ye^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$$

18. **a.** If  $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$ ,  $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$  and  $\vec{c} = 3\hat{i} + \hat{j} + \hat{k}$ , then find the vector  $\vec{d}$  which is perpendicular to both  $\vec{a}$  and  $\vec{b}$  and  $\vec{d} \cdot \vec{c} = 21$ .

Or 4

- **b.** If  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$  are three vectors with magnitudes 3, 5, 7 respectively such that  $\vec{a} + \vec{b} + \vec{c} = 0$ , then find the angle between  $\vec{a}$  and  $\vec{b}$ .
- 19. Find the Cartesian and vector equations of the plane through the points (-1, 1, 1) and (1, -1, 1) and perpendicular to the plane x + 2y + 2z = 5.
- 20. A random variable X has the following probability distribution:

$x_i$	0	1	2	3	4	5	6	7
$p_i$	0	k	2 <i>k</i>	2 <i>k</i>	3 <i>k</i>	$k^2$	$2k^2$	$7k^2 + k$

Determine: (i) k (ii) P(X < 3) (iii) P(0 < X < 3)

21.a. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the mean and variance of the number of kings.

Or 4

**b.** In a bolt factory, three machines A, B, C manufacture 25%, 35% and 40% of the total production respectively. Of their respective outputs, 5%, 4% and 2% are defective. A bolt is drawn at random from the total product and it is found to be defective. Find the probability that it was manufactured by the machine C.

### Section - D

22.a. Using elementary row transformations, find the inverse of the matrix  $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ 

Or 6

b. Using matrix method, solve the following system of linear equations:

$$x - y + z = 1$$

$$2x + y - z = 2$$

$$x - 2y - z = 4$$

23.a. Find the point on the curve  $y^2 = 4x$  which is nearest to the point (2, -8).

Or 6

- **b.** An open box with a square base is to be made out of a given quantity of sheet of area  $a^2$ . Show that the maximum volume of the box is  $\frac{a^3}{6\sqrt{3}}$ .
- 24. **a.** Using integration, find the area of the triangle whose vertices are A(1, 3), B(2, 5) and C(3, 4).

**b.** Using integration, find the area of the region bounded by the lines: y = 1 + |x + 1|, x = -2, x = 3 and y = 0

25. a. Show that the following lines intersect and find the point of their intersection:

$$\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda (3\hat{i} - \hat{j}) \text{ and}$$

$$\vec{r} = 4\hat{i} - \hat{k} + \mu (2\hat{i} + 3\hat{k})$$
Or

- **b.** Find the distance of the origin from the plane x + 2y + z = 4 measured parallel to the line  $\frac{x+1}{2} = \frac{y-1}{2} = \frac{z+2}{1}$
- 26. a. A gardener has a supply of fertilizers of type A which consists of 10% nitrogen and 5% phosphoric acid and of type B which consists of 5% nitrogen and 10% phosphoric acid. After testing the soil, it is found that at least 15 kg of nitrogen and 15 kg of phosphoric acid is required for a good crop. The fertilizer of type A costs Rs 3 per kg and type B costs Rs 4 per kg. How many kilograms of each fertilizer should be used to meet the requirements so that the cost is minimum?

Or 6

b. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a packet of nuts while it takes 3 hours on machine A and 1 hour on machine B to produce a packet of bolts. He earns a profit of Rs 17.50 per packet on nuts and Rs 7 per packet on bolts. How many packet of each should be produced each day so as to maximize his profit if he operates his machines for at most 12 hours a day? Also find the maximum profit.

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