2019

MATHEMATICS

Full marks: 100 Time: 3 hours

General instructions:

i) Approximately 15 minutes is allotted to read the question paper and revise the answers.
ii) The question paper consists of 26 questions. All questions are compulsory.
iii) Marks are indicated against each question.
iv) Internal choice has been provided in some questions.
v) Use of simple calculators (non-scientific and non-programmable) only is permitted.

N.B: Check that all pages of the question paper is complete as indicated on the top left side.

Section – A

1. **Choose the correct answer from the given alternatives:**

(a) If $f(x)=|x|$ and $g(x)=[x]$, then $g\circ f(-3.7)$ is equal to **1**

(i) -3.7 (ii) 3 (iii) 3.7 (iv) 4

(b) Consider the set **Q** with the binary operation * as $a*b=\frac{ab}{4}$. Then the identity element is **1**

(i) $\frac{1}{4}$ (ii) 1 (iii) 4 (iv) 16

(c) If a matrix A is both symmetric and skew-symmetric matric, then **1**

(i) A is a diagonal matrix (ii) A is a zero matrix
(iii) A is a square matrix (iv) none of these

(d) If $y=a^x x^a$ then $\frac{dy}{dx}$ is equal to **1**

(i) $a^x x^{a-1}(a-x\log a)$ (ii) $a^x x^{a-1}(a+x\log a)$
(iii) $a^x x^a(a+x\log a)$ (iv) $a^x x^{a-1}(x+a\log a)$

(e) The point on the curve $y=2x^2$, where the slope of the tangent is 8, is **1**

(i) (0, 2) (ii) (0, 8) (iii) (2, 8) (iv) (8, 2)

(f) The value of $\int \tan^2 x\,dx$ is **1**

(i) $x-\tan x+C$ (ii) $\tan x+x+C$ (iii) $\tan x-x+C$ (iv) $x\tan x+C$

(g) The value of $\int_{-1}^{1} \log\left(\frac{2-x}{2+x}\right)dx$ is **1**

(i) −1 (ii) 0 (iii) 1 (iv) 2

(h) If $p\hat{i} + 3\hat{j}$ is a vector of magnitude 5, then the value of p is **1**

(i) 0 (ii) 1 (iii) ± 3 (iv) ± 4

(i) Let A and B be events such that P(A) = $\frac{7}{13}$, P(B) = $\frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, then P(A |B) is equal to **1**

(i) $\frac{4}{9}$ (ii) $\frac{7}{13}$ (iii) $\frac{2}{3}$ (iv) $\frac{9}{4}$

(j) If A & B are two events such that P(A) = $\frac{1}{4}$, P(B) = $\frac{1}{3}$, P(A ∪ B) = $\frac{1}{2}$, then the events A and B are **1**

(i) independent (ii) dependent

(iii) mutually exclusive (iv) none of these

Section – B

2. Consider the set of real numbers **R**. Define the relation R on **R** as "a R b if and only if $a^2 + b^2 = 1$". Write the domain of R. Also, prove that R is not transitive. **2**

3. Find $f \circ g$ and $g \circ f$ if $f(x) = |x|$ and $g(x) = |4x + 3|$. Are they equal? **2**

4. Find the value of $\tan\left(\tan^{-1}\sqrt{3} + \sin^{-1}\frac{1}{\sqrt{2}} - \cot^{-1} 1\right)$ **2**

5. Solve the following equation for x: $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1} x\right) = 1$ **2**

6. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, show that $A'A = I_2$ **2**

7. Differentiate $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x. **2**

8. If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$ **2**

9. Evaluate $\int \sin^4 x\, dx$ **2**

10. Form a differential equation representing the given curve, $y = a e^{bx}$, where a & b are arbitrary constants. **2**

11. Find the value of λ for which $\vec{a}$ and $\vec{b}$ are perpendicular if $\vec{a} = 7\hat{i} - \lambda\hat{j} - 7\hat{k}$ and $\vec{b} = 4\hat{i} + 5\hat{j} - \hat{k}$ **2**

Section – C

12. **a.** If $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, show that $2A^{-1} = 9I - A$.

Or **4**

b. Using properties of determinants, prove that:

$$\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

13. **a.** If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_2 + x y_1 + y = 0$

Or **4**

b. Find the coordinates of the point at which the tangent to the curve $f(x) = x^2 - 6x + 1$ is parallel to the chord joining the points (1, –4) and (3, –8)

14. **a.** If $x = a\sin 2t\,(1 + \cos 2t)$, $y = b\cos 2t\,(1 - \cos 2t)$, show that $\frac{dy}{dx} = \frac{b}{a}$ at $t = \frac{\pi}{4}$

Or **4**

b. If $f(x) = \left(\frac{3+x}{1+x}\right)^{2+3x}$, find $f'(0)$.

15. Evaluate $\int \frac{\cos^5 x}{\sin x}\,dx$ **4**

16. **a.** Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos x}{(1+\sin x)(2+\sin x)}\,dx$

Or **4**

b. Evaluate $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x}\,dx = \pi\left(\sqrt{2} - 1\right)$

17. Solve the differential equation $x \sin\frac{y}{x}\frac{dy}{dx} + x - y\sin\frac{y}{x} = 0$, given that $y(1) = \frac{\pi}{2}$ **4**

18. **a.** If $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = 3\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - \hat{k}$, find the vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 4$

Or **4**

b. Show that the points A, B, C, D with position vectors $4\hat{i} + 8\hat{j} + 12\hat{k}$, $2\hat{i} + 4\hat{j} + 6\hat{k}$, $3\hat{i} + 5\hat{j} + 4\hat{k}$ and $5\hat{i} + 8\hat{j} + 5\hat{k}$ respectively are coplanar.

19. Find the foot and the length of the perpendicular drawn from the point (3, 4, 5) to the plane $2x - 5y + 3z = 39$ **4**

20. In a bulb factory, machines A, B and C manufacture 60%, 30% and 10% bulbs respectively. Out of these bulbs, 1%, 2% and 3% of the bulbs produced respectively by A, B and C are found to be defective. A bulb is picked up at random from the total production and found to be defective. Find the probability that this bulb was produced by the machine A. **4**

21. A die is tossed once. If the random variable X is defined as:

$$X = \begin{cases} 1, \text{ if the die results in an even number} \\ 0, \text{ if the die results in an odd number} \end{cases}$$ **4**

Then, find the mean and variance of X.

Section – D

22. **a.** Using elementary row transformations, find the inverse of the matrix $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

Or **6**

b. Solve the following system of linear equations using matrix method:

$$\begin{aligned} 2x + 3y + 3z &= 5 \\ x - 2y + z &= -4 \\ 3x - y - 2z &= 3 \end{aligned}$$

23. **a.** Show that the semi-vertical angle of a cone maximum volume and given slant height is $\tan^{-1}\sqrt{2}$

Or **6**

b. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

24. **a.** Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line $\frac{x}{a}+\frac{y}{b}=1$

Or **6**

b. Using the method of integration, find the area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$

25. **a.** Find the image of the point (1, 3, 4) in the plane $2x - y + z + 3 = 0$. Also, find the distance of the point from its image.

Or **6**

b. Show that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ intersect. Also, find their point of intersection.

26. **a.** A housewife wishes to mix two types of food X and Y in such a way that the mixture contains at least 8 units of vitamin A and 10 units of vitamin B. X contains 2 units/kg of vitamin A and 1 unit/kg of vitamin B. While Y contains 1 unit/kg of vitamin A and 2 units/kg of vitamin B. It costs Rs 60/kg of X and Rs 80/kg of Y. Formulate this problem as a linear programming problem to minimize the cost of such a mixture and solve it.

Or **6**

b. A shopkeeper wants to invest Rs 5400 on two types of pens. Type A costs Rs 180 per packet and type B costs Rs 60 per packet. He can get a profit of Rs 15 on type A and Rs 10 on type B. He has a space for 50 packets only. Formulate this as an LPP so as to get the number of each type of packets and the maximum profit. Also, find the maximum profit.

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