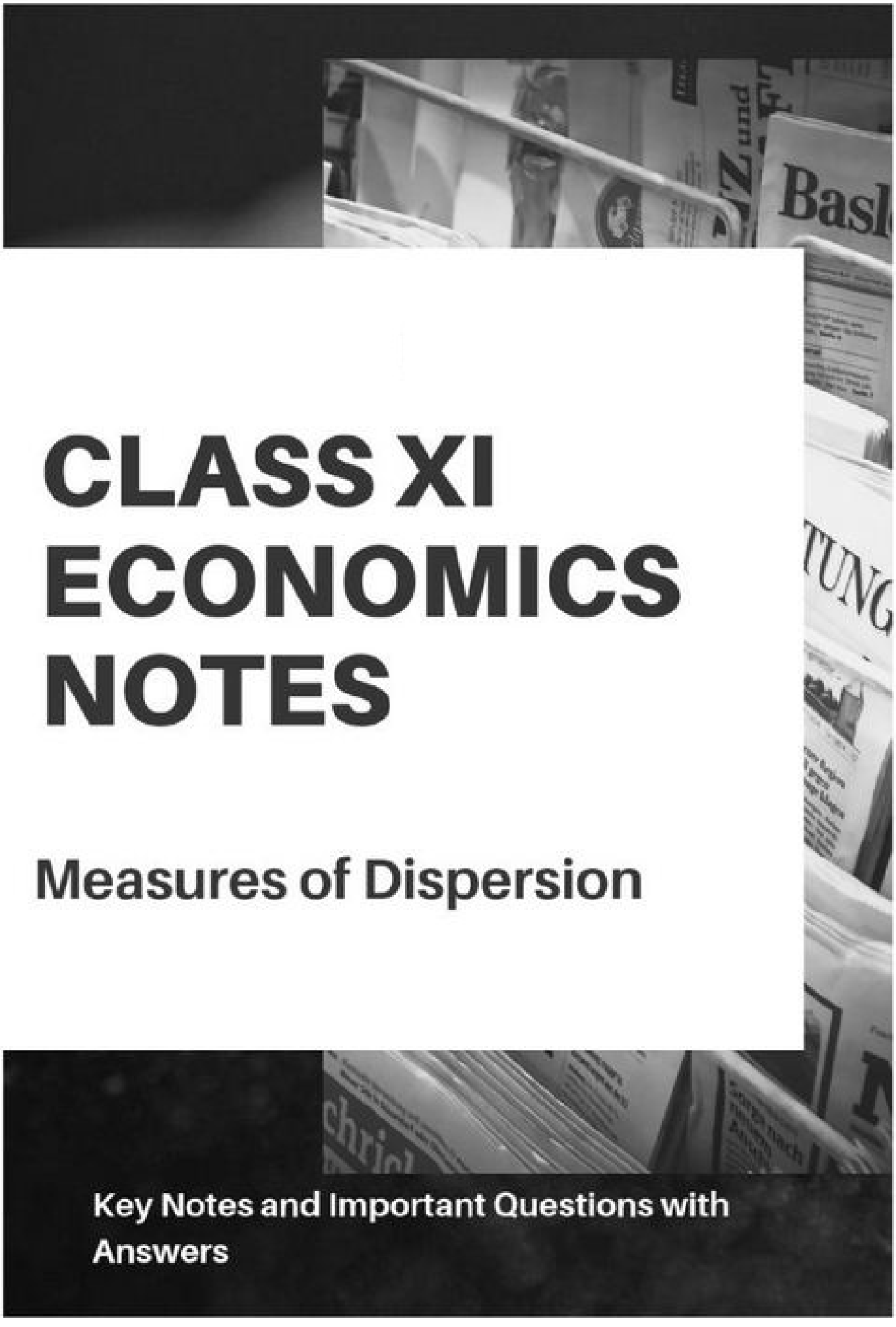


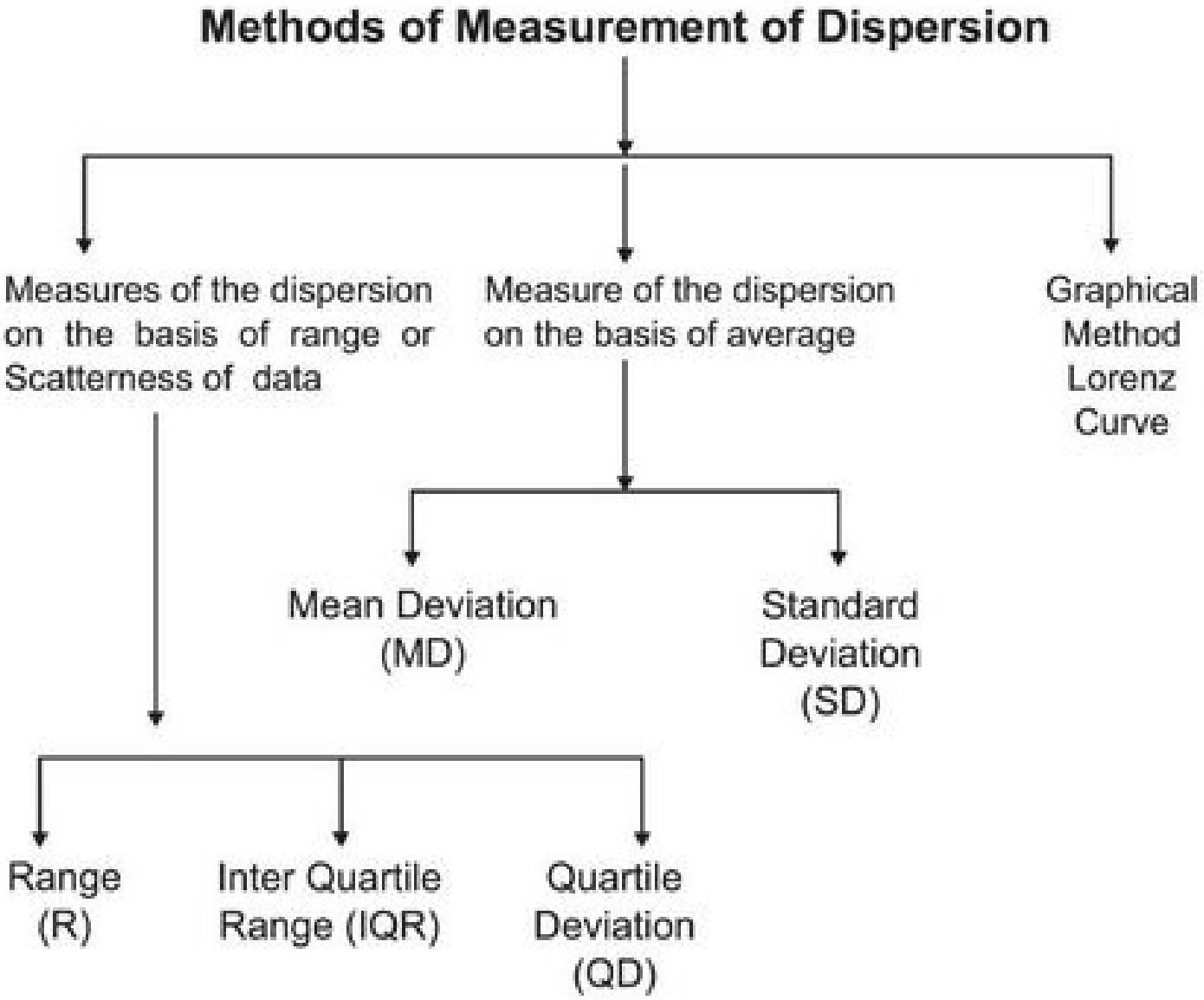
CLASS XI ECONOMICS NOTES

Measures of Dispersion

Key Notes and Important Questions with Answers

(ii) MEASURES OF DISPERSION

Scatterness of data from central value is known as dispersion. It indicates that how value of Variable or Items are different from its average value.



- Absolute and Relative measures of Dispersion :

Absolute measures of Dispersion : When the scatter or spread of data is meausred in the same units of original data. It is used only wihtin a series and measured in the same unit as that of series. It is not, suitable for comparision between two series.

Relative measures of Disperions : When the scatter or spread of data is measured as ratios or percentage, they are also known as coefficients of dispersion. These are independent of the units of measurement and used to compare two or more series where unit of measure are different.

Absolute Measures

1. Range (R) = L – S
2. Quartile deviation
$$= \frac{Q_3 - Q_1}{2}$$
3. Mean Deviation M.D
$$= \frac{\Sigma | dx |}{N}$$
4. Standard deviation

$$S.D \text{ or } \sigma = \sqrt{\frac{\Sigma dx^2}{N}}$$

Relative Measures

1. Coefficient of range = $\frac{L - S}{L + S}$
2. Coefficient of Q.D.
$$= \frac{Q3 - Q1}{Q3 + Q1}$$
3. Coefficient of M.D
$$= \frac{MD_{\bar{x}}}{\bar{X}}$$
4. Coefficient of S.D. = $\frac{S.D.}{\bar{X}}$
5. Coefficient of Variation

$$CV = \frac{SD}{\bar{X}} \times 100$$

1) Range : It is the difference between largest and smallest value of the series.

Range (R) = Largest value (L) – Smallest value (S)

$$\text{Coefficient of Range} = \frac{L - S}{L + S}$$

Note : More value of range means more dispersion and vice-versa.

2. Inter-Quartile Range (IQR)

$IQR = Q_3 - Q_1$

Where Q3 = Upper Quartile or Third Quartile

Q1 = Lower Quartile or First Quartile

Note: It is based on 50% of average value of distribution. It does not influence with extreme value.

3. Quartile Deviation (QD)

It is also called semi Inter Quartile Range.

$$QD = \frac{Q_3 - Q_1}{2}$$

$$\text{Coefficient of QD} = \frac{Q3 - Q1}{Q3 + Q1}$$

- **Measures of Dispersion on the Basis of Average**

It includes the following two methods.

1. Mean Deviation (MD)

   It is calculated as an average on the basis of deviation obtained from any central value such mean, median and mode. Here we calculate it from deviation taken from mean on median. It takes only the absolute value which is indicated by two | | bars. It is also known as second degree average. It is based on all the values. If dispersion is measured by the mean then the dispersion will be minimum. It ignore signs.

A. Mean Deviation from Mean ($MD_{\bar{x}}$)

Individual series	Discrete and continuous series
$MD_{\bar{x}} = \frac{\Sigma \mid X - \bar{X} \mid}{N} = \frac{\Sigma \mid dx \mid}{N}$	$MD_{\bar{x}} = \frac{\Sigma f \mid X - \bar{X} \mid}{\Sigma f} = \frac{\Sigma f \mid dx \mid}{\Sigma f}$
$\bar{X} = \frac{\Sigma X}{N}$	Σf = Summation of frequencies
N = No. of items	f = Frequencies
	$\bar{X} = \frac{\Sigma fx}{\Sigma f}$
Coeff. of MD = $\frac{MD_{\bar{x}}}{\bar{X}}$	Coeff. of MD = $\frac{MD_{\bar{x}}}{\bar{X}}$

B. Mean Deviation from Median MDm

- First of all, we should arrange the items and classes into ascending or descending order.
- Calculate cumulative frequency in discrete series and continuous series
- Find out MDm in different series by using the following formula :

<table>
<tr><th>Individual Series</th><th colspan="2">Discrete and continuous series</th></tr>
<tr><td>$MD_m = \frac{\Sigma \mid X - M \mid}{N}$
$= \frac{\Sigma \mid d\bar{x} \mid}{N}$</td><td colspan="2">$MD_m = \frac{\Sigma f \mid X - M \mid}{\Sigma f} = \frac{\Sigma f \mid dm \mid}{\Sigma f}$</td></tr>
<tr><td>$M = \text{size of } \left(\frac{N+1}{2}\right)^{th} \text{ items}$</td><td>$M = \text{size of } \left(\frac{N+1}{2}\right)^{th} \text{ items}$</td><td>$M = L_1 + \frac{\frac{N}{2} - C_f}{f} \times i$</td></tr>
<tr><td>N = no. of items.</td><td>in discrete series
$N = \Sigma f$</td><td>in continuous series
where
L_1 = Lower limit of median class</td></tr>
<tr><td>Coeff. of MD = $\frac{MD_m}{M}$</td><td colspan="2">Coeff. of MD = $\frac{MD_m}{M}$</td></tr>
</table>

2. Standard Deviation (SD)

   It is the square root of the arithmetic average of the square of the deviations measure from mean. It is also know as root mean square deviation. it is indicated by Greek Letter Sigma (σ). it is the best measure of dispersion. It removes the mathematical errors of mean deviation. It is based on all the vlaues of the series and is rigidly defined.

- Methods of Measurement of SD

  There are following four methods of measurement of standard deviation.

  (i) Actual mean method.

  (ii) Assumed mean method.

  (iii) Direct method.

  (iv) Step-Deviation method

INDIVIDUAL SERIES

Actual Mean Method	Assumed Mean Method	Step-Deviation method	Direct method
$\bar{X} = \frac{\Sigma X}{N}$	$\bar{X} = A + \frac{\Sigma d}{N}$	$\bar{X} = \frac{A + \Sigma d'}{N} \times i$	$\bar{X} = \frac{\Sigma X}{N}$
$SD = \sqrt{\frac{\Sigma (X - \bar{X})^2}{N}}$	$SD = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$	$SD = \sqrt{\frac{\Sigma d'^2}{N} - \left(\frac{\Sigma d'}{N}\right)^2} \times c$	$SD = \sqrt{\frac{\Sigma X^2}{N} - \left(\frac{\Sigma X}{N}\right)^2}$
or			or
$SD = \sqrt{\frac{\Sigma x^2}{N}}$	$d = X - A$	$d' = \frac{X - A}{c}$	$SD = \sqrt{\frac{\Sigma X^2}{N} - (\bar{X})^2}$
$x = X - \bar{X}$	d = deviation from assumed mean		
x = deviation from actual mean	A = assumed mean		

DISCREET AND CONTINUOUS SERIES

Actual Mean Method	Assumed Mean Method	Step-Deviation method	Direct method
$\bar{X} = \frac{\Sigma fx}{\Sigma f}$	$\bar{X} = A + \frac{\Sigma fd}{\Sigma f}$	$\bar{X} = \frac{\Sigma fd'}{\Sigma f} \times i$	$\bar{X} = \frac{\Sigma fx}{\Sigma f}$
$SD = \sqrt{\frac{\Sigma f(X - \bar{X})^2}{\Sigma f}}$	$SD = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2}$	$SD = \sqrt{\frac{\Sigma fd'^2}{\Sigma N} - \left(\frac{\Sigma fd'}{\Sigma f}\right)^2} \times c$	$SD = \sqrt{\frac{\Sigma fX^2}{\Sigma f} - \left(\frac{\Sigma fX}{\Sigma f}\right)^2}$
or			or
$SD = \sqrt{\frac{\Sigma fx^2}{\Sigma f}}$	$d = X - A$	$d' = \frac{X - A}{c}$	$SD = \sqrt{\frac{\Sigma fX^2}{\Sigma f} - (\bar{X})^2}$

NOTE : X = Value of item in discrete series and it is the median value of C.I. i.e. Class Interval

Coff. of SD = $\frac{SD}{\bar{X}}$ (in all series)

Coefficient of Variation (CV) : It is the most important relative measures of dispersion. When we multiply coefficient of standard deviation by 100 then we get coefficient of variation.

$$CV = \frac{SD}{\bar{X}} X\ 100 \quad = \frac{\sigma}{\bar{X}} X\ 100$$

more the value of CV means more variation, and less consistency, less uniformity, less homogeneity and vice versa.

- **Graphical Method : Lorenz Curve**

This method was developed by Dr. Max. O. Lorenz it estimates dispersion. It is a graphical method. It is useful for the study of distribution of income, wealth, profits, wages, sale, purchase turnover etc.

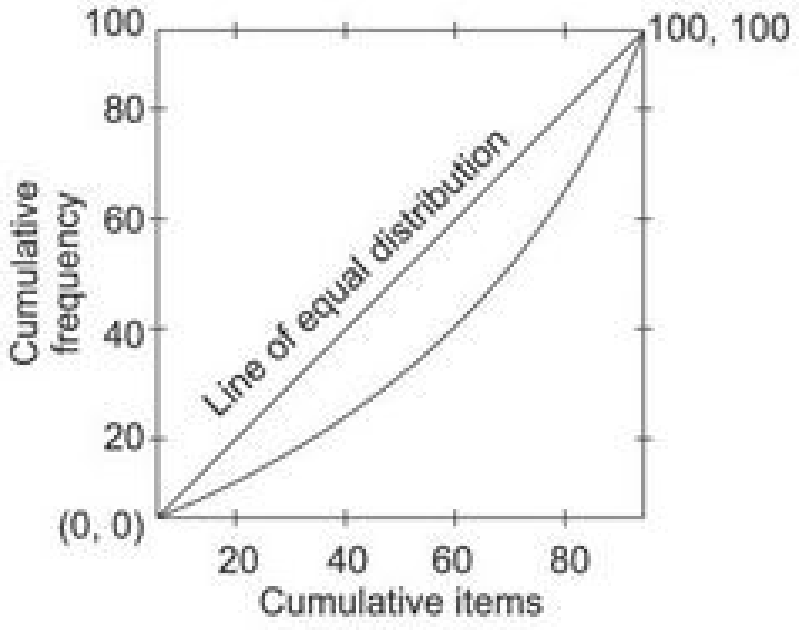
In this method, value the frequencies are cumulative and their percentage are calculated. These values are plotted on the graph paper and to join all the points with a curve. Thus the obtained curve is called Lorenz Curve.

The nearer the curve is to the line of equal distribution, lesser will be dispersion and the farther the curve is from the line of equal distribution, the greater will be dispersion.

The most important draw back of this curve is that is does not give a quantitative measure of dispersion.

- Construction of Lorenz Curve

1) Series is converted into a cumulative frequency series, the cumulative sum of items is assumed to be 100 and different items are converted into percentage of the cumulative sum.



2) Cumulative sum of frequency is assumed to be 100 and different frequencies are converted into percentage of sum of frequency.
3) Cumulative frequencies are plotted on X-axis and cumulative items are plotted on Y-axis of graph.
4) On both axis values are plotted of own 1–100.
5) A diagonal line joining (0, 0) with cumulative frequency (100,100) is called line of equal distribution.
6) Actual data are plotted by joining different points. This is the Lorenz Curve.

QUESTION BANK

ONE (1) MARK QUESTIONS :-

1) What is inter quartile range?
2) Give the formula of calculating coefficient of variation.
3) What is Lorenz Curve?
4) Calculate Range –

   22, 35, 32, 45, 42, 48, 39
5) Which graphical method is used to measure dispersion?
6) Give meaning of dispersion.
7) How is coefficient of mean deviation computed?
8) Which measure of dispersion covers middle 50% of the items?
9) Write one major demerit of mean deviation.
10) Define relative measure of dispersion?
11) Define Range.
12) Why standard deviation is better t han mean deviation?
13) If each item of a series is increased by 5, then effect on standard deviation is :

   (i) increase by 5 (ii) decrease by 5

   (iii) increase by 25 (iv) No change

14) If each item of a series is multiplied by 2, then effect on standard deviation is :

(i) increased by 2 (ii) Multiplied by 2

(ii) decreased by 2 (iv) become half.

15) Income of top 10% earner become double, then the quartile deviation also become :

(i) double (ii) half

(iii) do not change (iv) insufficient information

THREE & FOUR (3 & 4) QUESTIONS :-

1) Mention important measures of dispersion.
2) Mention any two merits and two demerits of mean deviation.
3) Distinguish between mean deviation and standard deviation.
4) What do you understand by dispersion?
5) Discuss the relative merits of range, mean deviation, deviation and standard as measures of dispersion.
6) Find the range and coefficient of range of the following :

Marks	10	20	30	40	50	60	70
No. of students	8	12	7	30	10	5	2

(Range = 60 marks; coefficient of range = 0.75)

7) Why do we calculate the coefficients of the measure of dispersion? Explain with example.
8) Discuss the importance of measures of dispersion.
9) Why standard deviation is regarded as the better measure of dispersion?
10) Find quartile deviation and its coefficient :

Wages	50	55	58	60	70	90	95	110
Frequency	5	7	12	10	8	6	2	9

11) From the price of gold and silver, state which is more stable in value—

Gold : 410 440 430 480 450 460 490 500 420 400

Silver : 910 930 960 920 900 970 990 940 980 950

12) Why is range not calculated for openended frequency distribution?

13) Explain two merits and demerits of Lorenz curve.

14) Calculate mean deviation from mean and coefficient of mean deviation.

X :	10-20	20-30	30-50	50-70	70-80
f :	5	8	16	8	3

15) Calculate standard deviation (direct method)

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	5	10	25	30	20	10

(Range = 60 marks; coefficient of range = 0.75)

SIX (6) MARKS QUESTIONS :-

1) The sum of 10 values is 100 and the sum of their squares is 1090. Find the Co-efficient of variation.

Ans. 30%

2) Calculate mean deviation and compare the variability of two series A and B.

Series A	10	12	16	20	25	27	30
Series B	10	20	22	25	27	31	40

Ans. $M.D_A = 6.28$
CMD = 0.30
$MD_B = 6.57$
CMD = 0.26
A more variable

3) Calculate SD and its Co-efficient.

Class	0-5	5-10	10-15	15-20	20-25	25-30	30-35
Freq.	4	6	10	16	12	8	4

Ans. $\overline{X} = 18$, SD = 7.89, CSD = 0.44

4) Calculate IQR, QD and CQD

Class Interval (X)	0-5	5-10	10-15	15-20	20-25	25-30
Freq.	3	9	15	23	30	20

Ans. IQR = 9.84, QD = 4.92, CQD = 0.25

5) Draw the Lorenz Curve from the given data as below :

Income	Below 200	200-500	500-1000	1000-2000	2000-3000
Factory A	6	10	14	12	8
Factory B	7	12	15	10	6

ANSWERS OF ONE (1) MARK QUESTIONS

1) The difference in the two values of quartile is called inter quartile range $(Q_3 - Q_1)$.

2) Coefficient of variation $= \frac{\sigma}{\bar{X}} \times 100$

3) Lorenz Curve is the graphic presentation of studying dispersion.

4) Range = Largest Value – Smallest Value

$= 48 - 22 = 26$

5) Lorenz Curve method is used to measure dispersion.

6) Dispersion is a measure of the variation of the item from a central value.

7) Mean deviation $= \frac{\Sigma f\ (D)}{\bar{X}}$

8) Inter quartile range.

9) The major demerit of mean deviation is that it ignores $\pm$ sings.

10) Relative measures are expressed in ratios or percentage, also knows as coefficient of dispersion.

11) Range is the difference between largest and smallest value of series.

12) Standard deviation is rigidly defined and fit for algebric operators.

13) (iv)

14) (ii)

15) (iii)

Exam Oriented Questions with Answers

Q1. What do you mean by Lorenz Curve?

Ans. It is a graphic representation of dispersion, which studies about distribution of income, wealth, profit, wages etc.

Q2. What is variance?

Ans. The square of standard deviation is known as variance

$$\text{Variance} = \sigma^2 = \frac{\Sigma x^2}{N}$$

Q3. From the following data find out which factory may be considered more uniform.

	No. of Workers	
Wages (in Rs.)	**Factory A**	**Factory B**
20	30	45
60	25	35
100	30	25
140	45	40
180	25	25
220	13	20
260	24	5
300	8	5

Hints : Calculate $\overline{X}$ and standard deviations of both factory.

Then calculate co-efficient of variation of both factory with help of

$$\text{Formula C.V.} = \frac{\sigma}{\overline{X}} \times 100 = \frac{\text{Standard Deviation}}{\overline{X}} \times 100$$

Factory A $\longrightarrow$ $\overline{X}_A = 137$, $\sigma_A = \text{S.D.} = 80.8$, C.V. = 59%

Factory B $\longrightarrow$ $\overline{X}_B = 114$, $\sigma_B = \text{S.D.} = 75.6$, C.V. = 66%

Hence, factory A is more uniform than B.