

National Council of Educational Research and Training (NCERT)

Book for Class IX

Subject: Maths

Chapter 9 – Areas of Parallelograms and Triangles

9.1 Introduction

In Chapter 5, you have seen that the study of Geometry, originated with the measurement of earth (lands) in the process of recasting boundaries of the fields and dividing them into appropriate parts. For example, a farmer *Budhia* had a triangular field and she wanted to divide it equally among her two daughters and one son. Without actually calculating the area of the field, she just divided one side of the triangular field into three equal parts and joined the two points of division to the opposite vertex. In this way, the field was divided into three parts and she gave one part to each of her children. Do you think that all the three parts so obtained by her were, in fact, equal in area? To get answers to this type of questions and other related problems, there is a need to have a relook at areas of plane figures, which you have already studied in earlier classes.

You may recall that the part of the plane enclosed by a simple closed figure is called a *planar region* corresponding to that figure. The magnitude or measure of this planar region is called its *area*. This magnitude or measure is always expressed with the help of a number (in some unit) such as 5 cm^2 , 8 m^2 , 3 hectares etc. So, we can say that area of a figure is a number (in some unit) associated with the part of the plane enclosed by the figure.

We are also familiar with the concept of congruent figures from earlier classes and from Chapter 7. *Two figures are called congruent, if they have the same shape and the same size.* In other words, if two figures A and B are congruent (see Fig. 9.1), then using a tracing paper,

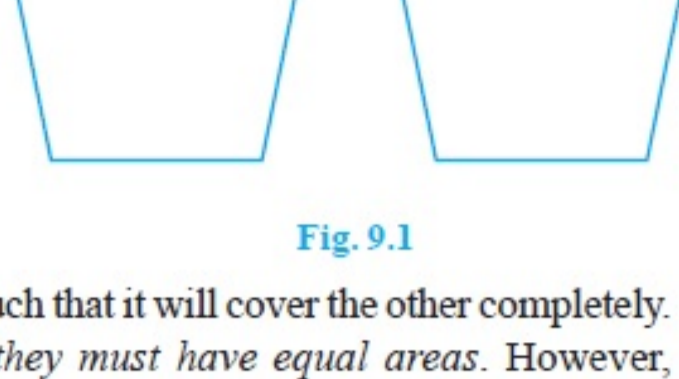


Fig. 9.1

you can superpose one figure over the other such that it will cover the other completely. So if two figures A and B are congruent, they must have equal areas. However, the converse of this statement is not true. In other words, two figures having equal areas need not be congruent. For example, in Fig. 9.2, rectangles ABCD and EFGH have equal areas ($9 \times 4 \text{ cm}^2$ and $6 \times 6 \text{ cm}^2$) but clearly they are not congruent. (Why?)

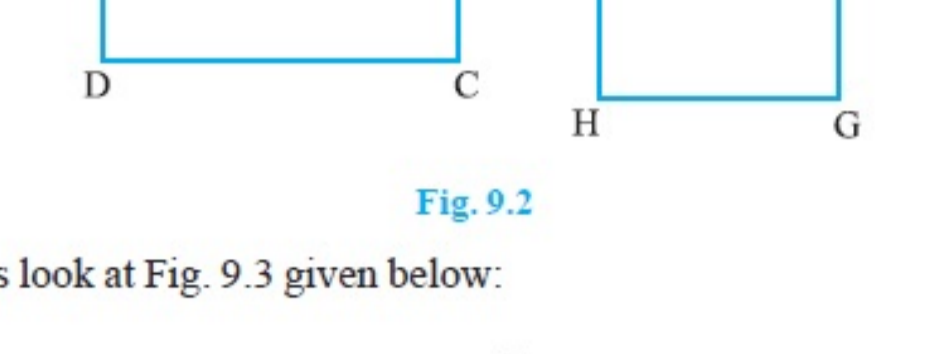


Fig. 9.2

Now let us look at Fig. 9.3 given below:

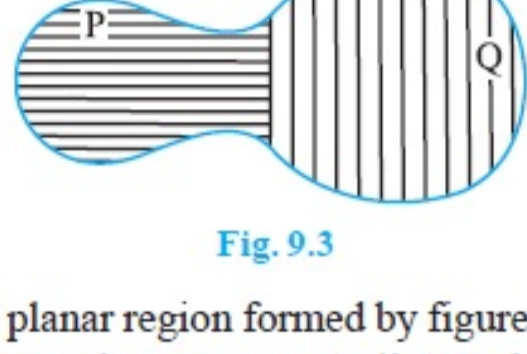


Fig. 9.3

You may observe that planar region formed by figure T is made up of two planar regions formed by figures P and Q. You can easily see that

$$\text{Area of figure T} = \text{Area of figure P} + \text{Area of figure Q.}$$

You may denote the area of figure A as $\text{ar}(A)$, area of figure B as $\text{ar}(B)$, area of figure T as $\text{ar}(T)$, and so on. Now you can say that *area of a figure is a number (in some unit) associated with the part of the plane enclosed by the figure with the following two properties:*

- (1) If A and B are two congruent figures, then $\text{ar}(A) = \text{ar}(B)$;
- (2) if a planar region formed by a figure T is made up of two non-overlapping planar regions formed by figures P and Q, then $\text{ar}(T) = \text{ar}(P) + \text{ar}(Q)$.

You are also aware of some formulae for finding the areas of different figures such as rectangle, square, parallelogram, triangle etc., from your earlier classes. In this chapter, attempt shall be made to consolidate the knowledge about these formulae by studying some relationship between the areas of these geometric figures under the condition when they lie on the same base and between the same parallels. This study will also be useful in the understanding of some results on 'similarity of triangles'.

9.2 Figures on the Same Base and Between the Same Parallels

Look at the following figures:

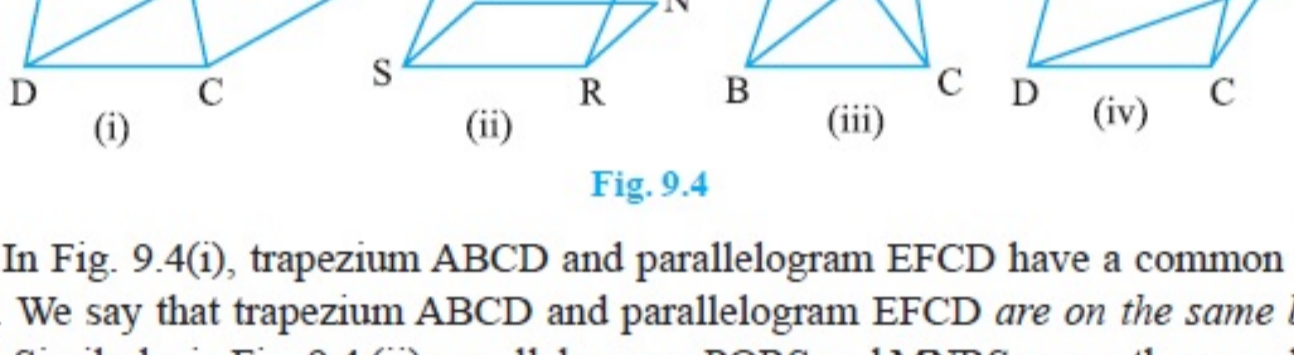


Fig. 9.4

In Fig. 9.4(i), trapezium ABCD and parallelogram EFCD have a common side DC. We say that trapezium ABCD and parallelogram EFCD are on the same base DC. Similarly, in Fig. 9.4(ii), parallelograms PQRS and MNRS are on the same base SR; in Fig. 9.4(iii), triangles ABC and DBC are on the same base BC and in Fig. 9.4(iv), parallelogram ABCD and triangle PDC are on the same base DC.

Now look at the following figures:

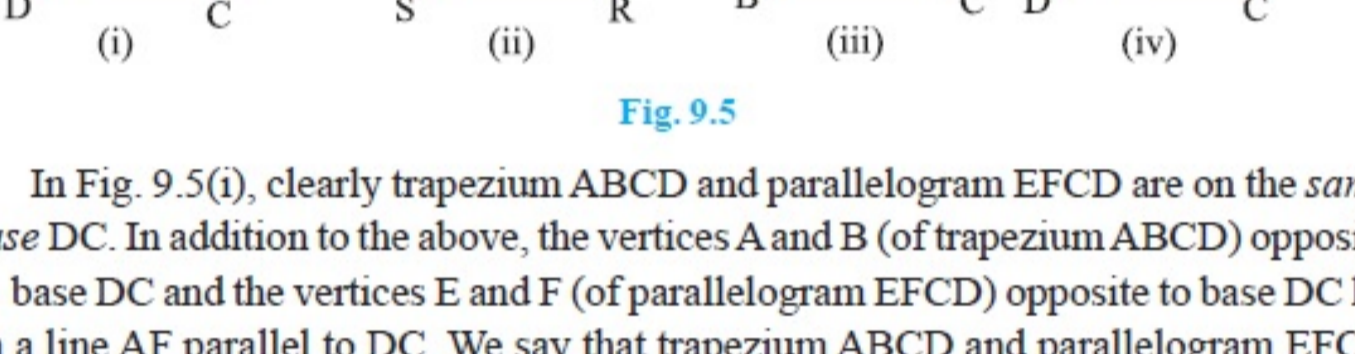


Fig. 9.5

In Fig. 9.5(i), clearly trapezium ABCD and parallelogram EFCD are on the same base DC. In addition to the above, the vertices A and B (of trapezium ABCD) opposite to base DC and the vertices E and F (of parallelogram EFCD) opposite to base DC lie on a line AF parallel to DC. We say that trapezium ABCD and parallelogram EFCD are on the same base DC and between the same parallels AF and DC. Similarly, parallelograms PQRS and MNRS are on the same base SR and between the same parallels PN and SR [see Fig. 9.5(ii)] as vertices P and Q of PQRS and vertices M and N of MNRS lie on a line PN parallel to base SR. In the same way, triangles ABC and DBC lie on the same base BC and between the same parallels AD and BC [see Fig. 9.5(iii)] and parallelogram ABCD and triangle PCD lie on the same base DC and between the same parallels AP and DC [see Fig. 9.5(iv)].

So, two figures are said to be on the same base and between the same parallels, if they have a common base (side) and the vertices (or the vertex) opposite to the common base of each figure lie on a line parallel to the base.

Keeping in view the above statement, you cannot say that ΔPQR and ΔDQR of Fig. 9.6(i) lie between the same parallels l and QR. Similarly, you cannot say that

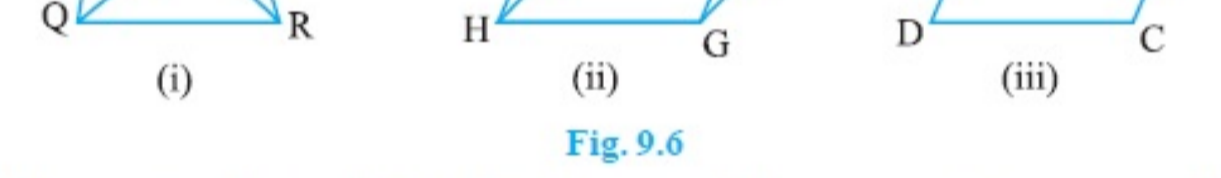


Fig. 9.6

parallelograms EFGH and MNGH of Fig. 9.6(ii) lie between the same parallels EF and HG and that parallelograms ABCD and EFCD of Fig. 9.6(iii) lie between the same parallels AB and DC (even though they have a common base DC and lie between the parallels AD and BC). So, it should clearly be noted that out of the two parallels, one must be the line containing the common base. Note that ΔABC and ΔDBE of Fig. 9.7(i) are not on the common base. Similarly, ΔABC and parallelogram PQRS of Fig. 9.7(ii) are also not on the same base.

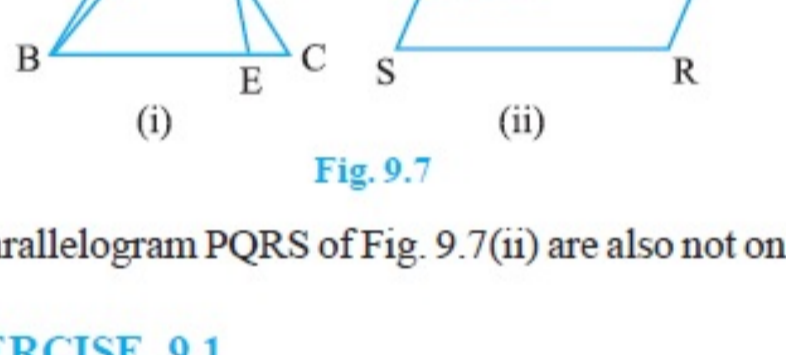


Fig. 9.7

EXERCISE 9.1

- Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.

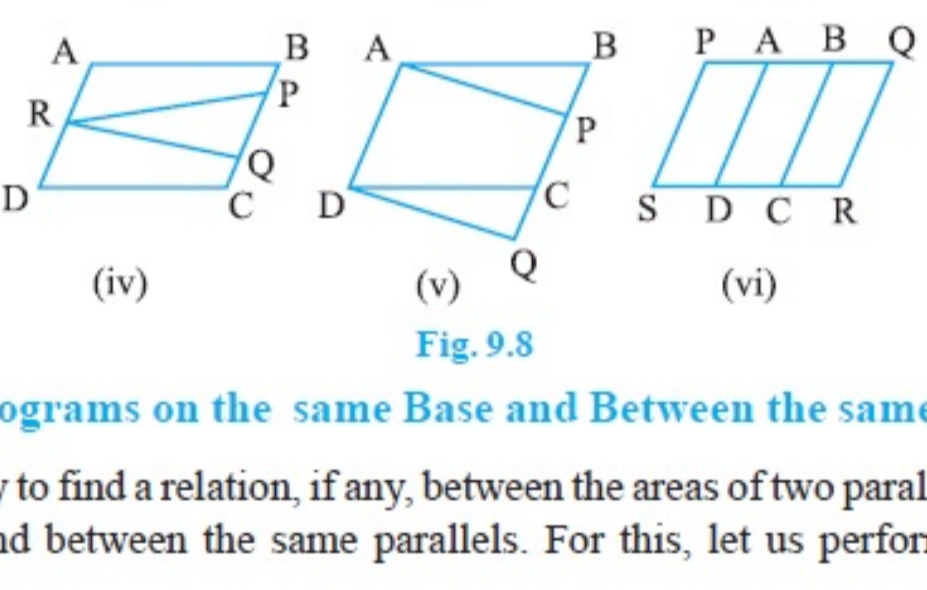


Fig. 9.8

9.3 Parallelograms on the same Base and Between the same Parallels

Now let us try to find a relation, if any, between the areas of two parallelograms on the same base and between the same parallels. For this, let us perform the following activities:

Activity 1 : Let us take a graph sheet and draw two parallelograms ABCD and PQCD on it as shown in Fig. 9.9.

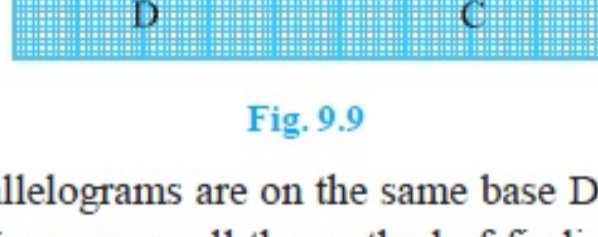


Fig. 9.9

The above two parallelograms are on the same base DC and between the same parallels PB and DC. You may recall the method of finding the areas of these two parallelograms by counting the squares.

In this method, the area is found by counting the number of complete squares enclosed by the figure, the number of squares having more than half their parts enclosed by the figure and the number of squares having half their parts enclosed by the figure. The squares whose less than half parts are enclosed by the figure are ignored. You will find that areas of both the parallelograms are (approximately) 15 cm^2 . Repeat this activity* by drawing some more pairs of parallelograms on the graph sheet. What do you observe? Are the areas of the two parallelograms different or equal? If fact, they are equal. So, this may lead you to conclude that *parallelograms on the same base and between the same parallels are equal in area.* However, remember that this is just a verification.

Activity 2 : Draw a parallelogram ABCD on a thick sheet of paper or on a cardboard sheet. Now, draw a line-segment DE as shown in Fig. 9.10.

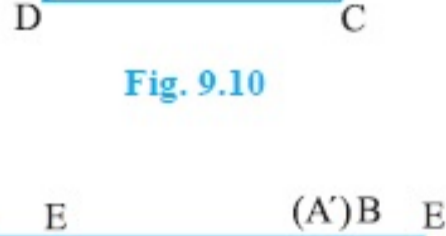


Fig. 9.10

*This activity can also be performed by using a Geoboard.

Next, cut a triangle $A'D'E'$ congruent to triangle ADE on a separate sheet with the help of a tracing paper and place $\Delta A'D'E'$ in such a way that $A'D'$ coincides with BC as shown in Fig 9.11. Note that there are two parallelograms ABCD and $EE'CD$ on the same base DC and between the same parallels AE' and DC. What can you say about their areas?

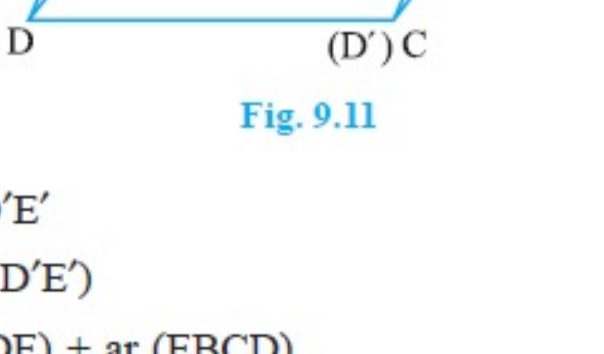


Fig. 9.11

As $\Delta ADE \cong \Delta A'D'E'$
Therefore $\text{ar}(ADE) = \text{ar}(A'D'E')$
Also $\text{ar}(ABCD) = \text{ar}(ADE) + \text{ar}(EBCD)$
 $= \text{ar}(A'D'E') + \text{ar}(EBCD)$
 $= \text{ar}(EE'CD)$

So, the two parallelograms are equal in area.

Let us now try to prove this relation between the two such parallelograms.

Theorem 9.1 : Parallelograms on the same base and between the same parallels are equal in area.

Proof : Two parallelograms ABCD and EFCD, on the same base DC and between the same parallels AF and DC are given (see Fig. 9.12).

We need to prove that $\text{ar}(ABCD) = \text{ar}(EFCD)$.

In ΔADE and ΔBCF ,

$$\angle DAE = \angle CBF \text{ (Corresponding angles from } AD \parallel BC \text{ and transversal AF)} \quad (1)$$

$$\angle AED = \angle BFC \text{ (Corresponding angles from } ED \parallel FC \text{ and transversal AF)} \quad (2)$$

$$\text{Therefore, } \angle ADE = \angle BCF \text{ (Angle sum property of a triangle)} \quad (3)$$

$$\text{Also, } AD = BC \text{ (Opposite sides of the parallelogram ABCD)} \quad (4)$$

$$\text{So, } \Delta ADE \cong \Delta BCF \quad [\text{By ASA rule, using (1), (3), and (4)}]$$

$$\text{Therefore, } \text{ar}(ADE) = \text{ar}(BCF) \text{ (Congruent figures have equal areas)} \quad (5)$$

$$\text{Now, } \text{ar}(ABCD) = \text{ar}(ADE) + \text{ar}(EDCB) = \text{ar}(BCF) + \text{ar}(EDCB) = \text{ar}(EFCD) \quad [\text{From(5)}]$$

So, parallelograms ABCD and EFCD are equal in area. ■

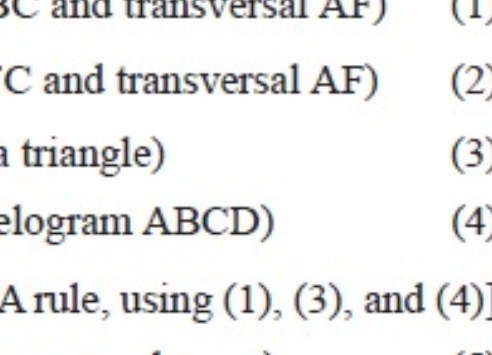


Fig. 9.12

Let us now take some examples to illustrate the use of the above theorem.

Example 1 : In Fig. 9.13, ABCD is a parallelogram and EFCD is a rectangle.

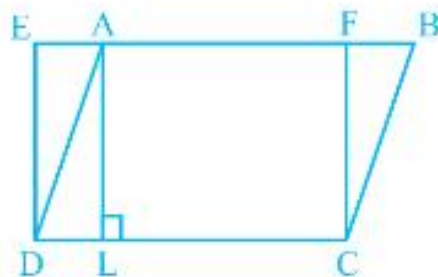


Fig. 9.13

Also, $AL \perp DC$. Prove that

- (i) $\text{ar}(\text{ABCD}) = \text{ar}(\text{EFCD})$
- (ii) $\text{ar}(\text{ABCD}) = DC \times AL$

Solution : (i) As a rectangle is also a parallelogram,

therefore, $\text{ar}(\text{ABCD}) = \text{ar}(\text{EFCD})$ (Theorem 9.1)

(ii) From above result,

$$\text{ar}(\text{ABCD}) = DC \times FC \text{ (Area of the rectangle = length} \times \text{breadth)} \quad (1)$$

As $AL \perp DC$, therefore, AFCL is also a rectangle

$$\text{So, } AL = FC \quad (2)$$

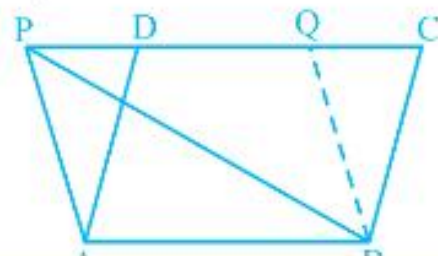
Therefore, $\text{ar}(\text{ABCD}) = DC \times AL$. [From (1) and (2)]

Can you see from the Result (ii) above that *area of a parallelogram is the product of its any side and the corresponding altitude*. Do you remember that you have studied this formula for area of a parallelogram in Class VII. On the basis of this formula, Theorem 9.1 can be rewritten as *parallelograms on the same base or equal bases and between the same parallels are equal in area*.

Can you write the converse of the above statement? It is as follows: *Parallelograms on the same base (or equal bases) and having equal areas lie between the same parallels*. Is the converse true? Prove the converse using the formula for area of the parallelogram.

Example 2 : If a triangle and a parallelogram are on the same base and between the same parallels, then prove that the area of the triangle is equal to half the area of the parallelogram.

Solution : Let ΔABP and parallelogram ABCD be on the same base AB and between the same parallels AB and PC (see Fig. 9.14).



[**Remark:** Note that by taking $BD = DE = EC$, the triangle ABC is divided into three triangles ABD , ADE and AEC of equal areas. In the same way, by dividing BC into n equal parts and joining the points of division so obtained to the opposite vertex of BC , you can divide $\triangle ABC$ into n triangles of equal areas.]

3. In Fig. 9.31, $ABCD$, $DCFE$ and $ABFE$ are parallelograms. Show that $\text{ar}(\triangle ADE) = \text{ar}(\triangle BCF)$.

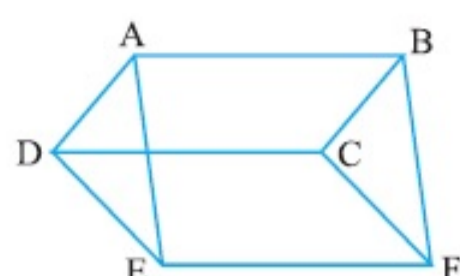


Fig. 9.31

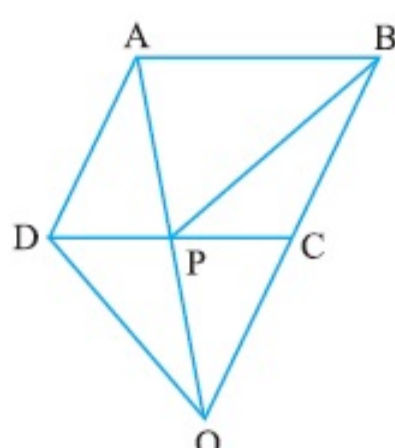


Fig. 9.32

5. In Fig. 9.33, ABC and BDE are two equilateral triangles such that D is the mid-point of BC . If AE intersects BC at F , show that

(i) $\text{ar}(\triangle BDE) = \frac{1}{4} \text{ar}(\triangle ABC)$

(ii) $\text{ar}(\triangle BDE) = \frac{1}{2} \text{ar}(\triangle BAE)$

(iii) $\text{ar}(\triangle ABC) = 2 \text{ar}(\triangle BEC)$

(iv) $\text{ar}(\triangle BFE) = \text{ar}(\triangle AFD)$

(v) $\text{ar}(\triangle BFE) = 2 \text{ar}(\triangle FED)$

(vi) $\text{ar}(\triangle FED) = \frac{1}{8} \text{ar}(\triangle AFC)$

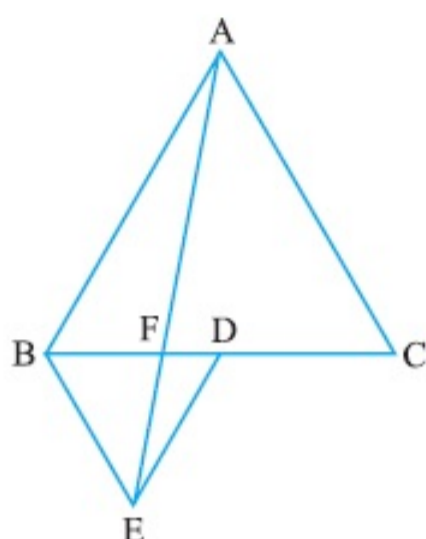


Fig. 9.33

[**Hint:** Join EC and AD . Show that $BE \parallel AC$ and $DE \parallel AB$, etc.]

6. Diagonals AC and BD of a quadrilateral $ABCD$ intersect each other at P . Show that $\text{ar}(\triangle APB) \times \text{ar}(\triangle CPD) = \text{ar}(\triangle APD) \times \text{ar}(\triangle BPC)$.

[**Hint:** From A and C , draw perpendiculars to BD .]

7. P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP , show that

(i) $\text{ar}(\triangle PRQ) = \frac{1}{2} \text{ar}(\triangle ARC)$

(ii) $\text{ar}(\triangle RQC) = \frac{3}{8} \text{ar}(\triangle ABC)$

(iii) $\text{ar}(\triangle PBQ) = \text{ar}(\triangle ARC)$

8. In Fig. 9.34, ABC is a right triangle right angled at A . $BCED$, $ACFG$ and $ABMN$ are squares on the sides BC , CA and AB respectively. Line segment $AX \perp DE$ meets BC at Y . Show that:

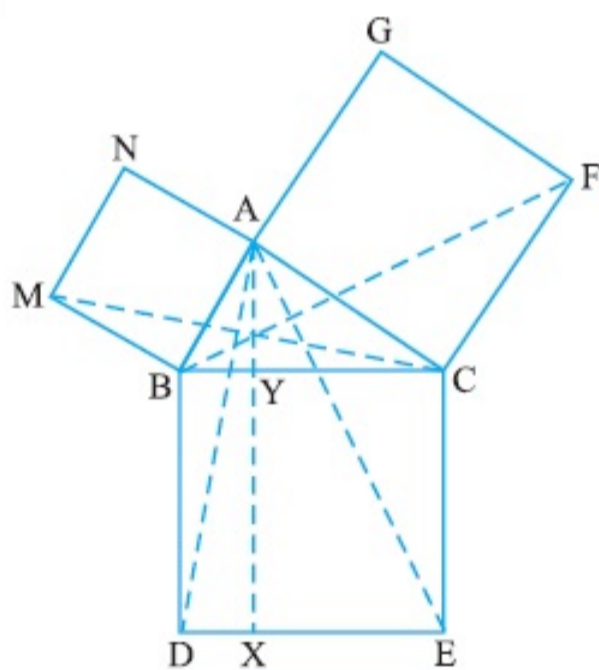


Fig. 9.34

(i) $\triangle MBC \cong \triangle ABD$

(ii) $\text{ar}(\triangle BYXD) = 2 \text{ar}(\triangle MBC)$

(iii) $\text{ar}(\triangle BYXD) = \text{ar}(\triangle ABMN)$

(iv) $\triangle FCB \cong \triangle ACE$

(v) $\text{ar}(\triangle CYXE) = 2 \text{ar}(\triangle FCB)$

(vi) $\text{ar}(\triangle CYXE) = \text{ar}(\triangle ACFG)$

(vii) $\text{ar}(\triangle BCED) = \text{ar}(\triangle ABMN) + \text{ar}(\triangle ACFG)$

Note: Result (vii) is the famous *Theorem of Pythagoras*. You shall learn a simpler proof of this theorem in Class X.

9.5 Summary

In this chapter, you have studied the following points :

1. Area of a figure is a number (in some unit) associated with the part of the plane enclosed by that figure.
2. Two congruent figures have equal areas but the converse need not be true.
3. If a planar region formed by a figure T is made up of two non-overlapping planar regions formed by figures P and Q , then $\text{ar}(T) = \text{ar}(P) + \text{ar}(Q)$, where $\text{ar}(X)$ denotes the area of figure X .
4. Two figures are said to be on the same base and between the same parallels, if they have a common base (side) and the vertices, (or the vertex) opposite to the common base of each figure lie on a line parallel to the base.
5. Parallelograms on the same base (or equal bases) and between the same parallels are equal in area.
6. Area of a parallelogram is the product of its base and the corresponding altitude.
7. Parallelograms on the same base (or equal bases) and having equal areas lie between the same parallels.
8. If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.
9. Triangles on the same base (or equal bases) and between the same parallels are equal in area.
10. Area of a triangle is half the product of its base and the corresponding altitude.
11. Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.
12. A median of a triangle divides it into two triangles of equal areas.