National Council of Educational Research and Training (NCERT)

Book for Class IX

Subject: Maths

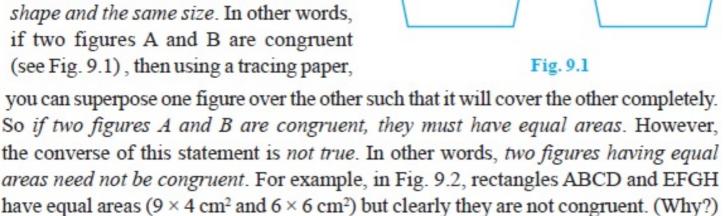
Chapter 9 - Areas of Parallelograms and Triangles

In Chapter 5, you have seen that the study of Geometry, originated with the measurement of earth (lands) in the process of recasting boundaries of the fields and dividing them into appropriate parts. For example, a farmer Budhia had a triangular field and she wanted to divide it equally among her two daughters and one son. Without actually calculating the area of the field, she just divided one side of the triangular field into three equal parts and joined the two points of division to the opposite vertex. In this way, the field was divided into three parts and she gave one part to each of her children. Do you think that all the three parts so obtained by her were, in fact, equal in area? To get answers to this type of questions and other related problems, there is a need to have a relook at areas of plane figures, which you have already studied in earlier classes. You may recall that the part of the plane enclosed by a simple closed figure is

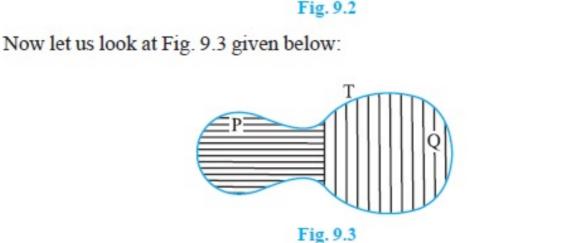
planar region is called its area. This magnitude or measure is always expressed with the help of a number (in some unit) such as 5 cm2, 8 m2, 3 hectares etc. So, we can say that area of a figure is a number (in some unit) associated with the part of the plane enclosed by the figure. We are also familiar with the concept В of congruent figures from earlier classes and from Chapter 7. Two figures are

called a planar region corresponding to that figure. The magnitude or measure of this

called congruent, if they have the same shape and the same size. In other words, if two figures A and B are congruent (see Fig. 9.1), then using a tracing paper,



6 cm E 9 cm В 6 cm 4 cm D H G



You may observe that planar region formed by figure T is made up of two planar

You may denote the area of figure A as ar(A), area of figure B as ar(B), area of

regions formed by figures P and Q. You can easily see that

the following two properties: If A and B are two congruent figures, then ar(A) = ar(B); and (2) if a planar region formed by a figure T is made up of two non-overlapping

Area of figure T = Area of figure P + Area of figure Q.

figure T as ar(T), and so on. Now you can say that area of a figure is a number

(in some unit) associated with the part of the plane enclosed by the figure with

You are also aware of some formulae for finding the areas of different figures such as rectangle, square, parallelogram, triangle etc., from your earlier classes. In

this chapter, attempt shall be made to consolidate the knowledge about these formulae by studying some relationship between the areas of these geometric figures under the

planar regions formed by figures P and Q, then ar(T) = ar(P) + ar(Q).

condition when they lie on the same base and between the same parallels. This study will also be useful in the understanding of some results on 'similarity of triangles'. 9.2 Figures on the Same Base and Between the Same Parallels Look at the following figures:

(iv) (iii)

Fig. 9.4

DC. We say that trapezium ABCD and parallelogram EFCD are on the same base

DC. Similarly, in Fig. 9.4 (ii), parallelograms PQRS and MNRS are on the same base

In Fig. 9.4(i), trapezium ABCD and parallelogram EFCD have a common side

SR; in Fig. 9.4(iii), triangles ABC and DBC are on the same base BC and in Fig. 9.4(iv), parallelogram ABCD and triangle PDC are on the same base DC.

M

Now look at the following figures:

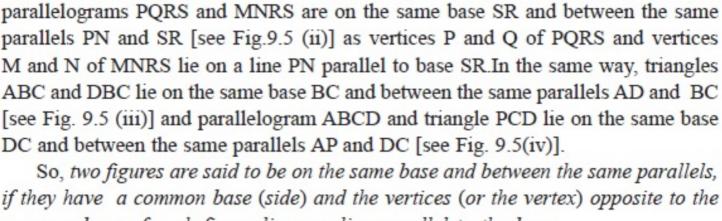
ΒЕ

(ii)

(i)

В R C (i) (ii) (iii) (iv) Fig. 9.5 In Fig. 9.5(i), clearly trapezium ABCD and parallelogram EFCD are on the same base DC. In addition to the above, the vertices A and B (of trapezium ABCD) opposite to base DC and the vertices E and F (of parallelogram EFCD) opposite to base DC lie on a line AF parallel to DC. We say that trapezium ABCD and parallelogram EFCD

are on the same base DC and between the same parallels AF and DC. Similarly,



(iii) (i) (ii) Fig. 9.6 parallelograms EFGH and MNGH of Fig. 9.6(ii) lie between the same parallels EF and HG and that parallelograms ABCD and EFCD of Fig. 9.6(iii) lie between the

EXERCISE 9.1

(ii)

In such a case, write the common base and the two parallels.

Which of the following figures lie on the same base and between the same parallels.

D (v) (iv) Fig. 9.8 9.3 Parallelograms on the same Base and Between the same Parallels Now let us try to find a relation, if any, between the areas of two parallelograms on the activities:

parallelograms by counting the squares.

(i)

(iii)

A

(vi)

R

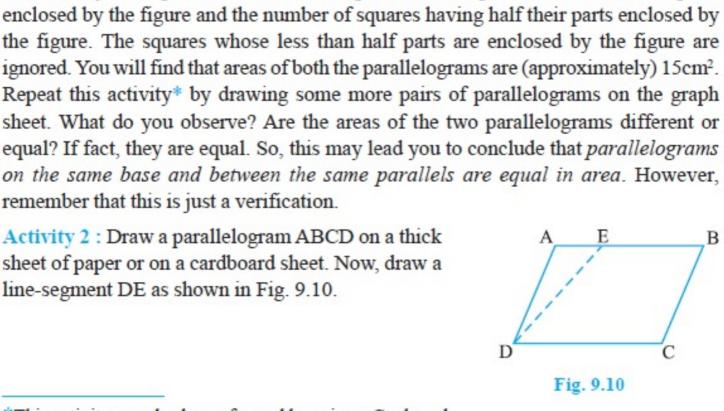


Fig. 9.9

parallels PB and DC. You may recall the method of finding the areas of these two

enclosed by the figure, the number of squares a having more than half their parts

The above two parallelograms are on the same base DC and between the same

In this method, the area is found by counting the number of complete squares

 Δ ADE \cong Δ A'D'E' ar(ADE) = ar(A'D'E')Therefore Also ar (ABCD) = ar (ADE) + ar (EBCD)= ar (A'D'E') + ar (EBCD)= ar (EE'CD)Let us now try to prove this relation between the two such parallelograms. **Theorem 9.1**: Parallelograms on the same base and between the same parallels

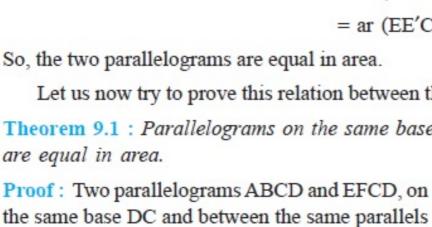
We need to prove that ar (ABCD) = ar (EFCD). In \triangle ADE and \triangle BCF,

∠DAE = ∠CBF (Corresponding angles from AD || BC and transversal AF) \angle AED = \angle BFC (Corresponding angles from ED || FC and transversal AF)

So, parallelograms ABCD and EFCD are equal in area.

DC and between the same parallels AP and DC [see Fig. 9.5(iv)]. common base of each figure lie on a line parallel to the base. Keeping in view the above statement, you cannot say that Δ PQR and Δ DQR of Fig. 9.6(i) lie between the same parallels l and QR. Similarly, you cannot say that Ε D M G same parallels AB and DC (even though they have a common base DC m and lie between the parallels AD and BC). So, it should clearly be noted that out of the two parallels, one must be the line containing the (ii) (i) common base. Note that ΔABC and Fig. 9.7 ΔDBE of Fig. 9.7(i) are not on the common base. Similarly, AABC and parallelogram PQRS of Fig. 9.7(ii) are also not on the same base.

same base and between the same parallels. For this, let us perform the following Activity 1: Let us take a graph sheet and draw two parallelograms ABCD and PQCD on it as shown in Fig. 9.9.



AF and DC are given (see Fig.9.12).

Therefore, $\angle ADE = \angle BCF$ (Angle sum property of a triangle)

= ar (EFCD)

remember that this is just a verification. Activity 2: Draw a parallelogram ABCD on a thick sheet of paper or on a cardboard sheet. Now, draw a line-segment DE as shown in Fig. 9.10. *This activity can also be performed by using a Geoboard. Next, cut a triangle A' D' E' congruent to triangle ADE on a separate sheet with the help of a tracing paper and place Δ A'D'E' in such a way that A'D' coincides with BC as shown in Fig 9.11. Note that there are two parallelograms ABCD and EE'CD on the same base DC and between the same parallels AE' and DC. What can you say about their areas? As

Fig. 9.12 [By ASA rule, using (1), (3), and (4)]

(1)

(2)

(3)

(4)

(5)

[From(5)]

В

(A')B

(D') C

Fig. 9.11

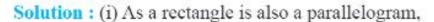
AD = BC (Opposite sides of the parallelogram ABCD) Also, \triangle ADE \cong \triangle BCF So. Therefore, ar (ADE) = ar (BCF) (Congruent figures have equal areas) ar(ABCD) = ar(ADE) + ar(EDCB)Now, = ar (BCF) + ar (EDCB)

Let us now take some examples to illustrate the use of the above theorem.

Example 1: In Fig. 9.13, ABCD is a parallelogram and EFCD is a rectangle.

Also, AL \(\perp\) DC. Prove that

- (i) ar (ABCD) = ar (EFCD)
- (ii) ar $(ABCD) DC \times AL$



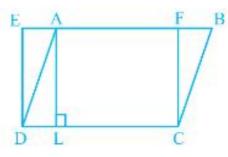


Fig. 9.13

ar (ABCD) = ar (EFCD)(ii) From above result,

therefore.

ar (ABCD) =
$$DC \times FC$$
 (Area of the rectangle = length × breadth) (1)

(Theorem 9.1)

AL \(\perp \) DC, therefore, AFCL is also a rectangle As

So,
$$AL - FC$$
 (2)

Therefore, ar $(ABCD) = DC \times AL$

[From (1) and (2)]

Can you see from the Result (ii) above that area of a parallelogram is the product of its any side and the coresponding altitude. Do you remember that you have studied this formula for area of a parallelogram in Class VII. On the basis of this formula, Theorem 9.1 can be rewritten as parallelograms on the same base or equal bases and between the same parallels are equal in area.

Can you write the converse of the above statement? It is as follows: Parallelograms on the same base (or equal bases) and having equal areas lie between the same parallels. Is the converse true? Prove the converse using the formula for area of the parallelogram.

Example 2: If a triangle and a parallelogram are on the same base and between the same parallels, then prove that the area of the triangle is equal to half the area of the parallelogram.

Solution : Let \triangle ABP and parallelogram ABCD be on the same base AB and between the same parallels AB and PC (see Fig. 9.14).

But Δ PAB ≅ Δ BQP (Diagonal PB divides parallelogram ABQP into two congruent triangles.)

ar(PAB) = ar(BQP)So,

Therefore,
$$\operatorname{ar}(PAB) = \frac{1}{2} \operatorname{ar}(ABQP)$$
 [From (2)]

This gives
$$\operatorname{ar}(PAB) = \frac{1}{2}\operatorname{ar}(ABCD)$$
 [From (1) and (3)

and CF \(\perp \) AD. If AB = 16 cm, AE = 8 cm and 2. If E,F,G and H are respectively the mid-points of

the sides of a parallelogram ABCD, show that $ar(EFGH) = \frac{1}{2} ar(ABCD)$.

Let us look at Fig. 9.18. In it, you have two triangles

between the same parallels on the graph sheet and

find their areas by the method of counting the

To obtain a logical answer to the above question,

From this, you obtain two parallelograms PBCR.

and ABCD on the same base BC and between the

the two areas are (approximately) equal.

you may proceed as follows:

same parallels BC and AR.

Therefore,

Therefore,

Now

So.

So,

CF = 10 cm, find AD.

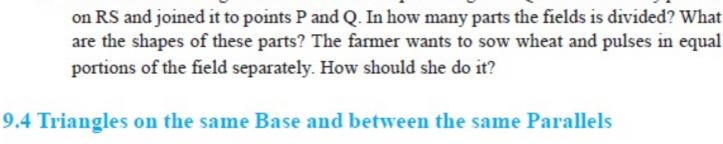
In Fig. 9.15, ABCD is a parallelogram, AE ⊥ DC

P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar(APB) = ar(BQC).

In Fig. 9.16, P is a point in the interior of a parallelogram ABCD. Show that

(i)
$$ar(APB) + ar(PCD) = \frac{1}{2}ar(ABCD)$$

- (ii) ar(APD) + ar(PBC) = ar(APB) + ar(PCD)Fig. 9.16
 - [Hint: Through P, draw a line parallel to AB.]
 - In Fig. 9.17, PQRS and ABRS are parallelograms Q В Α and X is any point on side BR. Show that
- (i) ar(PQRS) = ar(ABRS)(ii) ar (AX S) = $\frac{1}{2}$ ar (PQRS) R
- Fig. 9.17 A farmer was having a field in the form of a parallelogram PQRS. She took any point A



ABC and PBC on the same base BC and between the same parallels BC and AP. What can you say about the areas of such triangles? To answer this question, you may perform the activity of drawing several pairs of triangles on the same base and

squares. Each time, you will find that the areas of the two triangles are (approximately)

equal. This activity can be performed using a geoboard also. You will again find that

In Fig. 9.18, draw CD || BA and CR || BP such that D and R lie on line AP(see Fig.9.19).

Theorem 9.2: Two triangles on the same base (or equal bases) and between the same parallels are equal in area. Now, suppose ABCD is a parallelogram whose one of the diagonals is AC

In this way, you have arrived at the following theorem:

(see Fig. 9.20). Let AN⊥ DC. Note that

corresponding altitudes.

You wish to show that

quadrilateral ABCD.

areas.

areas lie between the same parallels.

 \triangle ADC \cong \triangle CBA

ar(ADC) = ar(CBA)

ar(ABC) = ar(PBC)

Therefore, ar $(ADC) = \frac{1}{2}$ ar (ABCD) $= \frac{1}{2} (DC \times AN) \qquad (Why?)$ Fig. 9.20

In other words, area of a triangle is half the product of its base (or any side) and

the corresponding altitude (or height). Do you remember that you have learnt this formula for area of a triangle in Class VII ? From this formula, you can see that two

triangles with same base (or equal bases) and equal areas will have equal

So, area of \triangle ADC = $\frac{1}{2}$ × base DC × corresponding altitude AN

(Why?)

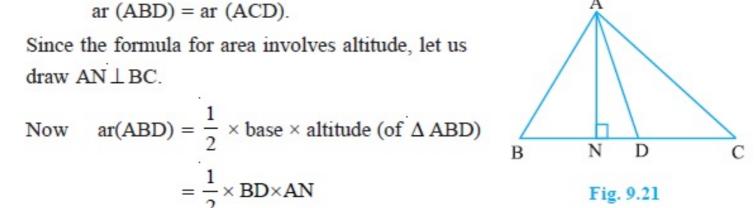
(Why?)

For having equal corresponding altitudes, the triangles must lie between the same parallels. From this, you arrive at the following converse of Theorem 9.2. Theorem 9.3: Two triangles having the same base (or equal bases) and equal

Example 3: Show that a median of a triangle divides it into two triangles of equal

Solution: Let ABC be a triangle and let AD be one of its medians (see Fig. 9.21).

Let us now take some examples to illustrate the use of the above results.



 $= \frac{1}{2} \times CD \times AN \qquad (As BD = CD)$

 $=\frac{1}{2} \times \text{base} \times \text{altitude (of } \Delta \text{ ACD)}$

= ar(ACD)

Example 4: In Fig. 9.22, ABCD is a quadrilateral

and BE | AC and also BE meets DC produced at E.

Show that area of Δ ADE is equal to the area of the

 Δ BAC and Δ EAC lie on the same base AC and

 \triangle ABC. Show that ar (ABE) = ar (ACE).

AD. Show that ar (BED) = $\frac{1}{4}$ ar (ABC).

it into four triangles of equal area.

(iii) ar (BDEF) = $\frac{1}{2}$ ar (ABC)

If AB = CD, then show that:

(i) ar (DOC) = ar (AOB)

(ii) ar(DCB) = ar(ACB)

8.

9.

6. In Fig. 9.25, diagonals AC and BD of quadrilateral

(iii) DA || CB or ABCD is a parallelogram.

ar(DBC) = ar(EBC). Prove that $DE \parallel BC$.

The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram

PBQR is completed (see Fig. 9.26). Show that

and F respectively, show that

ar(ABCD) = ar(PBQR).

at Y. Prove that ar (ADX) = ar(ACY).

ABCD intersect at O in such a way that ar (AOD) = ar (BOC). Prove that ABCD is a

ar(BDP) = ar(ARC). Show that both the quadrilaterals ABCD and DCPR are

16. In Fig. 9.29, ar (DRC) = ar (DPC) and

[Hint: Join CX.]

trapezium.

trapeziums.

ABCD intersect at O such that OB = OD.

[Hint: From D and B, draw perpendiculars to AC.]

In a triangle ABC, E is the mid-point of median

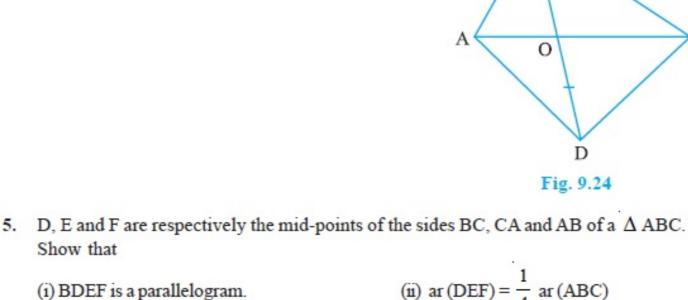
Show that the diagonals of a parallelogram divide

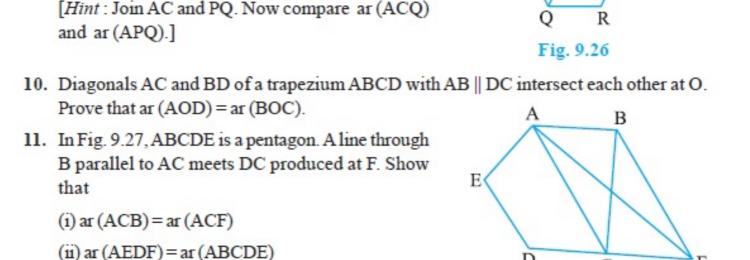
In Fig. 9.24, ABC and ABD are two triangles on the same base AB. If line- segment CD is bisected

by AB at O, show that ar(ABC) = ar(ABD).

Solution: Observe the figure carefully.

between the same parallels AC and BE.





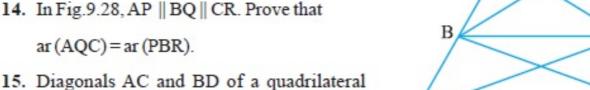
12. A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat

of the village decided to take over some portion of his plot from one of the corners to

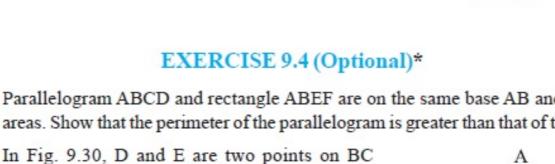
D and E are points on sides AB and AC respectively of ABC such that

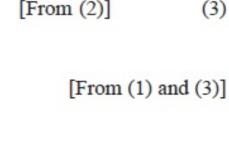
XY is a line parallel to side BC of a triangle ABC. If BE | AC and CF | AB meet XY at E

ar(ABE) = ar(ACF)



- Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.
 - ar(ABD) = ar(ADE) = ar(AEC). Can you now answer the question that you have left in the 'Introduction' of this chapter, whether the field of Budhia has been actually divided into three parts of equal area?

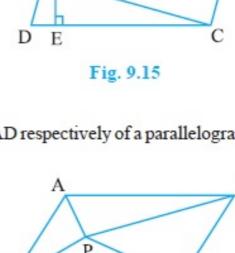


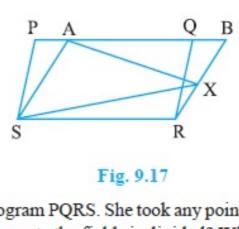


(2)

В

Fig. 9.30





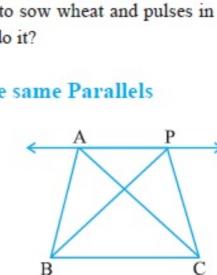
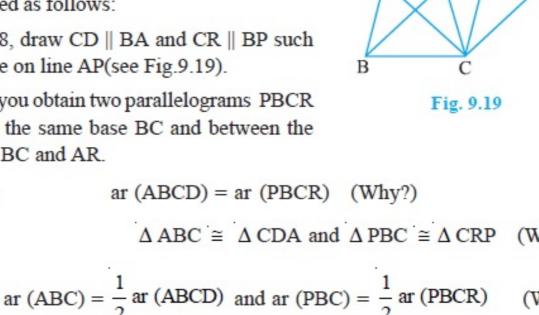
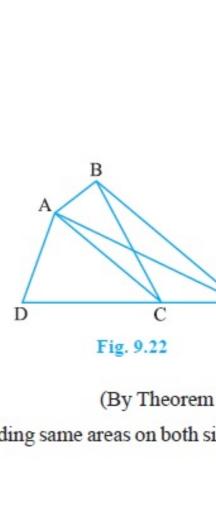


Fig. 9.18





Е

D

Fig. 9.23

В

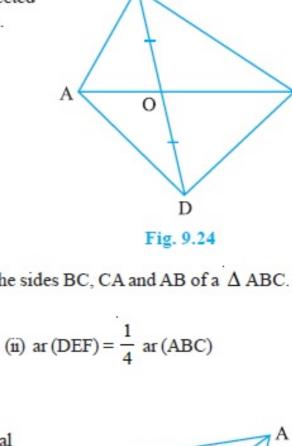


Fig. 9.25

В

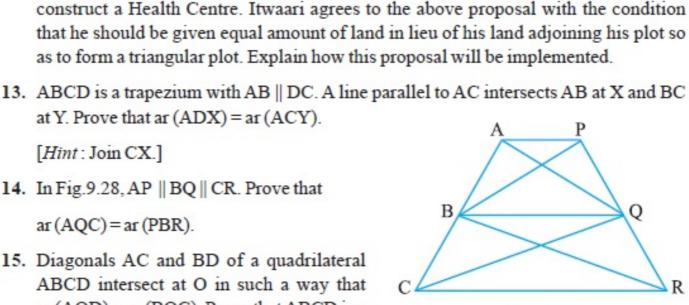


Fig. 9.28

В

D

Fig. 9.27

Fig. 9.29

such that BD = DE = EC. Show that

В

[Remark: Note that by taking BD = DE = EC, the triangle ABC is divided into three triangles ABD, ADE and AEC of equal areas. In the same way, by dividing BC into n equal parts and joining the points of division so obtained to the opposite vertex of BC, you can divide \triangle ABC into n triangles of equal areas.]

- 3. In Fig. 9.31, ABCD, DCFE and ABFE are parallelograms. Show that ar(ADE) = ar(BCF).
- 4. In Fig. 9.32, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersect DC at P, show that AP are AP are AP and AP are AP and AP are AP and AP are AP are AP and AP are AP are AP and AP are AP are AP and AP are AP

[Hint : Join AC.]

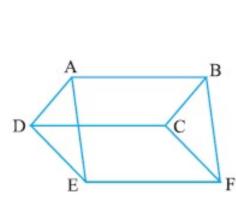


Fig. 9.31

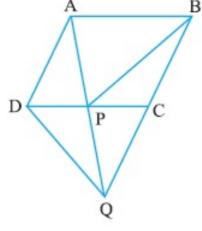
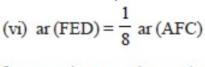
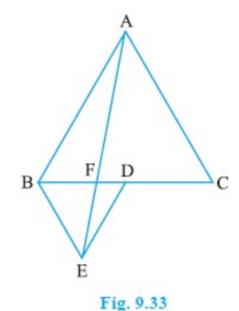


Fig. 9.32

- In Fig.9.33, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, show that
 - (i) $ar(BDE) = \frac{1}{4} ar(ABC)$
 - (ii) $ar(BDE) = \frac{1}{2} ar(BAE)$
 - (iii) ar (ABC) = 2 ar (BEC)
 - (iv) ar (BFE) = ar (AFD) (v) ar (BFE) = 2 ar (FED)





[Hint: Join EC and AD. Show that BE | AC and DE | AB, etc.]

Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that $ar(APB) \times ar(CPD) = ar(APD) \times ar(BPC)$.

[Hint: From A and C, draw perpendiculars to BD.] 7. P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R

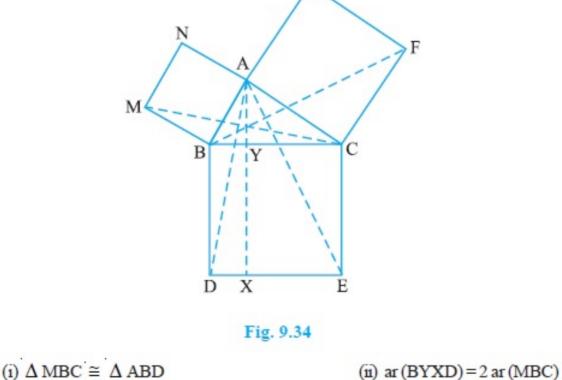
is the mid-point of AP, show that

(i) ar (PRQ) =
$$\frac{1}{2}$$
 ar (ARC)
(iii) ar (PBQ) = ar (ARC)

(ii)
$$ar(RQC) = \frac{3}{8} ar(ABC)$$

8. In Fig. 9.34, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are

squares on the sides BC, CA and AB respectively. Line segment AX \(\pextstyle DE\) meets BC at Y. Show that:



- (iii) ar (BYXD) = ar (ABMN)(v) ar (CYXE) = 2 ar (FCB)
- (vi) ar (CYXE) = ar (ACFG)

(iv) Δ FCB \cong Δ ACE

- (vii) ar (BCED) = ar (ABMN) + ar (ACFG)
- Note: Result (vii) is the famous Theorem of Pythagoras. You shall learn a simpler
- proof of this theorem in Class X.

9.5 Summary In this chapter, you have studied the following points:

Area of a figure is a number (in some unit) associated with the part of the plane enclosed by that figure.

- 2. Two congruent figures have equal areas but the converse need not be true. 3. If a planar region formed by a figure T is made up of two non-overlapping planar regions
- formed by figures P and Q, then ar (T) = ar(P) + ar(Q), where ar (X) denotes the area of figure X.
- Two figures are said to be on the same base and between the same parallels, if they have a common base (side) and the vertices, (or the vertex) opposite to the common base of
- each figure lie on a line parallel to the base. 5. Parallelograms on the same base (or equal bases) and between the same parallels are
- equal in area. Area of a parallelogram is the product of its base and the corresponding altitude. 6.
- Parallelograms on the same base (or equal bases) and having equal areas lie between the 7. same parallels.
- 8. If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.
- 9. Triangles on the same base (or equal bases) and between the same parallels are equal in
- area. Area of a triangle is half the product of its base and the corresponding altitude.
- Triangles on the same base (or equal bases) and having equal areas lie between the same
- parallels. 12. A median of a triangle divides it into two triangles of equal areas.