Note: This Question paper consists of three sections A. B and C.

Time: 3 Hrs   **Max. Marks:** 75

SECTION – A

I.   **Very Short Answer type Questions**

   (i) Answer all Questions

   (ii) Each Question carries 2 marks   10 x 2 = 20

1. If $ax^2 + bxy + 3y^2 - 5x + 2y - 3 = 0$ represents a circle, find the values of a and b. Also find its radius and centre.

2. State the necessary and sufficient condition for $lx + my + n = 0$ to be a, normal to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$

3. Find the angle between the circles $x^2 + y^2 - 12x - 6y + 41 = 0$ and $x^2 + y^2 + 4x + 6y - 59 = 0$

4. Find the equation of the parabola whose focus is S(1, -7) and vertex is A(1, -2).

5. Find the angle between the asymptotes of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$.

6. Evaluate $\displaystyle\int \dfrac{1}{(x+3)\sqrt{x+2}} dx$

7. Evaluate $\displaystyle\int \dfrac{\sin^4 x}{\cos^6 x} dx$

8. Evaluate $\displaystyle\int_0^1 \dfrac{x^2}{x^2+1} dx$

9. Evaluate $\displaystyle\int_0^\pi \dfrac{\sin^2 x - \cos^2 x}{\sin^3 x + \cos^3 x} dx$

10. Find the order and degree of the differential equation $\left[\dfrac{d^2y}{dx^2} - \left(\dfrac{dy}{dx} \right)^3 \right]^{6/5} = 6y$.

II. **Short Answer type Questions**
   (i) Answer any five Questions
   (ii) Each Question carries 4 marks   5 x 4 = 20

11. Show that the tangent at $(-1, 2)$ of circle $x^2 + y^2 - 4x - 8y + 7 = 0$ touches the circle $x^2 + y^2 + 4x + 6y = 0$. Also find its point of contact.

12. Find the equation of the circle passing through the points of intersection of the circles $x^2 + y^2 - 8x - 6y + 21 = 0$, $x^2 + y^2 - 2x - 15 = 0$ and $(1, 2)$.

13. Find the length of major axis, minor axis, latus rectum, eccentricity of the ellipse $9x^2 + 16y^2 = 144$.

14. Show that the point of intersection of the perpendicular tangents to an ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, $(a > b)$ lies on a circle.

15. Find the equation of the tangents to the hyperbola $3x^2 - 4y^2 = 12$ which are (i) Parallel to (ii) Perpendicular to the line $y = x - 7$.

16. Find the reduction formula for $\displaystyle\int_0^{\frac{\pi}{2}} \sin^n x \, dx$

17. Solve: $(1 + y^2) \, dx = (\text{Tan}^{-1} y - x)dy$

III. **Long Answer type Questions**
   (i) Answer any five Questions
   (ii) Each Question carries 7 marks   5 x 7 = 35

18. Show that the points $(1, 1)$, $(-6, 0)$, $(-2, 2)$ and $(-2, -8)$, are concyclic.

19. Find the direct common tangents to the circles $x^2 + y^2 + 22x - 4y - 100 = 0$, $x^2 + y^2 - 22x + 4y + 100 = 0$.

20. If y_1, y_2, y_3 are the y-coordinates of the vertices of the triangle in the parabola $y^2 = 4ax$ then show that the area of the triangle is $\dfrac{1}{8a} \left| (y_1 - y_2)(y_2 - y_3)(y_3 - y_1) \right|$ square units.

21. Evaluate $\displaystyle\int \dfrac{9\cos x - \sin x}{4\sin x + 5\cos x}$

22. Evaluate $\displaystyle\int \dfrac{dx}{(1 + x)\sqrt{3 + 2x - x^2}}$

23. Evaluate $\displaystyle\int_0^1 \dfrac{\log(1 + x)}{1 + x^2} dx$

24. Solve: $\dfrac{dy}{dx} = \dfrac{2x + y + 3}{2y + x + 1}$

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