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AJ-1545

M.A./M.Sc. (Final) MATHEMATICS

Term End Examination, 2021-22

INTEGRATION THEORY & FUNCTIONAL ANALYSIS

Time : Three hours] **[Maximum Marks : 100**

Note : *Answer any five questions. All questions carry equal marks.*

1. State and prove Radon-Nikondym Theorem.

2. (a) Let E be a measurable set of finite measure in (A, A, μ). Then prove that E contains a positive A with $\mu(A) > 0$.

(b) Prove that the union of finitely many inner regular sets is inner regular.

3. (a) Show that l_p^n is a Banach space.

(b) Let N be a non zero normed linear space and $S = \{x \quad N : ||x|| \leq 1\}$ be a linear subspace of N. Then N is Banach space if and only if S is complete.

4. Let X be a normed space over the field K and let M be a closed subspace of X. $|| \, ||_9 : \frac{X}{M} \to R$ by $||x + M||_9 = \inf \{||x + m|| : m \quad M\}$. Then $\left(\frac{X}{M}, || \, ||_9\right)$ is a normed space further if X is a Banach space then $\frac{X}{M}$ is a Banach space.

5. (a) State and prove open mapping theorem.

(b) State and prove Hahn-Banach Theorem.

6. Let $\{x_n\}$ be a weakly convergent sequence in a normed space X. Then prove that

(a) The weak limit of $\{x_n\}$ is unique.

(b) $\{||x_n||\}$ is a bounded sequence in R.

(c) Every subsequence $\{x_n\}$ of convergence weakly to the weak limit of $\{x_n\}$.

7. (a) Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.

(b) Let M be a proper closed linear subspace of a Hilbert space H then prove that there exists a non zero vector z_0 in H such that $Z_0 \quad M$.

8. (a) Let M and N be closed linear subspace of a Hilbert space H such that $M \quad N$. Then prove that linear space $M + N$ is also closed.

(b) State and prove projection theorem.

9. Prove that adjoint operator $T \to T^*$ on $B(H)$ has following properties.

(a) $(T_1 + T_2)^* = T_1^* + T_2^*$ (b) $(\alpha T)^* = \bar{\alpha} T^*$

(c) $(T_1 T_2)^* = T_2^* T_1^*$ (d) $||T^*|| = ||T||$

10. (a) Let H be a Hilbert space and T be a positive operator on H then $I + T$ is non singular.

(b) Any arbitrary operator T on a Hilbert space H can be unique expressed as $T = T_1 + iT_2$, where T_1 and T_2 are self adjoint operator.