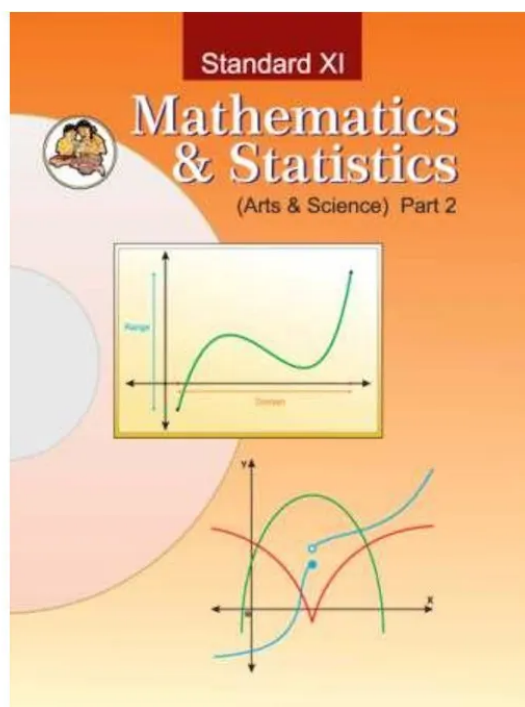


Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 9- Differentiation

Class 11 - Chapter 9 Differentiation



For any clarifications or questions you can write to info@indcareer.com

Postal Address

IndCareer.com, 52, Shilpa Nagar, Somalwada Nagpur - 440015
Maharashtra, India

WhatsApp: +91 9561 204 888, **Website:** <https://www.indcareer.com>

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-9-differentiation/>



Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 9- Differentiation

Class 11: Maths Chapter 9 solutions. Complete Class 11 Maths Chapter 9 Notes.

Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 9- Differentiation

Maharashtra Board 11th Maths Chapter 9, Class 11 Maths Chapter 9 solutions

Ex 9.1

Question 1.

Find the derivatives of the following w.r.t. x by using the method of the first principle.

(a) $x^2 + 3x - 1$

Solution:

$$\text{Let } f(x) = x^2 + 3x - 1$$

$$\therefore f(x + h) = (x + h)^2 + 3(x + h) - 1$$

$$= x^2 + 2xh + h^2 + 3x + 3h - 1$$

By first principle, we get

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-9-differentiation/>

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{[x^2 + 2xh + h^2 + 3x + 3h - 1] - [x^2 + 3x - 1]}{h} \\&= \lim_{h \rightarrow 0} \frac{2xh + h^2 + 3h}{h} \\&= \lim_{h \rightarrow 0} \frac{h(2x + h + 3)}{h} \\&= \lim_{h \rightarrow 0} (2x + h + 3) \quad \dots [\because h \rightarrow 0, \therefore h \neq 0] \\&= 2x + 3\end{aligned}$$

(b) $\sin(3x)$

Solution:

Let $f(x) = \sin 3x$

$f(x + h) = \sin 3(x + h) = \sin(3x + 3h)$

By first principle, we get

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sin(3x+3h) - \sin 3x}{h} \\
 &= \lim_{h \rightarrow 0} \left\{ \frac{2 \cos\left(\frac{3x+3h+3x}{2}\right) \sin\left(\frac{3x+3h-3x}{2}\right)}{h} \right\} \\
 &\quad \dots \left[\because \sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right) \right] \\
 &= \lim_{h \rightarrow 0} \left\{ 2 \cos\left(\frac{6x+3h}{2}\right) \left[\frac{\sin\left(\frac{3h}{2}\right)}{h} \right] \right\} \\
 &= \lim_{h \rightarrow 0} \left\{ 2 \cos\left(\frac{6x+3h}{2}\right) \left(\frac{\sin \frac{3h}{2}}{\frac{3h}{2}} \right) \times \left(\frac{3}{2} \right) \right\} \\
 &= 2 \times \frac{3}{2} \lim_{h \rightarrow 0} \cos \frac{6x+3h}{2} \lim_{h \rightarrow 0} \frac{\sin \frac{3h}{2}}{\frac{3h}{2}} \\
 &= 3 \cos\left(\frac{6x+0}{2}\right) (1) \quad \dots \left[\because \lim_{\theta \rightarrow 0} \frac{\sin p\theta}{p\theta} = 1 \right] \\
 &= 3 \cos 3x
 \end{aligned}$$

(c) e^{2x+1}

Solution:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-9-differentiation/>

$$\begin{aligned}\text{Let } f(x) &= e^{2x+1} \\ f(x+h) &= e^{2(x+h)+1} \\ &= e^{2x+2h+1}\end{aligned}$$

By first principle, we get

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{2x+2h+1} - e^{2x+1}}{h} \\ &= \lim_{h \rightarrow 0} e^{2x+1} \frac{(e^{2h} - 1)}{h} \\ &= e^{2x+1} \left(\lim_{h \rightarrow 0} \frac{e^{2h} - 1}{2h} \right) \times 2 \\ &= 2e^{2x+1} (1) \quad \dots \left[\because \lim_{x \rightarrow 0} \frac{e^{px} - 1}{px} = 1 \right] \\ &= 2e^{2x+1}\end{aligned}$$

(d) 3^x

Solution:

Let $f(x) = 3^x$

$f(x+h) = 3^{x+h}$

By first principle, we get

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3^{x+h} - 3^x}{h} \\ &= \lim_{h \rightarrow 0} 3^x \left(\frac{3^h - 1}{h} \right) \\ &= 3^x \lim_{h \rightarrow 0} \left(\frac{3^h - 1}{h} \right) \\ &= 3^x \log 3 \quad \dots \left[\because \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a \right] \end{aligned}$$

(e) $\log(2x + 5)$

Solution:

Let $f(x) = \log(2x + 5)$

$\therefore f(x+h) = \log[2(x+h) + 5] = \log(2x + 2h + 5)$

By first principle, we get

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\log(2x+2h+5) - \log(2x+5)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \log \left(\frac{2x+2h+5}{2x+5} \right) \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \log \left(\frac{2x+5+2h}{2x+5} \right) \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \log \left(1 + \frac{2h}{2x+5} \right) \\
 &= \lim_{h \rightarrow 0} \log \left(1 + \frac{2h}{2x+5} \right)^{\frac{1}{h}} \\
 &= \lim_{h \rightarrow 0} \log \left[\left(1 + \frac{2h}{2x+5} \right)^{\frac{2x+5}{2h}} \right]^{\frac{2}{2x+5}} \\
 &= \log e^{\frac{2}{2x+5}} \quad \dots \left[\because \lim_{x \rightarrow 0} (1+px)^{\frac{1}{px}} = e \right] \\
 &= \frac{2}{2x+5} \log e \\
 &= \frac{2}{2x+5} \quad \dots [\because \log e = 1]
 \end{aligned}$$

(f) $\tan(2x + 3)$

Solution:

Let $f(x) = \tan(2x + 3)$

$\therefore f(x+h) = \tan[2(x+h) + 3] = \tan(2x + 2h + 3)$

By first principle, we get

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\tan(2x+2h+3) - \tan(2x+3)}{h} \\
 &= \lim_{h \rightarrow 0} \left[\frac{\frac{\sin(2x+2h+3)}{\cos(2x+2h+3)} - \frac{\sin(2x+3)}{\cos(2x+3)}}{h} \right] \\
 &= \lim_{h \rightarrow 0} \left[\frac{\sin(2x+2h+3) \cdot \cos(2x+3) - \sin(2x+3) \cdot \cos(2x+2h+3)}{h \cos(2x+3) \cos(2x+2h+3)} \right] \\
 &= \lim_{h \rightarrow 0} \frac{\sin[(2x+2h+3) - (2x+3)]}{h \cos(2x+3) \cos(2x+2h+3)} \\
 &= \lim_{h \rightarrow 0} \left[\left(\frac{\sin 2h}{h} \right) \frac{1}{\cos(2x+3) \cos(2x+2h+3)} \right] \\
 &= \frac{2}{\cos(2x+3)} \lim_{h \rightarrow 0} \left(\frac{\sin 2h}{2h} \right) \lim_{h \rightarrow 0} \left(\frac{1}{\cos(2x+2h+3)} \right) \\
 &= \frac{2}{\cos(2x+3)} (1) \left(\frac{1}{\cos(2x+3)} \right) \dots \left[\because \lim_{\theta \rightarrow 0} \frac{\sin p\theta}{p\theta} = 1 \right] \\
 &= 2 \sec^2(2x+3)
 \end{aligned}$$

(g) $\sec(5x - 2)$

Solution:

Let $f(x) = \sec(5x - 2)$

$f(x+h) = \sec[5(x+h) - 2] = \sec(5x + 5h - 2)$

By first principle, we get

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sec(5x+5h-2) - \sec(5x-2)}{h} \\
 &= \lim_{h \rightarrow 0} \left[\frac{\frac{1}{\cos(5x+5h-2)} - \frac{1}{\cos(5x-2)}}{h} \right] \\
 &= \lim_{h \rightarrow 0} \left[\frac{\cos(5x-2) - \cos(5x+5h-2)}{h \cos(5x-2) \cos(5x+5h-2)} \right] \\
 &= \lim_{h \rightarrow 0} \left[\frac{2 \sin\left(\frac{5x-2+5x+5h-2}{2}\right) \sin\left(\frac{5x+5h-2-5x+2}{2}\right)}{h \cos(5x-2) \cos(5x+5h-2)} \right] \\
 &\quad \dots \left[\because \cos C - \cos D = 2 \sin\left(\frac{C+D}{2}\right) \sin\left(\frac{D-C}{2}\right) \right] \\
 &= \frac{1}{\cos(5x-2)} \lim_{h \rightarrow 0} \frac{2 \sin\left(5x-2+\frac{5h}{2}\right) \sin \frac{5h}{2}}{h \cos(5x+5h-2)} \\
 &= \frac{5}{\cos(5x-2)} \lim_{h \rightarrow 0} \left[\frac{\sin\left(5x-2+\frac{5h}{2}\right)}{\cos(5x+5h-2)} \cdot \frac{\sin\left(\frac{5h}{2}\right)}{\frac{5h}{2}} \right] \\
 &= \frac{5}{\cos(5x-2)} \left[\frac{\lim_{h \rightarrow 0} \sin\left(5x-2+\frac{5h}{2}\right)}{\lim_{h \rightarrow 0} \cos(5x+5h-2)} \lim_{h \rightarrow 0} \frac{\sin\left(\frac{5h}{2}\right)}{\frac{5h}{2}} \right] \\
 &= \frac{5}{\cos(5x-2)} \cdot \frac{\sin(5x-2)}{\cos(5x-2)} (1) \quad \dots \left[\because \lim_{\theta \rightarrow 0} \frac{\sin p\theta}{p\theta} = 1 \right] \\
 &= 5 \sec(5x-2) \tan(5x-2)
 \end{aligned}$$

(h) $x\sqrt{x}$

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-9-differentiation/>

Solution:

$$\text{Let } f(x) = x\sqrt{x} = x^{3/2}$$

$$\therefore f(x+h) = (x+h)^{3/2}$$

By first principle, we get

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^{3/2} - x^{3/2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{[(x+h)^{3/2} - x^{3/2}][(x+h)^{3/2} + x^{3/2}]}{h[(x+h)^{3/2} + x^{3/2}]} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h[(x+h)^{3/2} + x^{3/2}]} \\ &= \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h[(x+h)^{3/2} + x^{3/2}]} \\ &= \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2)}{h[(x+h)^{3/2} + x^{3/2}]} \\ &= \lim_{h \rightarrow 0} \frac{3x^2 + 3xh + h^2}{(x+h)^{3/2} + x^{3/2}} \quad \dots [\because h \rightarrow 0, h \neq 0] \\ &= \frac{\lim_{h \rightarrow 0} (3x^2 + 3xh + h^2)}{\lim_{h \rightarrow 0} (x+h)^{3/2} + \lim_{h \rightarrow 0} x^{3/2}} \\ &= \frac{3x^2 + 3x \times 0 + 0^2}{(x+0)^{3/2} + x^{3/2}} = \frac{3x^2}{2x^{3/2}} \\ &= \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x} \end{aligned}$$

Question 2.

Find the derivatives of the following w.r.t. x . at the points indicated against them by using the method of the first principle.

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-2-chapter-9-differentiation/>

(i) $2x+5$ ----- $\sqrt{\quad}$ at $x = 2$

Solution:

$$\text{Let } f(x) = \sqrt{2x + 5}$$

$$\therefore f(2) = \sqrt{2(2)+5} = \sqrt{9} = 3 \text{ and}$$

$$f(2+h) = \sqrt{2(2+h)+5} = \sqrt{2h+9}$$

By first principle, we get

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h)-f(a)}{h}$$

$$\therefore f'(2) = \lim_{h \rightarrow 0} \frac{f(2+h)-f(2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{2h+9}-3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{2h+9}-3}{h} \times \frac{\sqrt{2h+9}+3}{\sqrt{2h+9}+3}$$

$$= \lim_{h \rightarrow 0} \frac{2h+9-9}{h(\sqrt{2h+9}+3)}$$

$$= \lim_{h \rightarrow 0} \frac{2h}{h(\sqrt{2h+9}+3)}$$

$$= \lim_{h \rightarrow 0} \frac{2}{\sqrt{2h+9}+3} \quad \dots [\because h \rightarrow 0, h \neq 0]$$

$$= \frac{2}{\sqrt{0+9}+3}$$

$$= \frac{2}{3+3} = \frac{1}{3}$$

(ii) $\tan x$ at $x = \pi/4$

Solution:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-9-differentiation/>

Let $f(x) = \tan x$

$$\therefore f\left(\frac{\pi}{4}\right) = \tan \frac{\pi}{4} = 1 \text{ and } f\left(\frac{\pi}{4}+h\right) = \tan\left(\frac{\pi}{4}+h\right)$$

By first principle, we get

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$\therefore f'\left(\frac{\pi}{4}\right) = \lim_{h \rightarrow 0} \frac{f\left(\frac{\pi}{4}+h\right) - f\left(\frac{\pi}{4}\right)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\tan\left(\frac{\pi}{4}+h\right) - 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{\sin\left(\frac{\pi}{4}+h\right)}{\cos\left(\frac{\pi}{4}+h\right)} - 1}{h}$$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{\sin\left(\frac{\pi}{4}+h\right) - \cos\left(\frac{\pi}{4}+h\right)}{h \cos\left(\frac{\pi}{4}+h\right)} \\
 &= \lim_{h \rightarrow 0} \frac{\left[\left(\sin \frac{\pi}{4} \cos h + \cos \frac{\pi}{4} \sin h\right) - \left(\cos \frac{\pi}{4} \cos h - \sin \frac{\pi}{4} \sin h\right)\right]}{h \cos\left(\frac{\pi}{4}+h\right)} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{2}} \cos h + \frac{1}{\sqrt{2}} \sin h - \frac{1}{\sqrt{2}} \cos h + \frac{1}{\sqrt{2}} \sin h}{h \cos\left(\frac{\pi}{4}+h\right)} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{2}{\sqrt{2}} \sin h}{h \cos\left(\frac{\pi}{4}+h\right)} \\
 &= \frac{2}{\sqrt{2}} \lim_{h \rightarrow 0} \left(\frac{\sin h}{h}\right) \cdot \lim_{h \rightarrow 0} \frac{1}{\cos\left(\frac{\pi}{4}+h\right)} \\
 &= \sqrt{2} (1) \cdot \frac{1}{\cos\left(\frac{\pi}{4}+0\right)} \quad \dots \left[\because \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right] \\
 &= \sqrt{2} \times \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} \\
 &= 2
 \end{aligned}$$

(iii) 2^{3x+1} at $x = 2$

Solution:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-9-differentiation/>

Let $f(x) = 2^{3x+1}$

$\therefore f(2) = 2^{3(2)+1} = 2^7$ and

$f(2+h) = 2^{3(2+h)+1} = 2^{3h+7}$

By first principle, we get

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$\therefore f'(2) = \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$

$$= \lim_{h \rightarrow 0} \frac{2^{3h+7} - 2^7}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2^{3h} \cdot 2^7 - 2^7}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2^7(2^{3h} - 1)}{h}$$

$$= 2^7 \lim_{h \rightarrow 0} \left(\frac{2^{3h} - 1}{3h} \right) \times 3$$

$$= 2^7 (\log 2) \times 3 \quad \dots \left[\because \lim_{x \rightarrow 0} \left(\frac{a^{px} - 1}{px} \right) = \log a \right]$$

$$= 384 \log 2$$

(iv) $\log(2x + 1)$ at $x = 2$

Solution:

Let $f(x) = \log(2x + 1)$

$\therefore f(2) = \log [2(2) + 1] = \log 5$ and

$f(2+h) = \log [2(2+h) + 1] = \log(2h + 5)$

By first principle, we get

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-2-chapter-9-differentiation/>

$$\begin{aligned}
 f'(a) &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \\
 f'(2) &= \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\log(2h+5) - \log 5}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\log\left(\frac{2h+5}{5}\right)}{h} = \lim_{h \rightarrow 0} \frac{\log\left(1 + \frac{2h}{5}\right)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\log\left(1 + \frac{2h}{5}\right)}{\frac{2h}{5} \times \frac{5}{2}} \\
 &= \frac{2}{5} \lim_{h \rightarrow 0} \frac{\log\left(1 + \frac{2h}{5}\right)}{\frac{2h}{5}} = \frac{2}{5} \quad (1) \\
 &\quad \dots \left[\because \lim_{x \rightarrow 0} \frac{\log(1+px)}{px} = 1 \right] \\
 &= \frac{2}{5}
 \end{aligned}$$

(v) e^{3x-4} at $x = 2$

Solution:

$$\text{Let } f(x) = e^{3x-4}$$

$$f(2) = e^{3(2)-4} = e^2 \text{ and}$$

$$f(2+h) = e^{3(2+h)-4} = e^{3h+2}$$

By first principle, we get

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(2) = \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{e^{3h+2} - e^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{e^{3h} e^2 - e^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{e^2 (e^{3h} - 1)}{h}$$

$$= e^2 \lim_{h \rightarrow 0} \left(\frac{e^{3h} - 1}{3h} \right) \times 3$$

$$= 3e^2 (1) \quad \dots \left[\because \lim_{x \rightarrow 0} \frac{e^{px} - 1}{px} = 1 \right]$$

$$= 3e^2$$

(vi) $\cos x$ at $x = 5\pi/4$

Solution:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-9-differentiation/>

Let $f(x) = \cos x$

$$f\left(\frac{5\pi}{4}\right) = \cos\left(\frac{5\pi}{4}\right)$$

$$\text{and } f\left(\frac{5\pi}{4}+h\right) = \cos\left(\frac{5\pi}{4}+h\right)$$

By first principle, we get

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$f'\left(\frac{5\pi}{4}\right) = \lim_{h \rightarrow 0} \frac{f\left(\frac{5\pi}{4}+h\right) - f\left(\frac{5\pi}{4}\right)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cos\left(\frac{5\pi}{4}+h\right) - \cos\left(\frac{5\pi}{4}\right)}{h}$$

$$= 2 \sin\left[\frac{\frac{10\pi}{4}+h}{2}\right] \sin\left(\frac{h}{2}\right)$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin\left[\frac{\frac{10\pi}{4}+h}{2}\right] \sin\left(\frac{h}{2}\right)}{h}$$

$$\dots \left[\because \cos C - \cos D = -2 \sin\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right) \right]$$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{-2 \sin \left(\frac{5\pi}{4} + \frac{h}{2} \right) \sin \frac{h}{2}}{h} \\
 &= -2 \lim_{h \rightarrow 0} \sin \left(\frac{5\pi}{4} + \frac{h}{2} \right) \cdot \lim_{h \rightarrow 0} \frac{\sin \frac{h}{2}}{\frac{h}{2} \times 2} \\
 &= -2 \lim_{h \rightarrow 0} \sin \left(\frac{5\pi}{4} + \frac{h}{2} \right) \left(\frac{1}{2} \right) \lim_{h \rightarrow 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}} \\
 &= -2 \sin \left(\frac{5\pi}{4} + 0 \right) \cdot \frac{1}{2} (1) \\
 &\quad \dots \left[\because \lim_{\theta \rightarrow 0} \frac{\sin p\theta}{p\theta} = 1 \right] \\
 &= -\sin \frac{5\pi}{4} \\
 &= -\sin \left(\pi + \frac{\pi}{4} \right) \\
 &= \sin \frac{\pi}{4} \quad \dots [\because \sin(\pi + \theta) = -\sin \theta] \\
 &= \frac{1}{\sqrt{2}}
 \end{aligned}$$

Question 3.

Show that the function f is not differentiable at $x = -3$,

where $f(x) = x^2 + 2$ for $x < -3$

$= 2 - 3x$ for $x \geq -3$

Solution:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-9-differentiation/>

$$f(x) = x^2 + 2 \quad \text{for } x < -3$$
$$= 2 - 3x \quad \text{for } x \geq -3$$

$$\begin{aligned} L f'(-3) &= \lim_{h \rightarrow 0^-} \frac{f(-3+h) - f(-3)}{h} \\ &= \lim_{h \rightarrow 0^-} \frac{[(-3+h)^2 + 2] - [2 - 3(-3)]}{h} \\ &= \lim_{h \rightarrow 0^-} \frac{9 - 6h + h^2 + 2 - 11}{h} \\ &= \lim_{h \rightarrow 0^-} \frac{h^2 - 6h}{h} \\ &= \lim_{h \rightarrow 0^-} \frac{h(h-6)}{h} \\ &= \lim_{h \rightarrow 0^-} (h-6) \quad \dots [\because h \rightarrow 0, h \neq 0] \\ &= -6 \end{aligned}$$

$$\begin{aligned} R f'(-3) &= \lim_{h \rightarrow 0^+} \frac{f(-3+h) - f(-3)}{h} \\ &= \lim_{h \rightarrow 0^+} \frac{[2 - 3(-3+h)] - [2 - 3(-3)]}{h} \\ &= \lim_{h \rightarrow 0^+} \frac{(11 - 3h) - 11}{h} \\ &= \lim_{h \rightarrow 0^+} \frac{-3h}{h} \\ &= \lim_{h \rightarrow 0^+} (-3) \quad \dots [\because h \rightarrow 0, h \neq 0] \\ &= -3 \end{aligned}$$

$$\therefore L f'(-3) \neq R f'(-3)$$

$\therefore f$ is not differentiable at $x = -3$.

Question 4.

Show that $f(x) = x^2$ is continuous and differentiable at $x = 0$.

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-9-differentiation/>

Solution:

Continuity at $x = 0$:

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} x^2 = (0)^2 = 0$$

Also, $f(x) = x^2$

$$\therefore f(0) = (0)^2 = 0$$

$$\therefore \lim_{x \rightarrow 0} f(x) = f(0)$$

$\therefore f(x)$ is continuous at $x = 0$.

Differentiability at $x = 0$:

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$\therefore f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h^2 - 0}{h}$$

$$= \lim_{h \rightarrow 0} h \quad \dots [\because h \rightarrow 0, h \neq 0]$$

$$= 0$$

$\therefore f'(0)$ exists.

$\therefore f(x)$ is differentiable at $x = 0$.

Question 5.

Discuss the continuity and differentiability of

(i) $f(x) = x|x|$ at $x = 0$

Solution:

$$f(x) = x |x|$$

$$f(x) = x(-x), x < 0$$

$$= x(x), x \geq 0$$

Continuity at $x = 0$:

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (-x^2) = 0$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (x^2) = 0$$

$$f(0) = 0$$

$$\therefore \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0)$$

$\therefore f(x)$ is continuous at $x = 0$.

Differentiability at $x = 0$:

$$\begin{aligned} L f'(0) &= \lim_{h \rightarrow 0^-} \frac{f(0+h) - f(0)}{h} \\ &= \lim_{h \rightarrow 0^-} \frac{-h^2 - 0}{h} \\ &= \lim_{h \rightarrow 0^-} (-h) \quad \dots [\because h \rightarrow 0, h \neq 0] \\ &= 0 \end{aligned}$$

$$\begin{aligned} R f'(0) &= \lim_{h \rightarrow 0^+} \frac{f(0+h) - f(0)}{h} \\ &= \lim_{h \rightarrow 0^+} \frac{h^2 - 0}{h} \\ &= \lim_{h \rightarrow 0^+} (h) \quad \dots [\because h \rightarrow 0, h \neq 0] \\ &= 0 \end{aligned}$$

$$\therefore L f'(0) = R f'(0)$$

$\therefore f(x)$ is differentiable at $x = 0$.

(ii) $f(x) = (2x + 3) |2x + 3|$ at $x = -3/2$

Solution:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-9-differentiation/>

$$f(x) = (2x + 3) | 2x + 3|$$

$$\therefore f(x) = -(2x + 3)^2, \quad x < -\frac{3}{2}$$
$$= (2x + 3)^2, \quad x \geq -\frac{3}{2}$$

Continuity at $x = -\frac{3}{2}$:

$$\lim_{x \rightarrow -\frac{3}{2}^-} f(x) = \lim_{x \rightarrow -\frac{3}{2}^-} -(2x + 3)^2$$
$$= -\left(2 \times \frac{-3}{2} + 3\right)^2$$
$$= 0$$

$$\lim_{x \rightarrow -\frac{3}{2}^+} f(x) = \lim_{x \rightarrow -\frac{3}{2}^+} (2x + 3)^2$$
$$= \left(2 \times \frac{-3}{2} + 3\right)^2 = 0$$

$$f\left(-\frac{3}{2}\right) = \left(2 \times \frac{-3}{2} + 3\right)^2$$

$$\therefore f\left(-\frac{3}{2}\right) = 0$$

$$\therefore \lim_{x \rightarrow -\frac{3}{2}^-} f(x) = \lim_{x \rightarrow -\frac{3}{2}^+} f(x) = f\left(-\frac{3}{2}\right)$$

$$\therefore f(x) \text{ is continuous at } x = -\frac{3}{2}.$$

Differentiability at $x = -\frac{3}{2}$:

$$\begin{aligned} Lf'\left(-\frac{3}{2}\right) &= \lim_{h \rightarrow 0^-} \frac{f\left(-\frac{3}{2} + h\right) - f\left(-\frac{3}{2}\right)}{h} \\ &= \lim_{h \rightarrow 0^-} \frac{-\left[2\left(-\frac{3}{2} + h\right) + 3\right]^2 - 0}{h} \\ &= \lim_{h \rightarrow 0^-} -\left(\frac{4h^2}{h}\right) \\ &= -\lim_{h \rightarrow 0^-} 4h \quad \dots [\because h \rightarrow 0, h \neq 0] \\ &= 0 \end{aligned}$$

$$\begin{aligned} Rf'\left(-\frac{3}{2}\right) &= \lim_{h \rightarrow 0^+} \frac{f\left(-\frac{3}{2} + h\right) - f\left(-\frac{3}{2}\right)}{h} \\ &= \lim_{h \rightarrow 0^+} \frac{\left[2\left(-\frac{3}{2} + h\right) + 3\right]^2 - 0}{h} \\ &= \lim_{h \rightarrow 0^+} \frac{4h^2}{h} \\ &= \lim_{h \rightarrow 0^+} 4h \quad \dots [\because h \rightarrow 0, \therefore h \neq 0] \\ &= 0 \end{aligned}$$

$$\therefore Lf'\left(-\frac{3}{2}\right) = Rf'\left(-\frac{3}{2}\right)$$

$$\therefore f \text{ is differentiable at } x = -\frac{3}{2}.$$

Question 6.

Discuss the continuity and differentiability of $f(x)$ at $x = 2$.

$f(x) = [x]$ if $x \in [0, 4)$. [where $[]$ is a greatest integer (floor) function]

Solution:

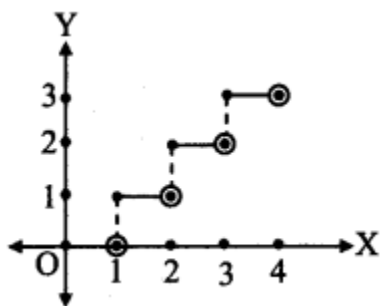
Explanation:

$$x \in [0, 4)$$

$$\therefore 0 \leq x < 4$$

We will plot graph for $0 \leq x < 4$

not for $x < 0$ and upto $x = 4$ making on X-axis.



$$f(x) = [x]$$

$\therefore$ Greatest integer function is discontinuous at all integer values of x and hence not differentiable at all integers.

$\therefore f$ is not continuous at $x = 2$.

$$\therefore f(x) = 1, x < 2$$

$$= 2, x \geq 2$$

$x \in$ neighbourhood of $x = 2$.

$$\therefore \text{L.H.L.} = 1, \text{R.H.L.} = 2$$

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-9-differentiation/>

$\therefore f$ is not continuous at $x = 2$.

$\therefore f$ is not differentiable at $x = 2$.

Question 7.

Test the continuity and differentiability of

$$f(x) = 3x + 2 \text{ if } x > 2$$

$$= 12 - x^2 \text{ if } x \leq 2 \text{ at } x = 2.$$

Solution:

$$f(x) = 3x + 2, \quad x > 2$$
$$= 12 - x^2, \quad x \leq 2$$

Continuity at $x = 2$:

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (12 - x^2) = 12 - (2)^2 = 8$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (3x + 2) = 3(2) + 2 = 8$$

$$f(2) = 12 - (2)^2 = 12 - 4 = 8$$

$$\therefore \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2)$$

$\therefore f(x)$ is continuous at $x = 2$.

Differentiability at $x = 2$:

$$\begin{aligned}Lf'(2) &= \lim_{h \rightarrow 0^-} \frac{f(2+h) - f(2)}{h} \\&= \lim_{h \rightarrow 0^-} \frac{12 - (2+h)^2 - 8}{h} \\&= \lim_{h \rightarrow 0^-} \frac{12 - (4 + 4h + h^2) - 8}{h} \\&= \lim_{h \rightarrow 0^-} \frac{(4h + h^2)}{h} \\&= - \lim_{h \rightarrow 0^-} (4 + h) \quad \dots [\because h \rightarrow 0, h \neq 0] \\&= -4 \\Rf'(2) &= \lim_{h \rightarrow 0^+} \frac{f(2+h) - f(2)}{h} \\&= \lim_{h \rightarrow 0^+} \frac{3(2+h) + 2 - 8}{h} \\&= \lim_{h \rightarrow 0^+} \frac{3h}{h} \\&= \lim_{h \rightarrow 0^+} (3) \quad \dots [\because h \rightarrow 0, h \neq 0] \\&= 3\end{aligned}$$

$$\therefore Lf'(2) \neq Rf'(2)$$

$\therefore f$ is not differentiable at $x = 2$.

Question 8.

If $f(x) = \sin x - \cos x$ if $x \leq \pi/2$

$= 2x - \pi + 1$ if $x > \pi/2$

Test the continuity and differentiability of f at $x = \pi/2$.

Solution:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-9-differentiation/>

$$f(x) = \sin x - \cos x, \quad x \leq \frac{\pi}{2}$$

$$= 2x - \pi + 1, \quad x > \frac{\pi}{2}$$

Continuity at $x = \frac{\pi}{2}$:

$$\lim_{x \rightarrow \frac{\pi}{2}^-} f(x) = \lim_{x \rightarrow \frac{\pi}{2}^-} (\sin x - \cos x)$$

$$= \sin \frac{\pi}{2} - \cos \frac{\pi}{2}$$

$$= 1 - 0 = 1$$

$$\lim_{x \rightarrow \frac{\pi}{2}^+} f(x) = \lim_{x \rightarrow \frac{\pi}{2}^+} (2x - \pi + 1)$$

$$= 2\left(\frac{\pi}{2}\right) - \pi + 1$$

$$= 1$$

$$f\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} - \cos \frac{\pi}{2} = 1 - 0 = 1$$

$$\therefore \lim_{x \rightarrow \frac{\pi}{2}^-} f(x) = \lim_{x \rightarrow \frac{\pi}{2}^+} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\therefore f(x) \text{ is continuous at } x = \frac{\pi}{2}.$$

Differentiability at $x = \frac{\pi}{2}$:

$$L f'\left(\frac{\pi}{2}\right) = \lim_{h \rightarrow 0^-} \frac{f\left(\frac{\pi}{2} + h\right) - f\left(\frac{\pi}{2}\right)}{h}$$

$$= \lim_{h \rightarrow 0^-} \frac{\sin\left(\frac{\pi}{2} + h\right) - \cos\left(\frac{\pi}{2} + h\right) - 1}{h}$$

$$= \lim_{h \rightarrow 0^-} \frac{\cos h + \sin h - 1}{h}$$

$$= \lim_{h \rightarrow 0^-} \left(\frac{\sin h}{h} - \frac{1 - \cos h}{h} \right)$$

$$= \lim_{h \rightarrow 0^-} \left(\frac{\sin h}{h} - \frac{2 \sin^2 \frac{h}{2}}{h} \right)$$

$$= 1 - \lim_{h \rightarrow 0^-} \left(\frac{\sin^2\left(\frac{h}{2}\right)}{\frac{h}{2}} \right)$$

$$= 1 - 0 = 1$$

$$\begin{aligned} Rf'\left(\frac{\pi}{2}\right) &= \lim_{h \rightarrow 0^+} \frac{f\left(\frac{\pi}{2} + h\right) - f\left(\frac{\pi}{2}\right)}{h} \\ &= \lim_{h \rightarrow 0^+} \frac{\left[2\left(\frac{\pi}{2} + h\right) - \pi + 1\right] - 1}{h} \\ &= \lim_{h \rightarrow 0^+} \left(\frac{2h}{h}\right) \\ &= \lim_{h \rightarrow 0^+} 2 \quad \dots [\because h \rightarrow 0, h \neq 0] \\ &= 2 \end{aligned}$$

$$\therefore Lf'\left(\frac{\pi}{2}\right) \neq Rf'\left(\frac{\pi}{2}\right)$$

$\therefore f(x)$ is not differentiable at $x = \frac{\pi}{2}$.

Question 9.

Examine the function

$$f(x) = x^2 \cos(1x), \text{ for } x \neq 0$$

$$= 0, \text{ for } x = 0$$

for continuity and differentiability at $x = 0$.

Solution:

$$f(x) = x^2 \cos \left(\frac{1}{x} \right), x \neq 0$$
$$= 0, x = 0$$

Continuity at $x = 0$:

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} x^2 \cos \frac{1}{x}$$
$$= 0 \quad \dots [-1 \leq \cos \frac{1}{x} \leq 1]$$

$$f(0) = 0$$

$$\therefore \lim_{x \rightarrow 0} f(x) = f(0)$$

$\therefore f(x)$ is continuous at $x = 0$.

Differentiability at $x = 0$:

$$f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}$$
$$= \lim_{h \rightarrow 0} \left(\frac{h^2 \cos \left(\frac{1}{h} \right) - 0}{h} \right)$$
$$= \lim_{h \rightarrow 0} h \cos \frac{1}{h}$$
$$\dots [\because h \rightarrow 0, h \neq 0]$$
$$= 0$$

$\therefore f'(0)$ exists.

$\therefore f(x)$ is differentiable at $x = 0$.

Ex 9.2

(I) Differentiate the following w.r.t. x

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-2-chapter-9-differentiation/>

Question 1.

$$y = x^{\frac{4}{3}} + e^x - \sin x$$

Solution:

$$y = x^{\frac{4}{3}} + e^x - \sin x$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{d}{dx} \left(x^{\frac{4}{3}} + e^x - \sin x \right)$$

$$\frac{dy}{dx} = \frac{d}{dx} \left(x^{\frac{4}{3}} \right) + \frac{d}{dx} (e^x) - \frac{d}{dx} (\sin x)$$

$$= \frac{4}{3} x^{\frac{4}{3}-1} + e^x - \cos x$$

$$= \frac{4}{3} x^{\frac{1}{3}} + e^x - \cos x$$

Question 2.

$$y = \sqrt{x} + \tan x - x^3$$

Solution:

$$y = \sqrt{x} + \tan x - x^3$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{d}{dx} (\sqrt{x} + \tan x - x^3)$$

$$\frac{dy}{dx} = \frac{d}{dx} (\sqrt{x}) + \frac{d}{dx} (\tan x) - \frac{d}{dx} (x^3)$$

$$= \frac{1}{2\sqrt{x}} + \sec^2 x - 3x^2$$

Question 3.

$$y = \log x - \operatorname{cosec} x + 5^x - \frac{3}{x^{\frac{3}{2}}}$$

Solution:

$$y = \log x - \operatorname{cosec} x + 5^x - \frac{3}{x^{\frac{3}{2}}}$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{d}{dx} \left(\log x - \operatorname{cosec} x + 5^x - \frac{3}{x^{\frac{3}{2}}} \right)$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}(\log x) - \frac{d}{dx}(\operatorname{cosec} x) + \frac{d}{dx}(5^x) \\ &\quad - 3 \frac{d}{dx} \left(x^{-\frac{3}{2}} \right) \end{aligned}$$

$$\begin{aligned} &= \frac{1}{x} - (-\operatorname{cosec} x \cot x) + 5^x \log 5 \\ &\quad - 3 \left(-\frac{3}{2} x^{-\frac{3}{2}-1} \right) \end{aligned}$$

$$= \frac{1}{x} - (-\operatorname{cosec} x \cot x) + 5^x \log 5$$

$$-3\left(-\frac{3}{2}x^{\frac{-5}{2}}\right)$$
$$= \frac{1}{x} + \operatorname{cosec} x \cot x + 5^x \log 5 + \frac{9}{2x^{\frac{5}{2}}}$$

Question 4.

$$y = x^{\frac{7}{3}} + 5x^{\frac{4}{5}} - \frac{5}{x^{\frac{2}{5}}}$$

Solution:

$$y = x^{\frac{7}{3}} + 5x^{\frac{4}{5}} - \frac{5}{x^{\frac{2}{5}}}$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{d}{dx} \left(x^{\frac{7}{3}} + 5x^{\frac{4}{5}} - \frac{5}{x^{\frac{2}{5}}} \right)$$

$$\frac{dy}{dx} = \frac{d}{dx} \left(x^{\frac{7}{3}} \right) + 5 \frac{d}{dx} \left(x^{\frac{4}{5}} \right) - 5 \frac{d}{dx} \left(x^{-\frac{2}{5}} \right)$$

$$= \frac{7}{3}x^{\frac{7}{3}-1} + 5 \left(\frac{4}{5}x^{\frac{4}{5}-1} \right) - 5 \left(-\frac{2}{5}x^{-\frac{2}{5}-1} \right)$$

$$= \frac{7}{3}x^{\frac{4}{3}} + 4x^{\frac{-1}{5}} + 2x^{\frac{-7}{5}}$$

$$= \frac{7}{3}x^{\frac{4}{3}} + \frac{4}{x^{\frac{1}{5}}} + \frac{2}{x^{\frac{7}{5}}}$$

Question 5.

$$y = 7^x + x^7 - \frac{2}{3}x\sqrt{x} - \log x + 7^7$$

Solution:

$$y = 7^x + x^7 - \frac{2}{3}x\sqrt{x} - \log x + 7^7$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{d}{dx}(7^x + x^7 - \frac{2}{3}x\sqrt{x} - \log x + 7^7)$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(7^x) + \frac{d}{dx}(x^7) - \frac{2}{3} \cdot \frac{d}{dx}\left(x^{\frac{3}{2}}\right) - \frac{d}{dx}(\log x) \\ &\quad + \frac{d}{dx}(7^7)\end{aligned}$$

$$= 7^x \log 7 + 7x^6 - \frac{2}{3} \times \frac{3}{2}x^{\frac{3}{2}-1} - \frac{1}{x} + 0$$

$$= 7^x \log 7 + 7x^6 - x^{\frac{1}{2}} - \frac{1}{x}$$

$$= 7^x \log 7 + 7x^6 - \sqrt{x} - \frac{1}{x}$$

Question 6.

$$y = 3 \cot x - 5e^x + 3 \log x - \frac{4}{x^{\frac{3}{4}}}$$

Solution:

$$y = 3 \cot x - 5e^x + 3 \log x - \frac{4}{x^{\frac{3}{4}}}$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{d}{dx} \left(3 \cot x - 5e^x + 3 \log x - \frac{4}{x^{\frac{3}{4}}} \right)$$

$$\begin{aligned} \frac{dy}{dx} &= 3 \frac{d}{dx} (\cot x) - 5 \frac{d}{dx} (e^x) + 3 \frac{d}{dx} (\log x) \\ &\quad - 4 \frac{d}{dx} \left(x^{-\frac{3}{4}} \right) \end{aligned}$$

$$= 3(-\operatorname{cosec}^2 x) - 5e^x + 3 \left(\frac{1}{x} \right) - 4 \left(-\frac{3}{4} x^{-\frac{3}{4}-1} \right)$$

$$= -3 \operatorname{cosec}^2 x - 5e^x + \frac{3}{x} + 3x^{-\frac{7}{4}}$$

$$= -3 \operatorname{cosec}^2 x - 5e^x + \frac{3}{x} + \frac{3}{x^{\frac{7}{4}}}$$

(II) Differentiate the following w.r.t. x

Question 1.

$$y = x^5 \tan x$$

Solution:

$$y = x^5 \tan x$$

Differentiating w.r.t. x , we get

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(x^5 \tan x) \\ &= x^5 \frac{d}{dx} \tan x + \tan x \frac{d}{dx} x^5 \\ &= x^5 \sec^2 x + \tan x (5x^4) \\ &= x^4 (x \sec^2 x + 5 \tan x)\end{aligned}$$

Question 2.

$$y = x^3 \log x$$

Solution:

$$y = x^3 \log x$$

Differentiating w.r.t. x , we get

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(x^3 \log x) \\ \frac{dy}{dx} &= x^3 \left(\frac{d}{dx} \log x \right) + \log x \left(\frac{d}{dx} x^3 \right) \\ &= x^3 \left(\frac{1}{x} \right) + \log x (3x^2) \\ &= x^2 + 3x^2 \log x \\ &= x^2 (1 + 3 \log x)\end{aligned}$$

Question 3.

$$y = (x^2 + 2)^2 \sin x$$

Solution:

$$y = (x^2 + 2)^2 \sin x$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{d}{dx} [(x^2 + 2)^2 \sin x]$$

$$\frac{dy}{dx} = (x^2 + 2)^2 \frac{d}{dx} \sin x + \sin x \left[\frac{d}{dx} (x^2 + 2)^2 \right]$$

$$= (x^2 + 2)^2 \cos x + \sin x \frac{d}{dx} (x^4 + 4x^2 + 4)$$

$$= (x^2 + 2)^2 \cos x + \sin x (4x^3 + 8x)$$

$$= (x^2 + 2)^2 \cos x + \sin x (4x)(x^2 + 2)$$

$$= (x^2 + 2) [(x^2 + 2) \cos x + 4x \sin x]$$

Question 4.

$$y = e^x \log x$$

Solution:

$$y = e^x \log x$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{d}{dx} (e^x \log x)$$

$$\frac{dy}{dx} = e^x \frac{d}{dx} \log x + \log x \left(\frac{d}{dx} e^x \right)$$

$$\begin{aligned} &= e^x \left(\frac{1}{x} \right) + \log x (e^x) \\ &= e^x \left(\frac{1}{x} + \log x \right) \end{aligned}$$

Question 5.

$$y = x^{\frac{3}{2}} e^x \log x$$

Solution:

$$y = x^{\frac{3}{2}} (e^x \log x)$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{d}{dx} \left(x^{\frac{3}{2}} (e^x \log x) \right)$$

$$\begin{aligned} \frac{dy}{dx} &= x^{\frac{3}{2}} \frac{d}{dx} (e^x \log x) + (e^x \log x) \frac{d}{dx} \left(x^{\frac{3}{2}} \right) \\ &= x^{\frac{3}{2}} \left[e^x \frac{d}{dx} \log x + \log x \frac{d}{dx} e^x \right] + (e^x \log x) \left(\frac{3}{2} x^{\frac{3}{2}-1} \right) \\ &= x^{\frac{3}{2}} \left[e^x \left(\frac{1}{x} \right) + \log x (e^x) \right] + (e^x \log x) \left(\frac{3}{2} x^{\frac{1}{2}} \right) \\ &= e^x x^{\frac{3}{2}} \left(\frac{1}{x} + \log x \right) + (e^x \log x) \left(\frac{3}{2} x^{\frac{1}{2}} \right) \\ &= x^{\frac{1}{2}} e^x \left[x \left(\frac{1}{x} + \log x \right) + (\log x) \left(\frac{3}{2} \right) \right] \end{aligned}$$

$$= \sqrt{x}e^x \left[1 + x \log x + \frac{3}{2} \log x \right]$$

Question 6.

$$y = \log e^{x^3} \log x^3$$

Solution:

$$\begin{aligned} y &= \log e^{x^3} \log x^3 \\ &= x^3 (\log e) \cdot 3 (\log x) \quad \dots [\because \log m^n = n \log m] \\ &= 3x^3 \log x \quad \dots [\because \log e = 1] \end{aligned}$$

Differentiating w.r.t. x , we get

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} (3x^3 \log x) \\ \frac{dy}{dx} &= 3 \left[x^3 \frac{d}{dx} \log x + \log x \left(\frac{d}{dx} x^3 \right) \right] \\ &= 3 \left[x^3 \left(\frac{1}{x} \right) + \log x (3x^2) \right] \\ &= 3 [x^2 + \log x (3x^2)] \\ &= 3x^2 (1 + 3 \log x) \end{aligned}$$

(III) Differentiate the following w.r.t. x

Question 1.

$$y = x^2\sqrt{x} + x^4 \log x$$

Solution:

$$y = x^2\sqrt{x} + x^4 \log x$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{d}{dx}(x^2\sqrt{x} + x^4 \log x)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{\frac{5}{2}}\right) + \frac{d}{dx}(x^4 \log x)$$

$$= \frac{5}{2}x^{\frac{5}{2}-1} + \left[x^4 \frac{d}{dx} \log x + \log x \left(\frac{d}{dx} x^4 \right) \right]$$

$$= \frac{5}{2}x^{\frac{3}{2}} + \left[x^4 \left(\frac{1}{x} \right) + \log x (4x^3) \right]$$

$$= \frac{5}{2}x^{\frac{3}{2}} + x^3(1 + 4\log x)$$

Question 2.

$$y = e^x \sec x - x^{\frac{5}{3}} \log x$$

Solution:

$$y = e^x \sec x - x^{\frac{5}{3}} \log x$$

Differentiating w.r.t. x , we get

Differentiating w.r.t. x , we get

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(e^x \sec x - x^{\frac{5}{3}} \log x \right) \\ \frac{dy}{dx} &= \frac{d}{dx} (e^x \sec x) - \frac{d}{dx} \left(x^{\frac{5}{3}} \log x \right) \\ &= \left[e^x \left(\frac{d}{dx} \sec x \right) + \sec x \frac{d}{dx} (e^x) \right] \\ &\quad - \left[x^{\frac{5}{3}} \left(\frac{d}{dx} \log x \right) + \log x \left(\frac{d}{dx} x^{\frac{5}{3}} \right) \right] \\ &= [e^x \sec x \tan x + \sec x (e^x)] \\ &\quad - \left[x^{\frac{5}{3}} \left(\frac{1}{x} \right) + \log x \left(\frac{5}{3} x^{\frac{5}{3}-1} \right) \right] \\ &= e^x \sec x (\tan x + 1) - \left[x^{\frac{2}{3}} + (\log x) \frac{5}{3} x^{\frac{2}{3}} \right] \\ &= e^x \sec x (\tan x + 1) - x^{\frac{2}{3}} \left(1 + \frac{5}{3} \log x \right)\end{aligned}$$

Question 3.

$$y = x^4 + x\sqrt{x} \cos x - x^2 e^x$$

Solution:

$$\begin{aligned}y &= x^4 + x\sqrt{x} \cos x - x^2 e^x \\ &= x^4 + x^{\frac{3}{2}} \cos x - x^2 e^x\end{aligned}$$

$$= x^4 + x^{\frac{3}{2}} \cos x - x^2 e^x$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{d}{dx} \left(x^4 + x^{\frac{3}{2}} \cos x - x^2 e^x \right)$$

$$\frac{dy}{dx} = \frac{d}{dx} (x^4) + \frac{d}{dx} \left(x^{\frac{3}{2}} \cos x \right) - \frac{d}{dx} (x^2 e^x)$$

$$= 4x^3 + x^{\frac{3}{2}} \frac{d}{dx} \cos x + \cos x \frac{d}{dx} \left(x^{\frac{3}{2}} \right) - \left[x^2 \left(\frac{d}{dx} e^x \right) + e^x \left(\frac{d}{dx} x^2 \right) \right]$$

$$= 4x^3 + x^{\frac{3}{2}} (-\sin x) + \cos x \left(\frac{3}{2} x^{\frac{1}{2}} \right) - [x^2 e^x + e^x (2x)]$$

$$= 4x^3 - x^{\frac{3}{2}} \sin x + \frac{3}{2} \sqrt{x} \cos x - x e^x (x + 2)$$

Question 4.

$$y = (x^3 - 2) \tan x - x \cos x + 7^x \cdot x^7$$

Solution:

$$y = (x^3 - 2) \tan x - x \cos x + 7^x \cdot x^7$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{d}{dx} [(x^3 - 2) \tan x] - \frac{d}{dx} (x \cos x) + \frac{d}{dx} (7^x x^7)$$

$$\begin{aligned}
 &= (x^3 - 2) \frac{d}{dx}(\tan x) + \tan x \frac{d}{dx}(x^3 - 2) \\
 &\quad - \left(x \frac{d}{dx} \cos x + \cos x \frac{d}{dx} x \right) + 7^x \left(\frac{d}{dx} x^7 \right) + x^7 \frac{d}{dx} (7^x) \\
 \therefore \frac{dy}{dx} &= (x^3 - 2) \sec^2 x + \tan x (3x^2) \\
 &\quad - (-x \sin x + \cos x) + 7^x (7x^6) + x^7 7^x (\log 7) \\
 &= (x^3 - 2) \sec^2 x + 3x^2 \tan x + x \sin x - \cos x \\
 &\quad + 7^x x^6 (7 + x \log 7)
 \end{aligned}$$

Question 5.

$$y = \sin x \log x + e^x \cos x - e^x \sqrt{x}$$

Solution:

$$y = \sin x \log x + e^x \cos x - e^x \sqrt{x}$$

Differentiating w.r.t. x , we get

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx}(\sin x \log x) + \frac{d}{dx}(e^x \cos x) - \frac{d}{dx}(e^x \sqrt{x}) \\
 &= \sin x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(\sin x) \\
 &\quad + e^x \frac{d}{dx}(\cos x) + \cos x \frac{d}{dx}(e^x) \\
 &\quad - \left[e^x \frac{d}{dx}(\sqrt{x}) + \sqrt{x} \left(\frac{d}{dx} e^x \right) \right]
 \end{aligned}$$

$$\begin{aligned}
 &= \sin x \left(\frac{1}{x} \right) + \log x (\cos x) \\
 &\quad - e^x \sin x + \cos x (e^x) - e^x \left(\frac{1}{2\sqrt{x}} \right) - \sqrt{x} e^x \\
 &= \frac{1}{x} \sin x + (\log x)(\cos x) + e^x(-\sin x + \cos x) \\
 &\quad - e^x \left(\frac{1}{2\sqrt{x}} + \sqrt{x} \right) \\
 &= \frac{\sin x}{x} + (\cos x)(\log x) + e^x(-\sin x + \cos x) \\
 &\quad - e^x \left(\frac{1+2x}{2\sqrt{x}} \right)
 \end{aligned}$$

Question 6.

$$y = e^x \tan x + \cos x \log x - \sqrt{x} 5^x$$

Solution:

$$y = e^x \tan x + (\cos x)(\log x) - (\sqrt{x})5^x$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{d}{dx}(e^x \tan x) + \frac{d}{dx}(\cos x \log x) - \frac{d}{dx}[(\sqrt{x})5^x]$$

$$\begin{aligned}
 &= \left(e^x \frac{d}{dx} \tan x + \tan x \frac{d}{dx} e^x \right) \\
 &\quad + \left[\cos x \frac{d}{dx} (\log x) + \log x \frac{d}{dx} (\cos x) \right] \\
 &\quad - \left[\sqrt{x} \frac{d}{dx} (5^x) + 5^x \frac{d}{dx} (\sqrt{x}) \right] \\
 &= e^x \sec^2 x + \tan x (e^x) + \cos x \left(\frac{1}{x} \right) \\
 &\quad + \log x (-\sin x) - \left[\sqrt{x} (5^x \log 5) + 5^x \frac{1}{2\sqrt{x}} \right] \\
 &= e^x (\sec^2 x + \tan x) + \frac{\cos x}{x} - \sin x \log x \\
 &\quad - \left[\frac{2x 5^x \log 5 + 5^x}{2\sqrt{x}} \right] \\
 &= e^x (\sec^2 x + \tan x) + \frac{\cos x}{x} - \sin x \log x \\
 &\quad - 5^x \left(\frac{2x \log 5 + 1}{2\sqrt{x}} \right)
 \end{aligned}$$

(IV) Differentiate the following w.r.t.x.

Question 1.

$$y = \frac{x^2+3}{x^2-5}$$

Solution:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-2-chapter-9-differentiation/>

$$y = \frac{x^2 + 3}{x^2 - 5}$$

Differentiating w.r.t. x , we get

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\frac{x^2 + 3}{x^2 - 5} \right) \\&= \frac{(x^2 - 5) \frac{d}{dx} (x^2 + 3) - (x^2 + 3) \frac{d}{dx} (x^2 - 5)}{(x^2 - 5)^2} \\&= \frac{(x^2 - 5)(2x) - (x^2 + 3)(2x)}{(x^2 - 5)^2} \\&= \frac{2x(x^2 - 5 - x^2 - 3)}{(x^2 - 5)^2} \\&= \frac{2x(-8)}{(x^2 - 5)^2} = \frac{-16x}{(x^2 - 5)^2}\end{aligned}$$

Question 2.

$$y = \frac{\sqrt{x}+5}{\sqrt{x}-5}$$

Solution:

$$y = \frac{\sqrt{x}+5}{\sqrt{x}-5}$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{\sqrt{x}+5}{\sqrt{x}-5} \right)$$

$$= \frac{(\sqrt{x}-5) \frac{d}{dx} (\sqrt{x}+5) - (\sqrt{x}+5) \frac{d}{dx} (\sqrt{x}-5)}{(\sqrt{x}-5)^2}$$

$$= \frac{(\sqrt{x}-5) \left(\frac{1}{2\sqrt{x}} \right) - (\sqrt{x}+5) \left(\frac{1}{2\sqrt{x}} \right)}{(\sqrt{x}-5)^2}$$

$$= \frac{\frac{1}{2\sqrt{x}} (\sqrt{x}-5 - \sqrt{x}-5)}{(\sqrt{x}-5)^2}$$

$$= \frac{\frac{1}{2\sqrt{x}} (-10)}{(\sqrt{x}-5)^2} = \frac{-5}{\sqrt{x} (\sqrt{x}-5)^2}$$

Question 3.

$$y = \frac{xe^x}{x+e^x}$$

Solution:

$$y = \frac{xe^x}{x+e^x}$$

Differentiating w.r.t. x , we get

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\frac{xe^x}{x+e^x} \right) \\&= \frac{(x+e^x) \frac{d}{dx} xe^x - xe^x \frac{d}{dx} (x+e^x)}{(x+e^x)^2} \\&= \frac{(x+e^x) \left(x \frac{d}{dx} e^x + e^x \frac{d}{dx} x \right) - xe^x \left(\frac{d}{dx} x + \frac{d}{dx} e^x \right)}{(x+e^x)^2} \\&= \frac{(x+e^x)(xe^x + e^x) - xe^x(1+e^x)}{(x+e^x)^2} \\&= \frac{(x+e^x)e^x(x+1) - xe^x(1+e^x)}{(x+e^x)^2} \\&= \frac{e^x[(x+e^x)(x+1) - x(1+e^x)]}{(x+e^x)^2} \\&= \frac{e^x[x^2 + x + xe^x + e^x - x - xe^x]}{(x+e^x)^2}\end{aligned}$$

$$= \frac{e^x (x^2 + e^x)}{(x + e^x)^2}$$

Question 4.

$$y = \frac{x \log x}{x + \log x}$$

Solution:

$$y = \frac{x \log x}{x + \log x}$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{x \log x}{x + \log x} \right)$$

$$= \frac{(x + \log x) \frac{d}{dx} (x \log x) - x \log x \frac{d}{dx} (x + \log x)}{(x + \log x)^2}$$

$$= \frac{(x + \log x) \left(x \frac{d}{dx} \log x + \log x \frac{d}{dx} x \right) - x \log x \left(\frac{d}{dx} x + \frac{d}{dx} \log x \right)}{(x + \log x)^2}$$

$$= \frac{(x + \log x) \left[x \left(\frac{1}{x} \right) + \log x (1) \right] - x \log x \left(1 + \frac{1}{x} \right)}{(x + \log x)^2}$$

$$= \frac{(x + \log x) (1 + \log x) - x \log x \left(1 + \frac{1}{x} \right)}{(x + \log x)^2}$$

$$= \frac{x + x \log x + \log x + (\log x)^2 - x \log x - \log x}{(x + \log x)^2}$$

$$= \frac{x + (\log x)^2}{(x + \log x)^2}$$

Question 5.

$$y = \frac{x^2 \sin x}{x + \cos x}$$

Solution:

$$y = \frac{x^2 \sin x}{x + \cos x}$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{x^2 \sin x}{x + \cos x} \right)$$

$$= \frac{(x + \cos x) \frac{d}{dx} (x^2 \sin x) - x^2 \sin x \frac{d}{dx} (x + \cos x)}{(x + \cos x)^2}$$

$$= \frac{(x + \cos x) \left(x^2 \frac{d}{dx} \sin x + \sin x \frac{d}{dx} x^2 \right) - x^2 \sin x \left(\frac{d}{dx} x + \frac{d}{dx} \cos x \right)}{(x + \cos x)^2}$$

$$= \frac{(x + \cos x) [x^2 \cos x + \sin x (2x)] - x^2 \sin x (1 - \sin x)}{(x + \cos x)^2}$$

$$= \frac{x^3 \cos x + 2x^2 \sin x + x^2 \cos^2 x + 2x \sin x \cos x - x^2 \sin x + x^2 \sin^2 x}{(x + \cos x)^2}$$

$$\begin{aligned} &= \frac{x^3 \cos x + x^2 \sin x + x^2 \cos^2 x + x^2 \sin^2 x + 2x \sin x \cos x}{(x + \cos x)^2} \\ &= \frac{x^3 \cos x + x^2 \sin x + x^2 (\sin^2 x + \cos^2 x) + x \sin 2x}{(x + \cos x)^2} \\ &= \frac{x^3 \cos x + x^2 \sin x + x^2 (1) + x \sin 2x}{(x + \cos x)^2} \\ &= \frac{x^2 + x^2 \sin x + x^3 \cos x + x \sin 2x}{(x + \cos x)^2} \\ &= \frac{x^2 (1 + \sin x + x \cos x) + x \sin 2x}{(x + \cos x)^2} \end{aligned}$$

Question 6.

$$y = \frac{5e^x - 4}{3e^x - 2}$$

Solution:

$$y = \frac{5e^x - 4}{3e^x - 2}$$

Differentiating w.r.t. x , we get

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left(\frac{5e^x - 4}{3e^x - 2} \right) \\ &= \frac{(3e^x - 2) \frac{d}{dx} (5e^x - 4) - (5e^x - 4) \frac{d}{dx} (3e^x - 2)}{(3e^x - 2)^2} \end{aligned}$$

$$\begin{aligned} &= \frac{(3e^x - 2)\left(5\frac{d}{dx}e^x - \frac{d}{dx}4\right) - (5e^x - 4)\left(3\frac{d}{dx}e^x - \frac{d}{dx}2\right)}{(3e^x - 2)^2} \\ &= \frac{(3e^x - 2)(5e^x - 0) - (5e^x - 4)(3e^x - 0)}{(3e^x - 2)^2} \\ &= \frac{15(e^x)^2 - 10e^x - 15(e^x)^2 + 12e^x}{(3e^x - 2)^2} = \frac{2e^x}{(3e^x - 2)^2} \end{aligned}$$

(V).

Question 1.

If $f(x)$ is a quadratic polynomial such that $f(0) = 3$, $f'(2) = 2$ and $f'(3) = 12$, then find $f(x)$.

Solution:

Let $f(x) = ax^2 + bx + c$ (i)

$$\therefore f(0) = a(0)^2 + b(0) + c$$

$$\therefore f(0) = c$$

But, $f(0) = 3$ (given)

$$\therefore c = 3$$
(ii)

Differentiating (i) w.r.t. x , we get

$$f'(x) = 2ax + b$$

$$\therefore f'(2) = 2a(2) + b$$

$$\therefore f'(2) = 4a + b$$

But, $f'(2) = 2$ (given)

$$\therefore 4a + b = 2$$
(iii)

$$\text{Also, } f'(3) = 2a(3) + b$$

$$\therefore f'(3) = 6a + b$$

But, $f'(3) = 12$ (given)

$$\therefore 6a + b = 12$$
(iv)

equation (iv) – equation (iii), we get

$$2a = 10$$

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-9-differentiation/>

$$\therefore a = 5$$

Substituting $a = 5$ in (iii), we get

$$4(5) + b = 2$$

$$\therefore b = -18$$

$$\therefore a = 5, b = -18, c = 3$$

$$\therefore f(x) = 5x^2 - 18x + 3$$

Check:

If $f(0) = 3$, $f(2) = 2$ and $f'(3) = 12$, then our answer is correct.

$$f(x) = 5x^2 - 18x + 3 \text{ and } f'(x) = 10x - 18$$

$$f(0) = 5(0)^2 - 18(0) + 3 = 3$$

$$f'(2) = 10(2) - 18 = 2$$

$$f'(3) = 10(3) - 18 = 12$$

Thus, our answer is correct.

Question 2.

If $f(x) = a \sin x - b \cos x$, $f'(\frac{\pi}{4}) = \sqrt{2}$ and $f'(\frac{\pi}{6}) = 2$, then find $f(x)$.

Solution:

$$f(x) = a \sin x - b \cos x$$

Differentiating w.r.t. x , we get

$$f'(x) = a \cos x - b(-\sin x)$$

$$\therefore f'(x) = a \cos x + b \sin x$$

$$\therefore f'\left(\frac{\pi}{4}\right) = a \cos\left(\frac{\pi}{4}\right) + b \sin\left(\frac{\pi}{4}\right)$$

$$\text{But, } f'\left(\frac{\pi}{4}\right) = \sqrt{2} \quad \dots(\text{given})$$

$$\therefore a \cos \frac{\pi}{4} + b \sin \frac{\pi}{4} = \sqrt{2}$$

$$\therefore a\left(\frac{1}{\sqrt{2}}\right) + b\left(\frac{1}{\sqrt{2}}\right) = \sqrt{2}$$

$$\therefore \frac{a+b}{\sqrt{2}} = \sqrt{2}$$

$$\therefore a+b=2 \quad \dots(i)$$

$$\text{Also, } f'\left(\frac{\pi}{6}\right) = a \cos \frac{\pi}{6} + b \sin \frac{\pi}{6}$$

$$\text{But, } f'\left(\frac{\pi}{6}\right) = 2 \quad \dots(\text{given})$$

$$\therefore a \cos \frac{\pi}{6} + b \sin \frac{\pi}{6} = 2$$

$$\therefore a\left(\frac{\sqrt{3}}{2}\right) + b\left(\frac{1}{2}\right) = 2$$

$$\therefore \sqrt{3}a + b = 4 \quad \dots(ii)$$

equation (ii) – equation (i), we get

$$\sqrt{3}a - a = 2$$

$$\therefore a(\sqrt{3} - 1) = 2$$

$$\therefore a = \frac{2}{\sqrt{3} - 1} = \frac{2(\sqrt{3} + 1)}{3 - 1}$$

$$\therefore a = \sqrt{3} + 1$$

Substituting $a = \sqrt{3} + 1$ in equation (i), we get

$$\sqrt{3} + 1 + b = 2$$

$$\therefore b = 1 - \sqrt{3}$$

Now, $f(x) = a \sin x - b \cos x$

$$\therefore f(x) = (\sqrt{3} + 1) \sin x + (\sqrt{3} - 1) \cos x$$

VI. Fill in the blanks. (Activity Problems)

Question 1.

$$y = e^x \cdot \tan x$$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}(e^x \tan x)$$

$$\begin{aligned} &= \square \frac{d}{dx} \tan x + \tan x \frac{d}{dx} \square \\ &= \square \square + \tan x \cdot \square \\ &= e^x [\square + \square] \end{aligned}$$

Solution:

$$y = e^x \cdot \tan x$$

Differentiating w.r.t. x , we get

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} (e^x \tan x) \\ &= \boxed{e^x} \frac{d}{dx} \tan x + \tan x \frac{d}{dx} \boxed{e^x} \\ &= \boxed{e^x} \boxed{\sec^2 x} + e^x \tan x \\ &= e^x [\boxed{\sec^2 x} + \boxed{\tan x}] \end{aligned}$$

Question 2.

$$y = \frac{\sin x}{x^2 + 2}$$

diff. w.r.t. x

$$\frac{dy}{dx} = \frac{\square \frac{d}{dx} (\sin x) - \sin x \frac{d}{dx} \square}{(x^2 + 2)^2}$$

$$= \frac{\square\square - \sin x \square}{(x^2 + 2)^2}$$

$$= \frac{\square - \square}{(x^2 + 2)^2}$$

Solution:

$$y = \frac{\sin x}{x^2 + 2}$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{\boxed{x^2 + 2} \frac{d}{dx}(\sin x) - \sin x \frac{d}{dx} \boxed{x^2 + 2}}{(x^2 + 2)^2}$$

$$= \frac{\boxed{(x^2 + 2)} \boxed{(\cos x)} - \sin x \boxed{(2x)}}{(x^2 + 2)^2}$$

$$= \frac{\boxed{(x^2 \cos x + 2 \cos x)} - \boxed{2x \sin x}}{(x^2 + 2)^2}$$

Question 3.

$$y = (3x^2 + 5) \cos x$$

Diff. w.r.t. x

Diff. *w.r.t. x*

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} [(3x^2 + 5) \cos x] \\
 &= (3x^2 + 5) \frac{d}{dx} [\square] + \cos x \frac{d}{dx} [\square] \\
 &= (3x^2 + 5) [\square] + \cos x [\square] \\
 \therefore \frac{dy}{dx} &= (3x^2 + 5) [\square] + [\square] \cos x
 \end{aligned}$$

Solution:

$$y = (3x^2 + 5) \cos x$$

Differentiating w.r.t. x , we get

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} [(3x^2 + 5) \cos x] \\
 &= (3x^2 + 5) \frac{d}{dx} [\cos x] \\
 &\quad + \cos x \frac{d}{dx} [3x^2 + 5] \\
 &= (3x^2 + 5) [-\sin x] + \cos x [6x] \\
 \frac{dy}{dx} &= (3x^2 + 5) [-\sin x] + [6x] \cos x
 \end{aligned}$$

Question 4.

Differentiate $\tan x$ and $\sec x$ w.r.t. x using the formulae for differentiation of $\frac{u}{v}$ and $\frac{1}{v}$ respectively.

Solution:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-9-differentiation/>

$$= \frac{\cos x (\cos x) - \sin x (-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\frac{dy}{dx} = \sec^2 x$$

Let $y = \sec x$

$$y = \frac{1}{\cos x}$$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{\cos x \frac{d}{dx}(1) - 1 \frac{d}{dx} \cos x}{\cos^2 x}$$

$$= \frac{\cos x (0) - 1(-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x}$$

$$= \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$$

$$\frac{dy}{dx} = \sec x \tan x$$

Let $y = \tan x$

Differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{d}{dx} (\tan x)$$

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{\sin x}{\cos x} \right)$$

$$= \frac{\cos x \frac{d}{dx} \sin x - \sin x \frac{d}{dx} \cos x}{\cos^2 x}$$



Maharashtra Board Solutions

Class 11 Arts & Science Maths

(Part 2)

- Chapter 1- Complex Numbers
- Chapter 2- Sequences and Series
- Chapter 3- Permutations and Combination
- Chapter 4- Methods of Induction and Binomial Theorem
- Chapter 5- Sets and Relations
- Chapter 6- Functions
- Chapter 7- Limits
- Chapter 8- Continuity
- Chapter 9- Differentiation

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-2-chapter-9-differentiation/>

About About Maharashtra State Board (MSBSHSE)

The Maharashtra State Board of Secondary and Higher Secondary Education or MSBSHSE (Marathi: महाराष्ट्र राज्य माध्यमिक आणि उच्च माध्यमिक शिक्षण मंडळ), is an **autonomous and statutory body established in 1965**. The board was amended in the year 1977 under the provisions of the Maharashtra Act No. 41 of 1965.

The Maharashtra State Board of Secondary & Higher Secondary Education (MSBSHSE), Pune is an independent body of the Maharashtra Government. There are more than 1.4 million students that appear in the examination every year. The Maha State Board conducts the board examination twice a year. This board conducts the examination for SSC and HSC.

The Maharashtra government established the Maharashtra State Bureau of Textbook Production and Curriculum Research, also commonly referred to as Ebalbharati, in 1967 to take up the responsibility of providing quality textbooks to students from all classes studying under the Maharashtra State Board. MSBHSE prepares and updates the curriculum to provide holistic development for students. It is designed to tackle the difficulty in understanding the concepts with simple language with simple illustrations. Every year around 10 lakh students are enrolled in schools that are affiliated with the Maharashtra State Board.

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-9-differentiation/>

FAQs

Where do I get the Maharashtra State Board Books PDF For free download?

You can download the Maharashtra State Board Books from the eBalbharti official website, i.e. cart.ebalbharati.in or from this article.

Add image

How to Download Maharashtra State Board Books?

Students can get the Maharashtra Books for primary, secondary, and senior secondary classes from here. You can view or download the Maharashtra State Board Books from this page or from the official website for free of cost. Students can follow the detailed steps below to visit the official website and download the e-books for all subjects or a specific subject in different mediums.

Step 1: Visit the official website ebalbharati.in

Step 2: On the top of the screen, select "Download PDF textbooks"

Step 3: From the "Classes" section, select your class.

Step 4: From "Medium", select the medium suitable to you.

Step 5: All Maharashtra board books for class 11th will now be displayed on the right side.

Step 6: Click on the "Download" option to download the PDF book.

Who developed the Maharashtra State board books?

As of now, the MSCERT and Balbharti are responsible for the syllabus and textbooks of Classes 1 to 8, while Classes 9 and 10 are under the Maharashtra State Board of Secondary and Higher Secondary Education (MSBSHSE).

How many state boards are there in Maharashtra?

The Maharashtra State Board of Secondary & Higher Secondary Education, conducts the HSC and SSC Examinations in the state of Maharashtra through its nine Divisional Boards located at Pune, Mumbai, Aurangabad, Nasik, Kolhapur, Amravati, Latur, Nagpur and Ratnagiri.

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-9-differentiation/>

About IndCareer

IndCareer.com is a leading developer of online career guidance resources for the Indian marketplace. Established in 2007, IndCareer.com is currently used by over thousands of institutions across India, including schools, employment agencies, libraries, colleges and universities.

IndCareer.com is designed to assist you in making the right career decision - a decision that meets your unique interests and personality.

For any clarifications or questions you can write to **info@indcareer.com**

Postal Address

IndCareer.com
52, Shilpa Nagar,
Somalwada
Nagpur - 440015
Maharashtra, India

WhatsApp: +91 9561 204 888

Website: <https://www.indcareer.com>

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-9-differentiation/>