

BH-5315  
**M.Sc. (I<sup>st</sup> Semester) Examination, 2022**  
**MATHEMATICS**

**(ADVANCED DISCRETE MATHEMATICS-I)**

*Time Allowed : Three Hours*

*Maximum Marks : 70*

**Note :** Attempt all sections as per instructions given.

**SECTION - A**

**(Objective Type / Multiple Choice / True or False)**

**Q. 1.** Answer the following questions in one sentence

(Attempt any ten questions) : **1×10=10**

(i) Let  $p \equiv$  mathematics is hard,  $q \equiv 4$  is even

number then write  $p \Rightarrow (\sim q)$  in proposition.

(ii) Write the negation of  $(\sim p) \wedge q$

**P.T.O.**

BH-5315

**(2)**

(iii)  $a(a + b) = a$  is known as:

(a) Idempotent law

(b) Absorption law

(c) Commutative law

(d) None of these

(iv) The number of element of any finite Boolean

algebra is \_\_\_\_\_

(v) Type-3 grammar is also called regular

grammar. [True/False]

(vi) Grammar of type-1 is often called \_\_\_\_\_

grammar.

(vii) The set of Natural numbers is a semigroup

under operation '\*' defined by  $a*b = \max \{a, b\}$

$\forall a, b \in \mathbb{N}$ . (True/False).

BH-5315

**(3)**

(viii) If  $Q_+$  is set of positive rational number and '\*'

is defined by  $a*b = \frac{ab}{2} \quad \forall a, b, \in Q_+$ . Write

identity element of  $(Q_+, *)$

(ix) If  $L = \{1, 2, 3, 4, 5\}$  write zero element in

$(L, I)$ .

(x) A complete disjunctive normal form of n

variable have \_\_\_\_\_ elements.

(xi) A Boolean algebra can not have 3 elements.

(True/False).

(xii) List all atoms of  $B_2^4$ .

#### SECTION - B

##### (Very Short Answer Type Questions)

**Q. 2.** Answer the following questions in two or three

sentences. (Attempt any five) :

**2×5=10**

**P.T.O.**

BH-5315

**(4)**

(i) Define Quantifiers.

(ii) Define semigroup homomorphism.

(iii) Define conjunctive normal form.

(iv) Define OR gate.

(v) Define Regular set.

(vi) Define Atom.

(vii) Define complete lattice.

#### SECTION - C

##### (Short Answer Type Questions)

**Note :** Attempt any five questions.

**4×5=20**

**Q. 3.** (i) Prove that the argument :

$p, p \rightarrow q, p \rightarrow r \vdash r$  is valid

BH-5315

**(5)**

(ii) Let  $(S, *)$  and  $(T, o)$  be semigroups prove

that  $S \times T$  and  $T \times S$  are isomorphic

semigroups.

(iii) Let  $(L, \leq)$  be a lattice then prove that :

$$a \leq b \Leftrightarrow a \wedge b = a, \forall a, b \in L$$

(iv) Prove that in a distributive lattice the

complement of an element is unique.

(v) Prove that the value of complete disjunctive

normal form is one.

**P.T.O.**

BH-5315

**(6)**

(vi) Construct a grammar generating the

language  $L = \{a^i b^n c^n : n \geq 1, i \geq 0\}$ .

(vii) Draw the switching circuit for the switching

function and replace it by simpler one :

$$f(x, y, z) = x \cdot z + [y \cdot (y'z) \cdot (x' + x \cdot z')]$$

#### **SECTION - D**

**(Long Answer Type Questions)**

**Note :** Attempt any three questions.

**3×10=30**

**Q. 4.** (i) State and prove fundamental theorem on

semigroup homomorphism.

(ii) Prove that language  $L = \{a^k b^k : k \geq 1\}$  is not

regular by using pumping lemma.

BH-5315

**(7)**

(iii) Let  $(L, \leq)$  is a lattice as poset then prove that :

$$(a) \quad (a \wedge b) \wedge c = a \wedge (b \wedge c) \quad \forall a, b, c \in L$$

$$(b) \quad (a \vee b) \vee c = a \vee (b \vee c) \quad \forall a, b, c \in L$$

(iv) In Boolean algebra  $(B, t, \cdot, ')$  prove that :

$$a + b = \ell \cup b\{a, b\} \quad \forall a, b \in B$$

—————