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K-1046

M.A./M.Sc. (Previous) Examination, 2022

MATHEMATICS

(Topology)

[Paper : Third]

Time Allowed : Three Hours

Maximum Marks : 100

Minimum Passing Marks : 36

Note : Attempt **any five** questions. All questions carry **equal** marks.

1. (a) Let $\{T_\lambda : \lambda \in \Lambda\}$ where Λ is an arbitrary set, be a collection of topologies for X . Then the intersection $\cap \{T_\lambda : \lambda \in \Lambda\}$ is also a topology for X .
- (b) Let X be a non empty set and $\mathcal{F}^*$ be a family of subset of X such that :

$$[F_1] \phi \in F^*, X \in F^*$$

$$[F_2] F_1, F_2 \in F^* \Rightarrow F_1 \cup F_2 \in F^*$$

$$[F_3] F_\lambda \in F^* \forall \lambda \in \Lambda \Rightarrow \cap \{F_\lambda : \lambda \in \Lambda\} \in F^*.$$

Then there exists a unique topology for X such that T -closed subset of X are precisely the members of F^* .

2. (a) Define the following terms :
 - (i) Base for a topology
 - (ii) Local Base
 - (iii) First countable
 - (iv) Second countable
- (b) If T is cofinite topology on $\mathbb{R}$, then show that $(\mathbb{R}, T)$ is not first countable space.
3. (a) Let X and Y be topological spaces. A mapping $f: X \rightarrow Y$ is continuous iff the inverse image under f of every closed set in Y is closed in X .

- (b) Show that a homeomorphic image of a first countable space is first countable.
- 4. (a) Show that regularity of a topological space is hereditary.
- (b) To show that every compact Hausdorff space is normal.
- 5. (a) Let Y be a subspace of a topological space X and let $A \subset Y$. Then A is compact relative to X iff A is compact relative to Y .
- (b) State and prove Lebesgue Covering Lemma.
- 6. (a) To show that every sequentially compact topological space is countably compact.
- (b) Show that compactness is a topological invariant.
- 7. (a) To show that the set of real numbers with the usual topology is connected.
- (b) To show that continuous image of connected space is connected.
- 8. (a) Let f be a map of a space Y onto a product space $X = \prod \{X_i : i \in I\}$. Then f is

continuous iff the composition map $\pi_i \circ f : Y \rightarrow X_i$ is continuous.

- (b) To show that product space $X = \prod \{X_i : i \in I\}$ is connected iff each coordinate space is connected.

9. State and prove Uryshon's Metrization theorem.

10. (a) Let (X, T) be a topological space and $Y \subset X$. Then a point $x_0 \in X$ is a limit point of Y iff T a net in $Y - \{x_0\}$ converges to $\{x_0\}$.

- (b) To show that every filter is contained in an ultrafilter.

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