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H-5321

M.A./M.Sc. (Semester-II) Examination, 2022

MATHEMATICS

(Advanced Abstract Algebra-II)

Time Allowed : Three Hours

Maximum Marks : 70

Minimum Passing Marks : 25

Note : Section-A is **compulsory** and attempt **any five** questions from Section-B and **any five** questions from Section-C and **any three** questions from Section-D.

SECTION-A

1. Choose the correct answer : [5x2=10]
- (i) A field F is called perfect if each of its algebraic extension is :

- (a) Perfect field
 - (b) Finite field
 - (c) Separable field
 - (d) Not separable field
- (ii) If E is a finite extension of a field F , then $|G(E/F)| \leq \dots\dots\dots$
- (a) $[E : F]$
 - (b) $[F : E]$
 - (c) $[E/F]$
 - (d) None of these
- (iii) The Galois group of $x^4 - 2 \in Q[x]$ is.....group.
- (a) Square
 - (b) Cube
 - (c) Octic
 - (d) None of these

(iv) $\mathbb{Q}(\sqrt[3]{2})$ is radical extension of $\mathbb{Q}$. Also.....is a radical extension of $\mathbb{Q}$.

(a) $\mathbb{Q}(\sqrt[3]{2}, \sqrt[5]{3})$

(b) $\mathbb{Q}(\sqrt[3]{3}, \sqrt[2]{3})$

(c) $\mathbb{Q}(\sqrt[3]{5}, \sqrt[5]{3})$

(d) None of these

(v) A field is called prime if it has no proper_____.

(a) Field

(b) Subfield

(c) Finite field

(d) None of these

SECTION-B

Note : Attempt **any five** questions. Each question carries **02** marks. [5x2=10]

2. (i) Define Artinian ring with example.

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(3)

[P.T.O.]

- (ii) Define primary module with example.
- (iii) Define extension field with example.
- (iv) Define inseparable extension.
- (v) Define splitting field.
- (vi) Define primitive elements.
- (vii) Define finite field with example.

SECTION-C

Note : Attempt **any five** questions. Each question carries **04** marks. [5x4=20]

3. (i) Every homomorphic image of a noetherian module is noetherian.
- (ii) State and prove Hilbert basis theorem.
- (iii) Every finite extension of a field is an algebraic extension.
- (iv) Show that the degree of splitting field of $x^3 - 2 \in \mathbb{Q}[x]$ is 6.

- (v) Prove that every extension of Q is separable.
- (vi) Let H be a finite subgroup of the group of automorphisms of a field E . Then $[E : E_H] = [H]$,

$$\text{where } E_H = \{x \in E \mid \sigma(x) = x \text{ for all } \sigma \in H\}.$$
- (vii) Explain Rational canonical form.

SECTION-D

Note : Attempt **any three** questions. Each question carries **10** marks. [3x10=30]

4.
 - (i) State and prove Wedderburn Artin theorem.
 - (ii) State and prove Hilbert basis theorem.
 - (iii) State and prove fundamental theorem of Galois theory.
 - (iv) Show that the splitting field of $x^4 + 1$ over Q is $Q(\sqrt{2}, i)$ whose degree over Q is 4.

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