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H-5322

**M.A./M.Sc. (Semester-II)
Examination, July, 2022**

MATHEMATICS

(Real Analysis-II)

Time Allowed : Three Hours

Maximum Marks : 70

Minimum Passing Marks : 25

Note : This question paper is divided into four sections. All sections are compulsory. Attempt questions as per instructions given in each section.

SECTION-A

(Objective Type Questions)

Note : Attempt **all** questions. Each question carries 1 mark.

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(1)

[P.T.O.]

1. Choose the correct answer : [1x10=10]

(i) If A_1 and A_2 are measurable subsets of $[a, b]$, then $A_1 \cup A_2$ is :

- (a) Non-measurable set
- (b) Measurable set
- (c) Integrable set
- (d) None of the above

(ii) There exists an uncountable set of measure. The set is called :

- (a) Lebesgue set
- (b) Riemann set
- (c) Cantor set
- (d) Cauchy set

(iii) Every continuous function is :

- (a) Non-measurable function
- (b) Derivable

- (c) Measurable function
 - (d) Integrable
- (iv) The cantor set C :
- (a) is countable
 - (b) is uncountable
 - (c) have measure
 - (d) Both (b) and (c)
- (v) If E is a measurable subset of $[a, b]$, then $\int_E 1 =$
- (a) 1
 - (b) $-m(E)$
 - (c) -1
 - (d) $m(E)$
- (vi) If $E \subset [a, b]$, then the outer measure of E is denoted by :
- (a) $\underline{m}E$
 - (b) $[mE]$

(c) $\|mE\|$

(d) $\bar{m}E$

(vii) If A_1 and A_2 are measurable subset of $[a, b]$, then :

I. $A_1 \cup A_2$ is measurable

II. $A_1 \cap A_2$ is measurable

(a) Only (I) is true

(b) Only (II) is true

(c) Both (I) and (II) are true

(d) Both (I) and (II) are false

(viii) If $f(x) = 1$. If x is an irrational number and $f(x) = 2$ if x is a rational number. Calculate the Lebesgue integral :

(a) ∞

(b) 1

(c) 2

(d) 0

(ix) If f_1 and f_2 are measurable function on $[a, b]$, then $f_1 + f_2$ is _____ on $[a, b]$.

- (a) Non-measurable function
- (b) Measurable function
- (c) Not defined
- (d) None of the above

(x) $L \int_a^b f \, dx$ is called :

- (a) Lebesgue upper integral
- (b) Lebesgue lower integral
- (c) Riemann upper integral
- (d) Riemann lower integral

SECTION-B

(Very Short Answer Type Questions)

Note : Attempt **any five** questions. Each question carries **02** marks. [2x5=10]

2. Answer the following questions in 2-3 sentences :

(i) A set A is μ -measurable, iff A^c is μ -measurable, μ being an outer measure.

(ii) Prove that the intersection of two measurable sets is outer measurable.

(iii) Let f and g be measurable functions defined over a measurable set E , then show that $f + g$ is measurable function over E .

(iv) Define Lebesgue integral.

(v) If A and B are disjoint measurable subsets of $[a, b]$ and if f is a bounded L -integral function on $[a, b]$, then :

$$\int_{A \cup B} f = \int_A f + \int_B f$$

(vi) Prove that $[f(x)]_n$ as defined above is bounded and measurable over $[a, b]$.

(vii) If $0 < q < p < \infty$ and $f \in L^p \cap L^q$ then $f \in L^r$ for all r . Such that $q < r < p$.

SECTION-C

(Short Answer Type Questions)

Note : Attempt **any five** questions. Each question carries **04** marks. [4x5=20]

3. (i) Show that (L^p, d) is a metric space.
- (ii) Prove that $L^\infty(0,1]$ is complete.
- (iii) If $\int_A f \, dx = 0$ for every measurable subset A of a measurable set E . Show that $f(x) = 0$ a.e. on E .
- (iv) If f is bounded real valued measurable function defined on a measurable set E of finite measure s.t. $a \leq f(x) \leq b$ then
- $$a \cdot m(E) \leq \int_E f \leq b \cdot m[E]$$
- (v) If f is a bounded function defined on $[a, b]$ and f is R -integrable on $[a, b]$ then f is also L -integrable on $[a, b]$ and $L \int_a^b f = R \int_a^b f$.
- (vi) Show that a function is simple, iff it is measurable and assumes only a finite number of values.

SECTION-D

(Long Answer Type Questions)

Note : Attempt **any three** questions. Each question carries **10** marks. [3x10=30]

4. (i) Let f and g be bounded functions in $L[a, b]$, then $(f + g) \in^2 (a, b)$ and

$$\int_a^b (f + g) = \int_a^b f + \int_a^b g .$$

- (ii) Let f be a bounded function defined on measurable set E with $m(E) < \infty$, show that

$$\inf_{f \leq \psi} \int_E \psi(x) dx = \sup_{f \geq \phi} \int_E \phi(x) dx$$

for all simple function ϕ and ψ iff f is measurable.

- (iii) Write and prove Riesz-Fischer theorem.
- (iv) Write and prove Lusin theorem.

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