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H-5328

M.A./M.Sc. (Semester-II) Examination, 2022

MATHEMATICS

(Complex Analysis - II)

Time Allowed : Three Hours

Maximum Marks : 70

Minimum Passing Marks : 25

Note : This question paper is divided into four sections.
Attempt questions as per given instructions in each section.

SECTION-A

**(Fill in the blank/Objective Type/
Multiple Choice Questions)**

Note : Attempt **any ten** questions. Each question carries **1** mark. [1x10=10]

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(1)

[P.T.O.]

1. (A) Fill in the blanks :
 - (i) Meromorphic function are those functions which has _____ as its only singularities in the finite part of the plane.
 - (ii) If $f(z)$ is an analytic function having $z = a$ as a pole of order m then $f(z) = \frac{\phi(z)}{(z-a)^m}$ and its residue is given by _____ .
 - (iii) Function elements (f_1, D_1) and (f_2, D_2) are direct analytic continuation of each other iff :
 - (a) $D_1 \cap D_2 \neq \phi$ and $f_1(z) = f_2(z); \forall z \in D_1 \cap D_2$
 - (b) $D_1 \cap D_2 = \phi$ and $f_1(z) = f_2(z); \forall z \in D_1 \cap D_2$
 - (c) $D_1 \cup D_2 \neq \phi$ and $f_1(z) = f_2(z); \forall z \in D_1 \cup D_2$
 - (d) $D_1 \cup D_2 = \phi$ and $f_1(z) = f_2(z); \forall z \in D_1 \cup D_2$
 - (iv) Schwartz's reflection principle states :
 - (a) $\overline{F(z)} = F(\bar{z})$

- (b) $\overline{F(z)} \neq F(\bar{z})$
- (c) $\overline{F(z)} = -F(\bar{z})$
- (d) None of the above
- (v) If $f: G \rightarrow \mathbb{C}$ is an analytic function then $u = \operatorname{Re} f$ and $v = \operatorname{Im} f$ are called harmonic ____.
- (vi) If $f(z)$ be analytic in the closed disc $|z| \leq R$ and let $f(z) \neq 0$ on $|z| = R$. If $z_1, z_2, \dots, z_n$ are the zeros of $f(z)$ in the open disc $|z| < R$ repeated according to their multiplicity and $z = re^{i\theta}$, $0 \leq r < R$, then $\log |f(z)| =$ ____.
- (vii) If F is an entire function of finite order ρ , then F has finite genus $\mu \leq \rho$. This statement is :
- (a) Great Picard theorem
- (b) Hadamard Factorization theorem
- (c) Schottky theorem
- (d) None of the above
- (viii) The order of the function e^{az} , $a \neq 0$ is :
- (a) e
- (b) 1

- (c) 0
- (d) None of the above
- (ix) The order of the function $\sin z$ is :
- (a) e
- (b) 0
- (c) 1
- (d) None of the above
- (x) If $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$, then $\psi(x, y)$ is called :
- (a) Harnack function
- (b) Harmonic function
- (c) Analytic function
- (d) All of the above
- (xi) If $f(z)$ is an entire transcendent function with maximum modules $m(r)$, then $\lim_{r \rightarrow \infty} \frac{\log m(r)}{\log r}$ is:
- (a) $\log r$
- (b) $\log m(r)$

- (c) $-\infty$
- (d) ∞
- (xii) If f is an entire function that omits two values, then f is a constant. This statement is :
 - (a) Little Picard theorem
 - (b) Bloch's theorem
 - (c) Schottky theorem
 - (d) Great Picard theorem

SECTION-B

(Very Short Answer Type Questions)

Note : Attempt **any five** questions. Each question carries **02** marks. Maximum word limit is **25-30** words.

[5x2=10]

- 2. (i) State and prove the Argument Principle.
- (ii) Define :
 - (a) Residue at pole
 - (b) Residue at infinity

- (iii) Given the identity $\sin^2 z + \cos^2 z = 1$ holds for real values of z , prove that it also holds for all complex values of z .
- (iv) Define subharmonic and superharmonic functions.
- (v) State the Hadamard's Three Circle theorem.
- (vi) Define the exponent of convergence for a sequence of non-zero complex numbers.
- (vii) State the Montel-Caratheodory theorem.

SECTION-C

(Short Answer Type Questions)

Note : Attempt **any five** questions. Each question carries **04** marks. Maximum word limit is **250** words. [5x4=20]

- 3.
 - (i) State and prove the Rouché's theorem.
 - (ii) State and prove Cauchy Residue theorem.
 - (iii) Show that analytic continuation of a given function element along a given curve is unique.

- (iv) Explain how it is possible to continue analytically the function $f(z) = 1 + z + z^2 + \dots + z^3 + \dots$ outside the circle of convergence of the power series.
- (v) State and prove the Jensen's formula.
- (vi) Let f be analytic function on a region containing the closure of $B(0; R)$; then show that $f(B(0; R))$ contains a disk of radius $\frac{1}{72} R |f'(0)|$.
- (vii) State and prove the Great Picard theorem.

SECTION-D

(Long Answer Type Questions)

Note : Attempt **any three** questions. Each question carries **10** marks. Maximum word limit is **500** words.

[3x10=30]

4. (i) Prove that $\int_0^\infty \frac{x^a}{1+x^2} dx = \frac{\pi}{2} \sec \frac{\pi a}{2}$

$$(-1 < a < 1).$$

- (ii) State and prove the Schwarz's Reflection Principle.

- (iii) State and prove the Borel's theorem.
- (iv) State and prove the Bloch's theorem.

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