2405

MATHEMATICS

(New/Old Course)

Time : 3 Hours] **[Maximum Marks : 100**

NOTE **:—** The questions in the question paper are based on revised and pre-revised syllabus marked as "**New Course**" and "**Old Course**" respectively and candidates are advised to appear in the relevant course meant for them. Candidates who may attempt the questions partly from "**New Course**" and partly from "**Old Course**" will not be awarded. Candidates are also advised to record "**New Course**" or "**Old Course**" as the case may be, on the front page of the answer-book.

(New Course)

Section–A

(Multiple Choice Questions) 1 each

1. Let $f : R \rightarrow R$ be defined as $f(x) = x^4$, then f is :

   (A) One-one onto (B) Many one onto

   (C) One-one but not onto (D) Neither one-one nor onto

2. Principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is :

(A) $\frac{\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{\pi}{6}$

(D) $\frac{5\pi}{6}$

3. Given a square matrix A, then (A + A') is :

(A) Symmetric matrix

(B) Skew-symmetric matrix

(C) Diagonal matrix

(D) Scalar matrix

4. If l, m, n are direction cosines of a line, then :

(A) $l^2 - m^2 + n^2 = 1$

(B) $l^2 + m^2 - n^2 = 1$

(C) $l^2 + m^2 + n^2 = 1$

(D) $l^2 - m^2 - n^2 = 1$

Section–B

(Very Short Answer Type Questions) 2 each

5. Given a square matrix $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ verify (A + A') is symmetric.

6. Evaluate

$$\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$$

7. Find the slope of the tangent to the curve $y = x^3 - x + 1$ at the point whose x-coordinate is 2.

8. Find the differential equation representing family of lines $y = mx$.

9. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having $\vec{a} \cdot \vec{b} = \sqrt{6}$.

10. What is LPP ? Define objective function.

11. If $P(A) = \frac{7}{13}$; $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, evaluate

$$P\left(\frac{A}{B}\right)$$

12. Given two independent events A and B such that P(A) = 0.3 and P(B) = 0.6, find P(A and B).

Section–C

(Short Answer Type Questions) 4 each

13. If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$, show that :

$$f\circ f(x) = x,\ x \neq \frac{2}{3}$$

What is f^{-1} ?

14. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, $-\frac{3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

15. Find $\frac{1}{2}(A + A')$ and $\frac{1}{2}(A - A')$, where :

$$A = \begin{bmatrix} 0 & 0 & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

16. If $y = 5 \cos x - 3 \sin x$, prove that :

$$\frac{d^2y}{dx^2} + y = 0$$

17. Find the intervals in which the function $f(x) = 2x^2 - 3x$ is strictly increasing and strictly decreasing.

18. Find the approximate value of $(255)^{1/4}$ upto 3 decimal place using differentials.

19. Using properties of definite integrals evaluate :

$$\int_0^{\pi/2} \frac{\cos^5 x}{\sin^5 x + \cos^5 x}\, dx$$

20. Find the general solution of the differential equation :

$$\frac{dy}{dx} = \sqrt{4 - y^2}, \; -2 < y < 2$$

21. Find a unit vector perpendicular to each of vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where :

$$\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$$

and $$\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}.$$

22. Show that the line joining (1, –1, 2) and (3, 4, –2) is perpendicular to the line joining (0, 3, 2) and (3, 5, 6).

23. Maximise :

$$Z = 5x + 3y$$

Subject to constraints :

$$3x + 5y \le 15,$$

$$5x + 2y \le 10$$

$$x \ge 0, y \ge 0$$

Section–D

(Long Answer Type Questions) 6 each

24. Using properties of determinants show that :

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc + bc + ca + ab$$

Or

Solve the system of equations by the matrix method :

$$x - y + z = 4;$$

$$2x + y - 3z = 0;$$

and $$x + y + z = 2$$

25. Find $\frac{dy}{dx}$ of the function $x^y + y^x = 1$

Or

Find $\frac{dy}{dx}$ if $x = a(\theta - \sin\theta)$ and $y = a(1 + \cos\theta)$ by eliminating parameter θ.

26. Using properties of definite integrals evaluate :

$$\int_0^1 x(1-x)^n dx$$

Or

Evaluate :

$$\int_2^3 x^2 dx$$

as the limit of a sum.

27. Find the particular solution of $\frac{dy}{dx} + 2y\tan x = \sin x$, $y = 0$, $x = \frac{\pi}{3}$.

Or

Show that differential equation is homogeneous and solve it :

$$(x^2 + xy)dy = (x^2 + y^2)dx$$

28. Find the shortest distance between the lines :

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$

and

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

Or

Find the equation of the plane through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and the point $(2, 2, 1)$.

29. A bag-I contains 3 red and 4 black balls while another bag-II contains 5 red and 6 black balls. One bag is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from bag-II.

Or

A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of getting two successes, at least two successes and at most two successes.