

AJ-1546

M.A./M.Sc. (Final) Mathematics

Term End Examination, 2021-22

PARTIAL DIFFERENTIAL EQUATION, MECHANICS & GRAVITATION

Time : Three hours]

[Maximum Marks : 100

Note: Answer any five questions. All questions carry equal marks.

1. (a) Solve the partial differential equation $\frac{\partial y}{\partial t} = 2 \frac{\partial^2 y}{\partial x^2}$ where $y(0, t) = 0 = y(5, t)$ and $y(x, 0) = 10 \sin 4\pi$ by using Laplace transform.

- (b) Find the fourier transform of $f(x)$ defined by $F(x) = \int_0^1, |x| < a$ and hence evaluate:

(i) $\int_{-\infty}^{\infty} \frac{\sin pa \cos px}{p} dx$ (ii) $\int_0^{\infty} \frac{\sin p}{p} dp$

2. (a) Find the complete integral of $(p^2 + q^2)x = pz$

- (b) Find the complete integral of the partial differential equation $p^2x + q^2y = z$

3. Derive two dimensional wave equation.

4. Obtain the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ using the method of separation of variables.

5. State and prove strong maximum principle for the heat equation.

6. Derive Hamilton canonical Equation.

7. Define cyclic coordinates and derive Rouths Equation.

8. State and prove Jacobi- poisson theorem.

9. Find the attraction and potential of spherical shell of finite thickness.

10. The values of V at any point at a distance r from a fixed point O are $V = 2\pi\gamma\rho(a^2 - b^2)$ if

$$r < b < a; V = 2\pi\gamma\rho\left(a^2 - \frac{r^2}{3} - \frac{2}{3}\frac{b^2}{r}\right), \text{ if } b < r < a \text{ and } v = \frac{4\pi\gamma\rho}{3} \frac{a^3 - b^3}{r}, \text{ if } b < a < r$$

Show that the attracting system is a spherical shell of density ρ , whose boundaries are spheres of centre O and radii a and b .