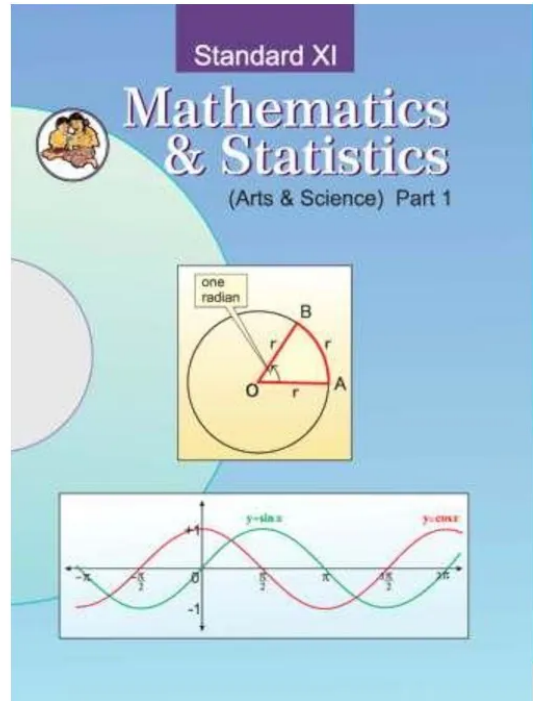


Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 1): Chapter 4- Determinants and Matrices

Class 11 - Chapter 4 Determinants and Matrices



For any clarifications or questions you can write to info@indcareer.com

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Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 1): Chapter 4- Determinants and Matrices

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Ex 4.1

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Question 1.

Find the values of the determinants.

i. $\begin{vmatrix} 2 & -4 \\ 7 & -15 \end{vmatrix}$

ii. $\begin{vmatrix} 2i & 3 \\ 4 & -i \end{vmatrix}$

iii. $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

iv. $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$

Solution:

i. $\begin{vmatrix} 2 & -4 \\ 7 & -15 \end{vmatrix}$

$$= 2(-15) - (-4)(7)$$

$$= -30 + 28$$

$$= -2$$

$$\begin{aligned}\text{ii. } & \begin{vmatrix} 2i & 3 \\ 4 & -i \end{vmatrix} \\ &= 2i(-i) - 3(4) \\ &= -2i^2 - 12 \\ &= -2(-1) - 12 \dots [\because i^2 = -1] \\ &= 2 - 12 \\ &= -10\end{aligned}$$

$$\begin{aligned}\text{iii. } & \begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix} \\ &= 3 \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} - (-4) \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 5 \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} \\ &= 3(1 + 6) + 4(1 + 4) + 5(3 - 2) \\ &= 3(7) + 4(5) + 5(1) \\ &= 21 + 20 + 5 \\ &= 46\end{aligned}$$

$$\begin{aligned}
 \text{iv. } \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} &= a \begin{vmatrix} b & f \\ f & c \end{vmatrix} - h \begin{vmatrix} h & f \\ g & c \end{vmatrix} + g \begin{vmatrix} h & b \\ g & f \end{vmatrix} \\
 &= a(bc - f^2) - h(hc - gf) + g(hf - gb) \\
 &= abc - af^2 - h^2c + fgh + fgh - g^2b \\
 &= abc + 2fgh - af^2 - bg^2 - ch^2 \\
 &= (-15) - (-4)(7) \\
 &= -30 + 28 \\
 &= -2
 \end{aligned}$$

Question 2.

Find the values of x, if

$$\begin{aligned}
 \text{i. } \begin{vmatrix} x^2 - x + 1 & x + 1 \\ x + 1 & x + 1 \end{vmatrix} &= 0 \\
 \text{ii. } \begin{vmatrix} x & -1 & 2 \\ 2x & 1 & -3 \\ 3 & -4 & 5 \end{vmatrix} &= 29
 \end{aligned}$$

Solution:

$$\begin{aligned}
 \text{i. } \begin{vmatrix} x^2 - x + 1 & x + 1 \\ x + 1 & x + 1 \end{vmatrix} &= 0 \\
 \therefore (x^2 - x + 1)(x + 1) - (x + 1)(x + 1) &= 0 \\
 \therefore (x + 1)[x^2 - x + 1 - (x + 1)] &= 0 \\
 \therefore (x + 1)(x^2 - x + 1 - x - 1) &= 0 \\
 \therefore (x + 1)(x^2 - 2x) &= 0
 \end{aligned}$$

$$\therefore (x+1)x(x-2) = 0$$

$$\therefore x = 0 \text{ or } x+1 = 0 \text{ or } x-2 = 0$$

$$\therefore x = 0 \text{ or } x = -1 \text{ or } x = 2$$

$$\text{ii. } \begin{vmatrix} x & -1 & 2 \\ 2x & 1 & -3 \\ 3 & -4 & 5 \end{vmatrix} = 29 = 29$$

$$\therefore x \begin{vmatrix} 1 & -3 \\ -4 & 5 \end{vmatrix} - (-1) \begin{vmatrix} 2x & -3 \\ 3 & 5 \end{vmatrix} + 2 \begin{vmatrix} 2x & 1 \\ 3 & -4 \end{vmatrix} = 29$$

$$x(5-12) + 1(10x+9) + 2(-8x-3) = 29$$

$$\therefore -7x + 10x + 9 - 16x - 6 = 29$$

$$\therefore -13x + 3 = 29$$

$$\therefore -13x = 26$$

$$\therefore x = -2$$

Question 3.

Find x and y if $\begin{vmatrix} 4i & i^3 & 2i \\ 1 & 3i^2 & 4 \\ 5 & -3 & i \end{vmatrix} = x + iy$, where $i^2 = -1$

Solution:

$$\begin{vmatrix} 4i & i^3 & 2i \\ 1 & 3i^2 & 4 \\ 5 & -3 & i \end{vmatrix}$$

$$= \begin{vmatrix} 4i & -i & 2i \\ 1 & -3 & 4 \\ 5 & -3 & i \end{vmatrix} \quad \dots [\because i^2 = -1]$$

$$= 4i \begin{vmatrix} -3 & 4 \\ -3 & i \end{vmatrix} - (-i) \begin{vmatrix} 1 & 4 \\ 5 & i \end{vmatrix} + 2i \begin{vmatrix} 1 & -3 \\ 5 & -3 \end{vmatrix}$$

$$= 4i(-3i + 12) + i(i - 20) + 2i(-3 + 15)$$

$$= 12i^2 + 48i + i^2 - 20i + 24i$$

$$= -11i^2 + 52i$$

$$= -11(-1) + 52i \dots [\because i^2 = -1]$$

$$= 11 + 52i$$

Comparing with $x + iy$, we get $x = 11$, $y = 52$

Question 4.

Find the minors and cofactors of elements of the determinant $D = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 2 & -1 \\ 5 & 7 & 2 \end{vmatrix}$

Soution:

Here, $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 2 & -1 \\ 5 & 7 & 2 \end{vmatrix}$

$$M_{11} = \begin{vmatrix} 2 & -1 \\ 7 & 2 \end{vmatrix} = 4 + 7 = 11$$

$$C_{11} = (-1)^{1+1} M_{11} = (1)(11) = 11$$

$$M_{12} = \begin{vmatrix} 1 & -1 \\ 5 & 2 \end{vmatrix} = 2 + 5 = 7$$

$$C_{12} = (-1)^{1+2} M_{12} = (-1)(7) = -7$$

$$M_{13} = \begin{vmatrix} 1 & 2 \\ 5 & 7 \end{vmatrix} = 7 - 10 = -3$$

$$C_{13} = (-1)^{1+3} M_{13} = (1)(-3) = -3$$

$$M_{21} = \begin{vmatrix} -1 & 3 \\ 7 & 2 \end{vmatrix} = -2 - 21 = -23$$

$$C_{21} = (-1)^{2+1} M_{21} = (-1)(-23) = 23$$

$$M_{22} = \begin{vmatrix} 2 & 3 \\ 5 & 2 \end{vmatrix} = 4 - 15 = -11$$

$$C_{22} = (-1)^{2+2} M_{22} = (1)(-11) = -11$$

$$M_{23} = \begin{vmatrix} 2 & -1 \\ 5 & 7 \end{vmatrix} = 14 + 5 = 19$$

$$C_{23} = (-1)^{2+3} M_{23} = (-1)(19) = -19$$

$$M_{31} = \begin{vmatrix} -1 & 3 \\ 1 & -1 \end{vmatrix} = 1 - 3 = -2$$

$$C_{31} = (-1)^{3+1} M_{31} = (1)(-2) = -2$$

$$M_{32} = \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = -2 - 3 = -5$$

$$C_{32} = (-1)^{3+2} M_{32} = (-1)(-5) = 5$$

$$M_{33} = \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 4 + 1 = 5$$

$$C_{33} = (-1)^{3+3} M_{33} = (1)(5) = 5$$

Question 5.

Evaluate $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ and cofactors of elements in the 2nd determinant and verify:

i. $-a_{21} \cdot M_{21} + a_{22} \cdot M_{22} - a_{23} \cdot M_{23}$ = value of A $a_{21} \cdot C_{21} + a_{22} \cdot C_{22} + a_{23} \cdot C_{23}$ — value of A where M_{21} , M_{22} , M_{23} are minors of a_{21} , a_{22} , a_{23} and C_{21} , C_{22} , C_{23} are cofactors of a_{21} , a_{22} , a_{23} .

Solution:

$$A = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 0 & 4 \\ 5 & -7 \end{vmatrix} - (-3) \begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix} + 5 \begin{vmatrix} 6 & 0 \\ 1 & 5 \end{vmatrix}$$

$$= 2(0 - 20) + 3(-42 - 4) + 5(30 - 0) = 2(-20) + 3(-46) + 5(30)$$

$$= 2(0 - 20) + 3(-42 - 4) + 5(30 - 0) = 2(-20) + 3(-46) + 5(30)$$

$$= -40 - 138 + 150 = -28$$

$$\text{Here, } \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

$$M_{21} = \begin{vmatrix} -3 & 5 \\ 5 & -7 \end{vmatrix} = 21 - 25 = -4$$

$$C_{21} = (-1)^{2+1} M_{21} = (-1)(-4) = 4$$

$$M_{22} = \begin{vmatrix} 2 & 5 \\ 1 & -7 \end{vmatrix} = -14 - 5 = -19$$

$$C_{22} = (-1)^{2+2} M_{22} = (1)(-19) = -19$$

$$M_{23} = \begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix} = 10 + 3 = 13$$

$$C_{23} = (-1)^{2+3} M_{23} = (-1)(13) = -13$$

$$- a_{21} \cdot M_{21} + a_{22} \cdot M_{22} - a_{23} \cdot M_{23}$$

$$= - (6)(-4) + (0)(-19) - (4)(13)$$

$$= 24 + 0 - 52$$

$$= -28$$

$$- a_{21} \cdot M_{21} + a_{22} \cdot M_{22} - a_{23} \cdot M_{23} = \text{value of A}$$

$$\text{ii. } a_{21} \cdot C_{21} + a_{22} \cdot C_{22} + a_{23} \cdot C_{23}$$

$$= (6)(4) + (0)(-19) + (4)(-13)$$

$$= 24 + 0 - 52$$

$$= -28$$

$$a_{21} \cdot C_{21} + a_{22} \cdot C_{22} + a_{23} \cdot C_{23} = \text{value of A}$$

Question 6.

Find the value of determinant expanding along third column $\begin{vmatrix} -1 & 1 & 2 \\ -2 & 3 & -4 \\ -3 & 4 & 0 \end{vmatrix}$

Solution:

$$\text{Here, } \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} -1 & 1 & 2 \\ -2 & 3 & -4 \\ -3 & 4 & 0 \end{vmatrix}$$

Expansion along the third column

$$= a_{13}C_{13} + a_{23}C_{23} + a_{33}C_{33}$$

$$= 2 \times (-1)^{1+3} \begin{vmatrix} -2 & 3 \\ -3 & 4 \end{vmatrix} - 4 \times (-1)^{2+3} \begin{vmatrix} -1 & 1 \\ -3 & 4 \end{vmatrix} + 0 \times (-1)^{3+3} \begin{vmatrix} -1 & 1 \\ -2 & 3 \end{vmatrix}$$

$$= 2(-8 + 9) + 4(-4 + 3) + 0$$

$$= 2 - 4$$

$$= -2$$

Ex 4.2

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Question 1.

Without expanding, evaluate the following determinants.

i.
$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$$

ii.
$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$$

iii.
$$\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$$

Solution:

i. Let $D = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

Applying $C_3 \rightarrow C_3 + C_2$, we get .

$$D = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$$

Taking $(a+b+c)$ common from C_3 , we get

$$D = (a+b+c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$$

$$= (a+b+c)(0)$$

... [$\because C_1$ and C_3 are identical]

$$= 0$$

$$\text{ii. } \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$$

Taking $(3x)$ common from R_3 , we get

$$D = 3x \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix}$$

$$= (3x)(0) = 0$$

... [$\because R_1$ and R_3 are identical]

$$= 0$$

$$\text{iii. Let } D = \begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$$

Applying $Cx_3 \rightarrow C_3 - 9C_2$, we get

$$D = \begin{vmatrix} 2 & 7 & 2 \\ 3 & 8 & 3 \\ 5 & 9 & 5 \end{vmatrix}$$

$$= 0 \dots [\because C_1 \text{ and } C_3 \text{ are identical}]$$

Question 2.

Prove that
$$\begin{vmatrix} x+y & y+z & z+x \\ z+x & x+y & y+z \\ y+z & z+x & x+y \end{vmatrix} = 2 \begin{vmatrix} x & y & z \\ z & x & y \\ y & z & x \end{vmatrix}$$

Solution:

$$\text{L.H.S.} = \begin{vmatrix} x+y & y+z & z+x \\ z+x & x+y & y+z \\ y+z & z+x & x+y \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$, we get

L.H.S.

$$= \begin{vmatrix} 2(x+y+z) & 2(x+y+z) & 2(x+y+z) \\ z+x & x+y & y+z \\ y+z & z+x & x+y \end{vmatrix}$$

Taking 2 common from R_1 , we get

$$\text{L.H.S.} = 2 \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z+x & x+y & y+z \\ y+z & z+x & x+y \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 - R_3$, we get

$$\text{L.H.S.} = 2 \begin{vmatrix} x & y & z \\ z+x & x+y & y+z \\ y+z & z+x & x+y \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$, we get

$$\text{L.H.S.} = 2 \begin{vmatrix} x & y & z \\ z & x & y \\ y+z & z+x & x+y \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - R_2$, we get

$$\text{L.H.S.} = 2 \begin{vmatrix} x & y & z \\ z & x & y \\ y & z & x \end{vmatrix} = \text{R.H.S.}$$

Question 3.

Using properties of determinant, show that

$$\begin{aligned} \text{i. } & \begin{vmatrix} a+b & a & b \\ a & a+c & c \\ b & c & b+c \end{vmatrix} = 4abc \\ \text{ii. } & \begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_x y & 1 \end{vmatrix} = 0 \end{aligned}$$

Solution:

i. L.H.S. =

Applying $C_1 \rightarrow C_1 - (C_2 + C_3)$, we get

$$\text{L.H.S.} = \begin{vmatrix} 0 & a & b \\ -2c & a+c & c \\ -2c & c & b+c \end{vmatrix}$$

Taking (-2) common from C_1 , we get

$$\text{L.H.S.} = -2 \begin{vmatrix} 0 & a & b \\ c & a+c & c \\ c & c & b+c \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we get

$$\text{L.H.S.} = -2 \begin{vmatrix} 0 & a & b \\ c & a & 0 \\ c & 0 & b \end{vmatrix}$$

$$= -2[0(ab - 0) - a(bc - 0) + b(0 - ac)]$$

$$= -2(0 - abc - abc)$$

$$= -2(-2abc)$$

$$= 4abc = \text{R.H.S.}$$

ii.

$$\begin{aligned} \text{L.H.S.} &= \begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix} \\ &= \begin{vmatrix} \frac{\log_e x}{\log_e x} & \frac{\log_e y}{\log_e x} & \frac{\log_e z}{\log_e x} \\ \frac{\log_e x}{\log_e y} & \frac{\log_e y}{\log_e y} & \frac{\log_e z}{\log_e y} \\ \frac{\log_e x}{\log_e z} & \frac{\log_e y}{\log_e z} & \frac{\log_e z}{\log_e z} \end{vmatrix} \\ &\quad \dots \left[\because \log_e b = \frac{\log_e b}{\log_e c} \right] \end{aligned}$$

Taking $\frac{1}{\log_e x}$, $\frac{1}{\log_e y}$, $\frac{1}{\log_e z}$ common from R_1 ,

R_2 , R_3 respectively, we get

L.H.S.

$$\begin{aligned} &= \frac{1}{\log_e x \cdot \log_e y \cdot \log_e z} \begin{vmatrix} \log_e x & \log_e y & \log_e z \\ \log_e x & \log_e y & \log_e z \\ \log_e x & \log_e y & \log_e z \end{vmatrix} \\ &= \frac{1}{\log_e x \cdot \log_e y \cdot \log_e z} (0) \\ &\quad \dots [\because R_1, R_2, R_3 \text{ are identical}] \\ &= 0 = \text{R.H.S.} \end{aligned}$$

Question 4.

Solve the following equations.

i.
$$\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$$

ii. *Solution : i.*
$$\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we get

$$\begin{vmatrix} x+2 & x+6 & x-1 \\ 4 & -7 & 3 \\ -3 & -4 & 7 \end{vmatrix} = 0$$

$$\therefore (x+2)(-49+12) - (x+6)(28+9) + (x-1)(-16-21) = 0$$

$$\therefore (x+2)(-37) - (x+6)(37) + (x-1)(-37) = 0$$

$$\therefore -37(x+2+x+6+x-1) = 0$$

$$\therefore 3x+7 = 0$$

$$\therefore x = \frac{-7}{3}$$

ii.
$$\begin{vmatrix} x-1 & x & x-2 \\ 0 & x-2 & x-3 \\ 0 & 0 & x-3 \end{vmatrix} = 0$$

Applying $R_2 \rightarrow R_2 - R_3$, we get

$$\begin{vmatrix} x-1 & x & x-2 \\ 0 & x-2 & 0 \\ 0 & 0 & x-3 \end{vmatrix} = 0$$

$$\therefore (x-1)(x-2)(x-3) - x(0-0) + (x-2)(0-0) =$$

$$\therefore (x-1)(x-2)(x-3) = 0$$

$$\therefore x-1 = 0 \text{ or } x-2 = 0 \text{ or } x-3 = 0$$

$$\therefore x = 1 \text{ or } x = 2 \text{ or } x = 3$$

Question 5.

If $\begin{vmatrix} 4+x & 4-x & 4-x \\ 4-x & 4+x & 4-x \\ 4-x & 4-x & 4+x \end{vmatrix} = 0$, then find the values of x .

Solution:

$$\begin{vmatrix} 4+x & 4-x & 4-x \\ 4-x & 4+x & 4-x \\ 4-x & 4-x & 4+x \end{vmatrix} = 0$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} 12-x & 4-x & 4-x \\ 12-x & 4+x & 4-x \\ 12-x & 4-x & 4+x \end{vmatrix} = 0$$

Taking $(12-x)$ common from C_1 , we get

$$(12-x) \begin{vmatrix} 1 & 4-x & 4-x \\ 1 & 4+x & 4-x \\ 1 & 4-x & 4+x \end{vmatrix} = 0$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we get

$$(12-x) \begin{vmatrix} 1 & 4-x & 4-x \\ 0 & 2x & 0 \\ 0 & 0 & 2x \end{vmatrix} = 0$$

$$(12-x)[1(4x^2-0) - (4-x)(0-0) + (4-x)(0-0)] = 0$$

$$\therefore (12-x)(4x^2) = 0$$

$$\therefore x^2(12-x) = 0$$

$$\therefore x = 0 \text{ or } 12-x = 0$$

$$\therefore x = 0 \text{ or } x = 12$$

Question 6.

Without expanding determinant, show that

$$\begin{vmatrix} 1 & 3 & 6 \\ 6 & 1 & 4 \\ 3 & 7 & 12 \end{vmatrix} + 4 \begin{vmatrix} 2 & 3 & 3 \\ 2 & 1 & 2 \\ 1 & 7 & 6 \end{vmatrix} = 10 \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 7 \\ 3 & 2 & 6 \end{vmatrix}$$

Solution:

$$\text{L.H.S.} = \begin{vmatrix} 1 & 3 & 6 \\ 6 & 1 & 4 \\ 3 & 7 & 12 \end{vmatrix} + 4 \begin{vmatrix} 2 & 3 & 3 \\ 2 & 1 & 2 \\ 1 & 7 & 6 \end{vmatrix}$$

In 1st determinant, taking 2 common from C_3 we get

$$\text{L.H.S.} = \begin{vmatrix} 1 & 3 & 6 \\ 6 & 1 & 4 \\ 3 & 7 & 12 \end{vmatrix} + 4 \begin{vmatrix} 2 & 3 & 3 \\ 2 & 1 & 2 \\ 1 & 7 & 6 \end{vmatrix}$$

In 1st determinant, taking 2 common from C_3 , we get

$$\begin{aligned} \text{L.H.S.} &= 2 \begin{vmatrix} 1 & 3 & 3 \\ 6 & 1 & 2 \\ 3 & 7 & 6 \end{vmatrix} + 4 \begin{vmatrix} 2 & 3 & 3 \\ 2 & 1 & 2 \\ 1 & 7 & 6 \end{vmatrix} \\ &= \begin{vmatrix} 2 & 3 & 3 \\ 12 & 1 & 2 \\ 6 & 7 & 6 \end{vmatrix} + \begin{vmatrix} 8 & 3 & 3 \\ 8 & 1 & 2 \\ 4 & 7 & 6 \end{vmatrix} \\ &= \begin{vmatrix} 2+8 & 3 & 3 \\ 12+8 & 1 & 2 \\ 6+4 & 7 & 6 \end{vmatrix} \end{aligned}$$

$$= \begin{vmatrix} 10 & 3 & 3 \\ 20 & 1 & 2 \\ 10 & 7 & 6 \end{vmatrix}$$

Interchanging rows and columns, we get

$$\text{L.H.S.} = \begin{vmatrix} 10 & 20 & 10 \\ 3 & 1 & 7 \\ 3 & 2 & 6 \end{vmatrix}$$

Taking 10 common from R_1 , we get

$$\text{L.H.S.} = 10 \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 7 \\ 3 & 2 & 6 \end{vmatrix} = \text{R.H.S.}$$

Ex 4.3

Question 1.

Solve the following linear equations by using Cramer's Rule.

$$x+y+z=6, x-y+z=2, x+2y-z=2$$

$$x+y-2z=-10,$$

$$2x+y-3z=-19, 4x+6y+z=2$$

$$x+z=1, y+z=1, x+y=4$$

$$\frac{-2}{x} - \frac{1}{y} - \frac{3}{z} = 3, \frac{2}{x} - \frac{3}{y} + \frac{1}{z} = -13 \text{ and } \frac{2}{x} - \frac{3}{z} = -11$$

Solution:

Given equations are

$$x+y+z=6,$$

$$x-y+z=2,$$

$$x+2y-z=2.$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix}$$

$$1 \cdot 2 \cdot (-1) = 1(1 \cdot (-2)) - 1(-1 \cdot (-1)) + 1(2 \cdot 1)$$

$$= 1(-1) - 1(-2) + 1(3)$$

$$= -1 + 2 + 3$$

$$= 4 \neq 0$$

$$D_x = \begin{vmatrix} 6 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 2 & -1 \end{vmatrix}$$

$$= 6(1 \cdot (-2)) - 1(-2 \cdot (-2)) + 1(4 \cdot 2)$$

$$= 6(-1) - 1(-4) + 1(6)$$

$$= -6 + 4 + 6$$

$$= 4$$

$$D_y = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & -1 \end{vmatrix}$$

$$= 1(-2 \cdot (-2)) - 6(-1 \cdot (-1)) + 1(2 \cdot (-4)) = 1(-4) - 6(-2) + 1(0)$$

$$= -4 + 12 + 0 = 8$$

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$$\begin{aligned} D_z &= \begin{vmatrix} 1 & 1 & 6 \\ 1 & -1 & 2 \\ 1 & 2 & 2 \end{vmatrix} \\ &= 1(-2-4) - 1(2-2) + 6(2+1) \\ &= 1(-6) - 1(0) + 6(3) \\ &= -6 + 0 + 18 = 12 \end{aligned}$$

By Cramer's Rule,

$$\begin{aligned} x &= \frac{D_x}{D} = \frac{4}{4} = 1, \quad y = \frac{D_y}{D} = \frac{8}{4} = 2 \quad \text{and} \\ z &= \frac{D_z}{D} = \frac{12}{4} = 3 \end{aligned}$$

$\therefore x = 1, y = 2$ and $z = 3$ are the solutions of the given equations.

ii. Given equations are $x+y-2z = -10$,

$$2x + y - 3z = -19,$$

$$4x + 6y + z = 2.$$

$$\begin{aligned} D &= \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 4 & 6 & 1 \end{vmatrix} \\ &= 1(1+18) - 1(2+12) - 2(12-4) \\ &= 1(19) - 1(14) - 2(8) \\ &= 19 - 14 - 16 = -11 \neq 0 \end{aligned}$$

$$D_x = \begin{vmatrix} -10 & 1 & -2 \\ -19 & 1 & -3 \\ 2 & 6 & 1 \end{vmatrix}$$
$$= -10(1 + 18) - 1(-19 + 6) - 2(-114 - 2)$$
$$= -10(19) - 1(-13) - 2(-116)$$
$$= -190 + 13 + 232 = 55$$

$$D_y = \begin{vmatrix} 1 & -10 & -2 \\ 2 & -19 & -3 \\ 4 & 2 & 1 \end{vmatrix}$$
$$= 1(-19 + 6) - (-10)(2 + 12) - 2(4 + 76) = 1(-13) + 10(14) - 2(80)$$
$$= -13 + 140 - 160 = -33$$

$$D_z = \begin{vmatrix} 1 & 1 & -10 \\ 2 & 1 & -19 \\ 4 & 6 & 2 \end{vmatrix}$$
$$= 1(2 + 114) - 1(4 + 76) - 10(12 - 4)$$
$$= 1(116) - 1(80) - 10(8)$$
$$= 116 - 80 - 80$$
$$= -44$$

By Cramer's Rule,

$$x = \frac{D_x}{D} = \frac{55}{-11} = -5, \quad y = \frac{D_y}{D} = \frac{-33}{-11} = 3,$$

$$z = \frac{D_z}{D} = \frac{-44}{-11} = 4$$

$\therefore x = -5, y = 3$ and $z = 4$ are the solutions of the given equations.

[Note: The question has been modified]

iii. Given equations are

$$x + z = 1, \text{ i.e., } x + 0y + z = 1,$$

$$y + z = 1, \text{ i.e., } 0x + y + z = 1,$$

$$x + y = 4, \text{ i.e., } x + y + 0z = 4.$$

$$D = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 1(0 - 1) - 0 + 1(0 - 1)$$

$$= 1(-1) + 1(-1)$$

$$= -1 - 1 = -2 \neq 0$$

$$D_x = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 1(0 - 1) - 0 + 1(1 - 4) = 1(-1) + 1(-3)$$

$$= -1 - 3$$

$$= -4$$

$$D_y = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 4 & 0 \end{vmatrix}$$

$$= 1(0 - 4) - 1(0 - 1) + 1(0 - 1)$$

$$= 1(-4) - 1(-1) + 1(-1)$$

$$= -4 + 1 - 1$$

$$= -4$$

$$\begin{aligned} D_z &= \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 4 \end{vmatrix} \\ &= 1(4 - 1) - 0 + 1(0 - 1) \\ &= 1(3) + 1(-1) \\ &= 3 - 1 \\ &= 2 \end{aligned}$$

By Cramer's Rule,

$$\begin{aligned} x &= \frac{D_x}{D} = \frac{-4}{-2} = 2, y = \frac{D_y}{D} = \frac{-4}{-2} = 2, \\ z &= \frac{D_z}{D} = \frac{2}{-2} = -1 \end{aligned}$$

$\therefore x = 2, y = 2$ and $z = -1$ are the solutions of the given equations.

$$\text{Let } \frac{1}{x} = p, \frac{1}{y} = q, \frac{1}{z} = r$$

$\therefore$ The given equations become

$$-2p - q - 3r = 3, \text{ i.e., } 2p + q + 3r = -3,$$

$$2p - 3q + r = -13,$$

$$2p - 3r = -11, \text{ i.e., } 2p + 0q - 3r = -11.$$

$$\begin{aligned} D &= \begin{vmatrix} 2 & 1 & 3 \\ 2 & -3 & 1 \\ 2 & 0 & -3 \end{vmatrix} \\ &= 2(9 - 0) - 1(-6 - 2) + 3(0 + 6) \\ &= 2(9) - 1(-8) + 3(6) \\ &= 18 + 8 + 18 \\ &= 44 \neq 0 \end{aligned}$$

$$\begin{aligned} D_p &= \begin{vmatrix} -3 & 1 & 3 \\ -13 & -3 & 1 \\ -11 & 0 & -3 \end{vmatrix} \\ &= -3(9 - 0) - 1(39 + 11) + 3(0 - 33) \\ &= -3(9) - 1(50) + 3(-33) \\ &= -27 - 50 - 99 \\ &= -176 \end{aligned}$$

$$\begin{aligned} D_q &= \begin{vmatrix} 2 & -3 & 3 \\ 2 & -13 & 1 \\ 2 & -11 & -3 \end{vmatrix} \\ &= 2(39 + 11) - (-3)(-6 - 2) + 3(-22 + 26) \\ &= 2(50) + 3(-8) + 3(4) \\ &= 100 - 24 + 12 \\ &= 88 \end{aligned}$$

$$\begin{aligned} D_r &= \begin{vmatrix} 2 & 1 & -3 \\ 2 & -3 & -13 \\ 2 & 0 & -11 \end{vmatrix} \\ &= 2(33 - 0) - 1(-22 + 26) - 3(0 + 6) \\ &= 2(33) - 1(4) - 3(6) \\ &= 66 - 4 - 18 \\ &= 44 \end{aligned}$$

By Cramer's Rule,

$$\begin{aligned} p &= \frac{D_p}{D} = \frac{-176}{44} = -4, \quad q = \frac{D_q}{D} = \frac{88}{44} = 2, \\ r &= \frac{D_r}{D} = \frac{44}{44} = 1 \\ \therefore \quad \frac{1}{x} &= -4, \quad \frac{1}{y} = 2, \quad \frac{1}{z} = 1 \end{aligned}$$

$\therefore x = -\frac{1}{4}, y = \frac{1}{2}, z = 1$ are the solutions of the given equations.

Question 2.

The sum of three numbers is 15. If the second number is subtracted from the sum of first and third numbers, then we get 5. When the third number is subtracted from the sum of twice the first number and the second number, we get 4. Find the three numbers.

Solution:

Let the three numbers be x , y and z .

According to the given conditions, $x + y + z = 15$,

$x + z - y = 5$, i.e., $x - y + z = 5$,

$2x + y - z = 4$.

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$
$$= 1(1 - 1) - 1(-1 - 2) + 1(1 + 2)$$
$$= 1(0) - 1(-3) + 1(3)$$
$$= 0 + 3 + 3$$
$$= 6 \neq 0$$

$$D_x = \begin{vmatrix} 15 & 1 & 1 \\ 5 & -1 & 1 \\ 4 & 1 & -1 \end{vmatrix}$$
$$= 15(1 - 1) - 1(-5 - 4) + 1(5 + 4)$$
$$= 15(0) - 1(-9) + 1(9)$$
$$= 0 + 9 + 9$$
$$= 18$$

$$D_y = \begin{vmatrix} 1 & 15 & 1 \\ 1 & 5 & 1 \\ 2 & 4 & -1 \end{vmatrix}$$
$$= 1(-5 - 4) - 15(-1 - 2) + 1(4 - 10)$$
$$= 1(-9) - 15(-3) + 1(-6)$$
$$= -9 + 45 - 6 = 30$$

$$D_z = \begin{vmatrix} 1 & 1 & 15 \\ 1 & -1 & 5 \\ 2 & 1 & 4 \end{vmatrix}$$
$$= 1(-4 - 5) - 1(4 - 10) + 15(1 + 2)$$
$$= 1(-9) - 1(-6) + 15(3)$$
$$= -9 + 6 + 45$$
$$= 42$$

By Cramer's Rule,

$$x = \frac{D_x}{D} = \frac{18}{6} = 3, y = \frac{D_y}{D} = \frac{30}{6} = 5,$$

$$z = \frac{D_z}{D} = \frac{42}{6} = 7$$

∴ The three numbers are 3, 5 and 7.

Question 3.

Examine the consistency of the following equations.

i. $2x - y + 3 = 0$, $3x + y - 2 = 0$, $11x + 2y - 3 = 0$

ii. $2x + 3y - 4 = 0$, $x + 2y = 3$, $3x + 4y + 5 = 0$

iii. $x + 2y - 3 = 0$, $7x + 4y - 11 = 0$, $2x + 4y - 6 = 0$

Solution:

i. Given equations are $2x - y + 3 = 0$,

$3x + y - 2 = 0$,

$11x + 2y - 3 = 0$.

$$D = \begin{vmatrix} 2 & -1 & 3 \\ 3 & 1 & -2 \\ 11 & 2 & -3 \end{vmatrix}$$

$$\begin{aligned} &= 2(-3 + 4) - (-1)(-9 + 22) + 3(6 - 11) \\ &= 2(1) + 1(13) + 3(-5) \\ &= 2 + 13 - 15 = 0 \end{aligned}$$

∴ The given equations are consistent.

ii. Given equations are $2x + 3y - 4 = 0$,

$$x + 2y = 3, \text{ i.e., } x + 2y - 3 = 0,$$

$$3x + 4y + 5 = 0.$$

$$\begin{vmatrix} 2 & 3 & -4 \\ 1 & 2 & -3 \\ 3 & 4 & 5 \end{vmatrix}$$

$$= 2(10 + 12) - 3(5 + 9) - 4(4 - 6)$$

$$= 2(22) - 3(14) - 4(-2)$$

$$= 44 - 42 + 8$$

$$= 10 \neq 0$$

∴ The given equations are not consistent.

iii. Given equations are $x + 2y - 3 =$

$$7x + 4y - 11 = 0,$$

$$2x + 4y - 6 = 0.$$

$$\begin{vmatrix} 1 & 2 & -3 \\ 7 & 4 & -11 \\ 2 & 4 & -6 \end{vmatrix}$$

$$= 1(-24 + 44) - 2(-42 + 22) - 3(28 - 8)$$

$$= 1(20) - 2(-20) - 3(20)$$

$$= 20 + 40 - 60$$

$$= 0$$

∴ The given equations are consistent.

Question 4.

Find k , if the following equations are consistent.

i. $2x + 3y - 2 = 0, 2x + 4y - k = 0, x - 2y + 3k = 0$

ii. $kx + 3y + 1 = 0, x + 2y + 1 = 0, x + y = 0$

Solution:

i. Given equations are $2x + 3y - 2 = 0,$

$$2x + 4y - k = 0,$$

$$x - 2y + 3k = 0.$$

Since these equations are consistent,

$$\begin{vmatrix} 2 & 3 & -2 \\ 2 & 4 & -k \\ 1 & -2 & 3k \end{vmatrix} = 0$$

$$\therefore 2(12k - 2k) - 3(6k + k) - 2(-4 - 4) = 0$$

$$\therefore 2(10k) - 3(7k) - 2(-8) = 0$$

$$\therefore 20k - 21k + 16 = 0$$

$$\therefore k = 16$$

Given equations are

$$kx + 3y + 1 = 0,$$

$$x + 2y + 1 = 0,$$

$$x + y = 0, \text{ i.e., } x + y + 0 = 0.$$

Since these equations are consistent,

$$\begin{vmatrix} k & 3 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 0$$

$$\therefore k(0 - 1) - 3(0 - 1) + 1(1 - 2) = 0$$

$$\therefore k(-1) - 3(-1) + 1(-1) = 0$$

$$\therefore -k + 3 - 1 = 0$$

$$\therefore k = 2.$$

Question 5.

Find the area of triangle whose vertices are

i. A (5,8), B (5,0), C (1,0)

ii. P(3/2, 1), Q(4,2), R(4, -1/2)

iii. M (0, 5), N (-2, 3), T (1, -4)

Solution:

i. Here, $A(x_1, y_1) \equiv A(5, 8)$, $B(x_2, y_2) \equiv B(5, 0)$, $C(x_3, y_3) \equiv C(1, 0)$

$$\text{Area of a triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\begin{aligned} A(\Delta ABC) &= \frac{1}{2} \begin{vmatrix} 5 & 8 & 1 \\ 5 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix} \\ &= \frac{1}{2} [5(0 - 0) - 8(5 - 1) + 1(0 - 0)] \\ &= \frac{1}{2} [0 - 8(4) + 0] \\ &= \frac{1}{2} (-32) \\ &= -16 \end{aligned}$$

Since area cannot be negative,

$A(\Delta ABC) = 16$ sq. units

ii. Here, $P(x_1, y_1) \equiv P(3/2, 1)$, $Q(x_2, y_2) \equiv Q(4, 2)$, $R(x_3, y_3) \equiv R(4, -\frac{1}{2})$

$$\text{Area of a triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\begin{aligned}\therefore A(\Delta PQR) &= \frac{1}{2} \begin{vmatrix} \frac{3}{2} & 1 & 1 \\ 4 & 2 & 1 \\ 4 & -\frac{1}{2} & 1 \end{vmatrix} \\ &= \frac{1}{2} \left[\frac{3}{2} \left(2 + \frac{1}{2} \right) - 1(4 - 4) + 1(-2 - 8) \right] \\ &= \frac{1}{2} \left[\frac{3}{2} \left(\frac{5}{2} \right) - 1(0) + 1(-10) \right] \\ &= \frac{1}{2} \left(\frac{15}{4} - 10 \right) \\ &= \frac{1}{2} \left(\frac{-25}{4} \right) \\ &= \frac{-25}{8}\end{aligned}$$

Since area cannot be negative

$$A(\Delta PQR) = 25/8 \text{ sq. units}$$

iii. Here, $M(x_1, y_1) \equiv M(0, 5)$, $N(x_2, y_2) \equiv N(-2, 3)$

$T(x_3, y_3) \equiv T(1, -4)$

$$\text{Area of a triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

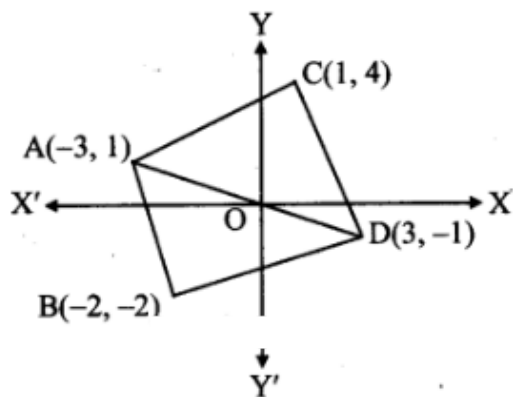
$$\begin{aligned}
 \therefore A(\Delta MNT) &= \frac{1}{2} \begin{vmatrix} 0 & 5 & 1 \\ -2 & 3 & 1 \\ 1 & -4 & 1 \end{vmatrix} \\
 &= \frac{1}{2} [0 - 5(-2 \cdot 1) + 1(8 - 3)] \\
 &= \frac{1}{2} [-5(-3) + 1(5)] \\
 &= \frac{1}{2} (15 + 5) \\
 &= \frac{1}{2} (20) \\
 &= 10 \text{ sq. units}
 \end{aligned}$$

Question 6.

Find the area of quadrilateral whose vertices are A (-3, 1), B (-2, -2), C (1, 4), D (3, -1).

Solution:

A(-3, 1), B(-2, -2), C(1, 4), D(3, -1)



$$A(\square ABDC) = A(\Delta ABD) + A(\Delta ADC)$$

$$\text{Area of triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}$$

$$\begin{aligned}
 A(\triangle ABD) &= \frac{1}{2} \begin{vmatrix} -3 & 1 & 1 \\ -2 & -2 & 1 \\ 3 & -1 & 1 \end{vmatrix} \\
 &= \frac{1}{2} [-3(-2+1) - 1(-2-3) + 1(2+6)] \\
 &= \frac{1}{2} [-3(-1) - 1(-5) + 1(8)] \\
 &= \frac{1}{2} (3+5+8) \\
 &= \frac{1}{2} (16) \\
 \therefore A(\triangle ABD) &= 8 \text{ sq. units} \\
 A(\triangle ADC) &= \frac{1}{2} \begin{vmatrix} -3 & 1 & 1 \\ 3 & -1 & 1 \\ 1 & 4 & 1 \end{vmatrix} \\
 &= \frac{1}{2} [-3(-1-4) - 1(3-1) + 1(12+1)] \\
 &= \frac{1}{2} [-3(-5) - 1(2) + 1(13)] \\
 &= \frac{1}{2} [15-2+13] \\
 &= \frac{1}{2} (26) \\
 \therefore A(\triangle ADC) &= 13 \text{ sq. units} \\
 \therefore A(\square ABDC) &= A(\triangle ABD) + A(\triangle ADC) \\
 &= 8 + 13 \\
 &= 21 \text{ sq. units}
 \end{aligned}$$

Question 7.

Find the value of k, if the area of triangle whose vertices are P (k, 0), Q (2,2), R (4,3) is $\frac{3}{2}$ sq. units.

Solution:

Here, $P(x_1, y_1) \equiv P(k, 0)$, $Q(x_2, y_2) \equiv Q(2, 2)$, $R(x_3, y_3) \equiv R(4, 3)$

Here, $P(x_1, y_1) = P(k, 0)$, $Q(x_2, y_2) = Q(2, 2)$, $R(x_3, y_3) = R(4, 3)$

$$\therefore A(\Delta PQR) = \frac{3}{2} \text{ sq. units}$$

$$\text{Area of triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\therefore \pm \frac{3}{2} = \frac{1}{2} \begin{vmatrix} k & 0 & 1 \\ 2 & 2 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

$$\therefore \pm [k(2-3) - 0 + 1(6-8)] \therefore \pm [k(-1) - 2] = \pm \frac{1}{2} (-k-2)$$

$$\therefore \pm 3 = -k - 2$$

$$\therefore 3 = -k - 2 \text{ or } -3 = -k - 2$$

$$\therefore k = -5 \text{ or } k = 1$$

Question 8.

Examine the collinearity of the following set of points:

i. A (3, -1), B (0, -3), C (12, 5)

ii. P (3, -5), Q (6, 1), R (4, 2)

iii. L(0, 1/2), M(2, -1), N(-4, 7/2)

Solution:

i. Here, $A(x_1, y_1) \equiv A(3, -1)$, $B(x_2, y_2) \equiv B(0, -3)$, $C(x_3, y_3) \equiv C(12, 5)$

If $A(\Delta ABC) = 0$, then the points A, B, C are collinear.

$$\begin{aligned} \therefore A(\Delta ABC) &= \frac{1}{2} \begin{vmatrix} 3 & -1 & 1 \\ 0 & -3 & 1 \\ 12 & 5 & 1 \end{vmatrix} \\ &= \frac{1}{2} [3(-3-5) - (-1)(0-12) + 1(0+36)] \\ &= \frac{1}{2} [3(-8) + 1(-12) + 1(36)] \\ &= \frac{1}{2} (-24 - 12 + 36) \end{aligned}$$

$$= \frac{1}{2}(-24 - 12 + 36)$$

$$= 0$$

∴ The points A, B, C are collinear.

ii. Here, $P(x_1, y_1) \equiv P(3, -5)$, $Q(x_2, y_2) \equiv Q(6, 1)$, $R(x_3, y_3) \equiv R(4, 2)$

∴ If $A(\Delta PQR) = 0$, then the points P, Q, R are collinear

$$\therefore A(\Delta PQR) = \frac{1}{2} \begin{vmatrix} 3 & -5 & 1 \\ 6 & 1 & 1 \\ 4 & 2 & 1 \end{vmatrix}$$

$$= \frac{1}{2} [3(1-2) - (-5)(6-4) + 1(12-4)]$$

$$= \frac{1}{2} [3(-1) + 5(2) + 1(8)]$$

$$= \frac{1}{2} (-3 + 10 + 8) = \frac{15}{2} \neq 0$$

∴ The points P, Q, R are non-collinear.

iii. Here, $L(x_1, y_1) \equiv L(0, 1/2)$, $M(x_2, y_2) \equiv M(2, -1)$, $N(x_3, y_3) \equiv N(-4, 7/2)$

If $A(\Delta LMN) = 0$, then the points L, M, N are collinear.

$$\therefore A(\Delta LMN) = \frac{1}{2} \begin{vmatrix} 0 & \frac{1}{2} & 1 \\ 2 & -1 & 1 \\ -4 & \frac{7}{2} & 1 \end{vmatrix}$$

$$= \frac{1}{2} [0 - \frac{1}{2}(2+4) + 1(7-4)]$$

$$= \frac{1}{2} [-\frac{1}{2}(6) + 1(3)]$$

$$= \frac{1}{2} (-3 + 3) = 0$$

∴ The points L, M, N are collinear.

Ex 4.4

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Question 1.

Construct a matrix $A = [a_{ij}]_{3 \times 2}$ whose elements a_{ij} are given by

i. $a_{ij} = \frac{(i-j)^2}{5-i}$

ii. $a_{ij} = i - 3j$

iii. $a_{ij} = \frac{(i+j)^3}{5}$

Solution:

$$A = [a_{ij}]_{3 \times 2}$$

$$\therefore A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

i. $a_{ij} = \frac{(i-j)^2}{5-i}$

$$\therefore a_{11} = \frac{(1-1)^2}{5-1} = 0, a_{12} = \frac{(1-2)^2}{5-1} = \frac{(-1)^2}{4} = \frac{1}{4},$$

$$a_{21} = \frac{(2-1)^2}{5-2} = \frac{1}{3}, a_{22} = \frac{(2-2)^2}{5-2} = 0,$$

$$a_{31} = \frac{(3-1)^2}{5-3} = \frac{2^2}{2} = 2, a_{32} = \frac{(3-2)^2}{5-3} = \frac{1}{2}$$

$$\therefore A = \begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{3} & 0 \\ 2 & \frac{1}{2} \end{bmatrix}$$

[Note: Answer given in the textbook is $A = \begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{2} & 0 \\ 2 & \frac{1}{2} \end{bmatrix}$

However, as per our calculation it is $\begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{3} & 0 \\ 2 & \frac{1}{2} \end{bmatrix}$].

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$$\therefore A = \begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{3} & 0 \\ 2 & \frac{1}{2} \end{bmatrix}$$

[Note: Answer given in the textbook is $A = \begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{2} & 0 \\ 2 & \frac{1}{2} \end{bmatrix}$

However, as per our calculation it is $\begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{3} & 0 \\ 2 & \frac{1}{2} \end{bmatrix}$].

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$$\text{ii. } a_{ij} = i - 3j$$

$$\therefore a_{11} = 1 - 3(1) = 1 - 3 = -2,$$

$$a_{12} = 1 - 3(2) = 1 - 6 = -5,$$

$$a_{21} = 2 - 3(1) = 2 - 3 = -1,$$

$$a_{22} = 2 - 3(2) = 2 - 6 = -4$$

$$a_{31} = 3 - 3(1) = 3 - 3 = 0,$$

$$a_{32} = 3 - 3(2) = 3 - 6 = -3$$

$$\therefore A = \begin{bmatrix} -2 & -5 \\ -1 & -4 \\ 0 & -3 \end{bmatrix}$$

$$\text{iii. } a_{ij} = \frac{(i+j)^3}{5}$$

$$\therefore a_{11} = \frac{(1+1)^3}{5} = \frac{2^3}{5} = \frac{8}{5}, a_{12} = \frac{(1+2)^3}{5} = \frac{3^3}{5} = \frac{27}{5}$$

$$a_{21} = \frac{(2+1)^3}{5} = \frac{3^3}{5} = \frac{27}{5}, a_{22} = \frac{(2+2)^3}{5} = \frac{4^3}{5} = \frac{64}{5}$$

$$a_{31} = \frac{(3+1)^3}{5} = \frac{4^3}{5} = \frac{64}{5}, a_{32} = \frac{(3+2)^3}{5} = \frac{5^3}{5} = \frac{125}{5}$$

$$\therefore A = \begin{bmatrix} \frac{8}{5} & \frac{27}{5} \\ \frac{27}{5} & \frac{64}{5} \\ \frac{64}{5} & \frac{125}{5} \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 8 & 27 \\ 27 & 64 \\ 64 & 125 \end{bmatrix}$$

Question 2.

Classify the following matrices as a row, a column, a square, a diagonal, a scalar, a unit, an upper triangular, a lower triangular, a symmetric or a skew-symmetric matrix.

i.
$$\begin{bmatrix} 3 & -2 & 4 \\ 0 & 0 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 3 & -2 & 4 \\ 0 & 0 & -5 \\ 0 & 0 & 0 \end{bmatrix}$

As every element below the diagonal is zero in matrix A.

$\therefore$ A is an upper triangular matrix.

ii.
$$\begin{bmatrix} 0 & 4 & 7 \\ -4 & 0 & -3 \\ -7 & 3 & 0 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 0 & 4 & 7 \\ -4 & 0 & -3 \\ -7 & 3 & 0 \end{bmatrix}$

$\therefore A^T = \begin{bmatrix} 0 & -4 & -7 \\ 4 & 0 & 3 \\ 7 & -3 & 0 \end{bmatrix}$

$\therefore A^T = - \begin{bmatrix} 0 & 4 & 7 \\ -4 & 0 & -3 \\ -7 & 3 & 0 \end{bmatrix}$

$$\therefore A^T = -A, \text{ i.e., } A = -A^T$$

$\therefore A$ is a skew-symmetric matrix.

iii. $\begin{bmatrix} 5 \\ 4 \\ -3 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 5 \\ 4 \\ -3 \end{bmatrix}$

$\therefore$ As matrix A has only one column.

$\therefore A$ is a column matrix.

iv. $[9 \quad \sqrt{2} \quad -3]$

Solution:

Let $A = [9 \quad \sqrt{2} \quad -3]$

As matrix A has only one row.

$\therefore A$ is a row matrix.

v. $\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

As matrix A has all its non-diagonal elements zero and diagonal elements same.

$\therefore$ A is a scalar matrix.

vi. $\begin{bmatrix} 2 & 0 & 0 \\ 3 & -1 & 0 \\ -7 & 3 & 1 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 2 & 0 & 0 \\ 3 & -1 & 0 \\ -7 & 3 & 1 \end{bmatrix}$

As every element above the diagonal is zero in matrix A.

$\therefore$ A is a lower triangular matrix.

vii. $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$

As matrix A has all its non-diagonal elements zero.

viii.
$$\begin{bmatrix} 10 & -15 & 27 \\ -15 & 0 & \sqrt{34} \\ 27 & \sqrt{34} & \frac{5}{3} \end{bmatrix}$$

Solution:

$$\text{Let } A = \begin{bmatrix} 10 & -15 & 27 \\ -15 & 0 & \sqrt{34} \\ 27 & \sqrt{34} & \frac{5}{3} \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 10 & -15 & 27 \\ -15 & 0 & \sqrt{34} \\ 27 & \sqrt{34} & \frac{5}{3} \end{bmatrix}$$

$$\therefore A^T = A, \text{ i/e., } A = A^T$$

$\therefore A$ is a symmetric matrix.

ix.
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In matrix A , all the non-diagonal elements are zero and diagonal elements are one.

$\therefore A$ is a unit (identity) matrix.

$$x. \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Solution:

$$\text{Let } A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\therefore A^T = A, \text{ i/e., } A = A^T$$

$\therefore A$ is a symmetric matrix.

$\therefore A$ is a symmetric matrix.

Question 3.

Which of the following matrices are singular or non-singular?

$$i. \begin{bmatrix} a & b & c \\ p & q & r \\ 2a - p & 2b - q & 2c - r \end{bmatrix}$$

$$ii. \begin{bmatrix} 5 & 0 & 5 \\ 1 & 99 & 100 \\ 6 & 99 & 105 \end{bmatrix}$$

$$iii. \begin{bmatrix} 3 & 5 & 7 \\ -2 & 1 & 4 \\ 3 & 2 & 5 \end{bmatrix}$$

$$iv. \begin{bmatrix} 7 & 5 \\ -4 & 7 \end{bmatrix}$$

Solution:

$$\text{Let } A = \begin{bmatrix} a & b & c \\ p & q & r \\ 2a-p & 2b-q & 2c-r \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} a & b & c \\ p & q & r \\ 2a-p & 2b-q & 2c-r \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 + R_2$, we get

$$|A| = \begin{vmatrix} a & b & c \\ p & q & r \\ 2a & 2b & 2c \end{vmatrix}$$

Taking 2 common from R_3 , we get

$$|A| = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ a & b & c \end{vmatrix}$$

$$= 2(0) \quad \dots [\because R_1 \text{ and } R_3 \text{ are identical}]$$

$$= 0$$

$\therefore$ A is a singular matrix.

$$\text{ii. Let } A = \begin{bmatrix} 5 & 0 & 5 \\ 1 & 99 & 100 \\ 6 & 99 & 105 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 5 & 0 & 5 \\ 1 & 99 & 100 \\ 6 & 99 & 105 \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 + C_1$

$$|A| = \begin{vmatrix} 5 & 5 & 5 \\ 1 & 100 & 100 \\ 6 & 105 & 105 \end{vmatrix}$$

$= 0 \dots [\because C_2 \text{ and } C_3 \text{ are identical}]$

$\therefore A$ is a singular matrix.

iii. Let $A = \begin{bmatrix} 3 & 5 & 7 \\ -2 & 1 & 4 \\ 3 & 2 & 5 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 3 & 5 & 7 \\ -2 & 1 & 4 \\ 3 & 2 & 5 \end{vmatrix}$$

$$= 3(5 - 8) - 5(-10 - 12) + 7(-4 - 3)$$

$$= -9 + 110 - 49 = 52 \neq 0$$

$\therefore A$ is a non-singular matrix.

iv. Let $A = \begin{bmatrix} 7 & 5 \\ -4 & 7 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 7 & 5 \\ -4 & 7 \end{vmatrix} = 49 + 20 = 69 \neq 0$$

Question 4.

Find k, if the following matrices are singular.

i. $\begin{bmatrix} 7 & 3 \\ -2 & k \end{bmatrix}$

ii. $\begin{bmatrix} 4 & 3 & 1 \\ 7 & k & 1 \\ 10 & 9 & 1 \end{bmatrix}$

iii. $\begin{bmatrix} k-1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & -2 & 4 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 7 & 3 \\ -2 & k \end{bmatrix}$

Since A is a singular matrix,

$$|A| = 0$$

$$\therefore \begin{vmatrix} 7 & 3 \\ -2 & k \end{vmatrix} = 0$$

$$\therefore 7k + 6 = 0$$

$$\therefore 7k = -6$$

$$k = -6/7$$

ii. Let $A = \begin{bmatrix} 4 & 3 & 1 \\ 7 & k & 1 \\ 10 & 9 & 1 \end{bmatrix}$

Since A is a singular matrix,

$$|A| = 0$$

$$\therefore \begin{vmatrix} 4 & 3 & 1 \\ 7 & k & 1 \\ 10 & 9 & 1 \end{vmatrix} = 0$$

$$\therefore 4(k - 9) - 3(7 - 10) + 1(63 - 10k) = 0$$

$$\therefore 4k - 36 + 9 + 63 - 10k = 0$$

$$\therefore -6k + 36 = 0$$

$$\therefore 6k = 36$$

$$\therefore k = 6$$

iii. Let $A = \begin{bmatrix} k-1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & -2 & 4 \end{bmatrix}$

Since A is a singular matrix

$$|A| = 0$$

$$\therefore \begin{vmatrix} k-1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & -2 & 4 \end{vmatrix}$$

$$\therefore (k-1)(4+4) - 2(12-2) + 3(-6-1) = 0$$

$$\therefore 8k-8-20-21=0$$

$$\therefore 8k = 49$$

$$\therefore k = 49/8$$

Question 5.

If $A = \begin{bmatrix} 5 & 1 & -1 \\ 3 & 2 & 0 \end{bmatrix}$, find $(A^T)^T$.

Solution:

$$A = \begin{bmatrix} 5 & 1 & -1 \\ 3 & 2 & 0 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 5 & 3 \\ 1 & 2 \\ -1 & 0 \end{bmatrix}$$

$$\therefore (A^T)^T = \begin{bmatrix} 5 & 1 & -1 \\ 3 & 2 & 0 \end{bmatrix}$$

Question 6.

If $A =$, find $(A^T)^T$.

Solution:

$$A = \begin{bmatrix} 7 & 3 & 1 \\ -2 & -4 & 1 \\ 5 & 9 & 1 \end{bmatrix}$$
$$\therefore A^T = \begin{bmatrix} 7 & -2 & 5 \\ 3 & -4 & 9 \\ 1 & 1 & 1 \end{bmatrix}$$
$$\therefore (A^T)^T = \begin{bmatrix} 7 & 3 & 1 \\ -2 & -4 & 1 \\ 5 & 9 & 1 \end{bmatrix}$$

Question 7.

Find a, b, c, if $\begin{bmatrix} 1 & \frac{3}{5} & a \\ b & -5 & -7 \\ -4 & c & 0 \end{bmatrix}$ is a symmetric matrix.

Solution:

$$\text{Let } A = \begin{bmatrix} 1 & \frac{3}{5} & a \\ b & -5 & -7 \\ -4 & c & 0 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 1 & b & -4 \\ \frac{3}{5} & -5 & c \\ a & -7 & 0 \end{bmatrix}$$

Since A is a symmetric matrix,

$$A = A^T$$

$$\therefore \begin{bmatrix} 1 & \frac{3}{5} & a \\ b & -5 & -7 \\ -4 & c & 0 \end{bmatrix} = \begin{bmatrix} 1 & b & -4 \\ \frac{3}{5} & -5 & c \\ a & -7 & 0 \end{bmatrix}$$

$\therefore$ By equality of matrices, we get

$$a = -4, b = \frac{3}{5}, c = -7$$

Question 8.

Find x, y, z, if is a symmetric matrix.

Solution:

$$\text{Let } A = \begin{bmatrix} 0 & -5i & x \\ y & 0 & z \\ \frac{3}{2} & -\sqrt{2} & 0 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 0 & y & \frac{3}{2} \\ -5i & 0 & -\sqrt{2} \\ x & z & 0 \end{bmatrix}$$

Since A is a skew-symmetric matrix,

$$A = -A^T$$

$$\therefore \begin{bmatrix} 0 & -5i & x \\ y & 0 & z \\ \frac{3}{2} & -\sqrt{2} & 0 \end{bmatrix} = - \begin{bmatrix} 0 & y & \frac{3}{2} \\ -5i & 0 & -\sqrt{2} \\ x & z & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 0 & -5i & x \\ y & 0 & z \\ \frac{3}{2} & -\sqrt{2} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -y & \frac{-3}{2} \\ 5i & 0 & \sqrt{2} \\ -x & -z & 0 \end{bmatrix}$$

$\therefore$ By equality of matrices, we get

$$x = \frac{-3}{2}, y = 5i, z = \sqrt{2}$$

Question 9.

For each of the following matrices, using its transpose, state whether it is symmetric, skew-symmetric or neither.

symmetric or neither.

i. $\begin{bmatrix} 1 & 2 & -5 \\ 2 & -3 & 4 \\ -5 & 4 & 9 \end{bmatrix}$

ii. $\begin{bmatrix} 2 & 5 & 1 \\ -5 & 4 & 6 \\ -1 & -6 & 3 \end{bmatrix}$

iii. $\begin{bmatrix} 0 & 1+2i & i-2 \\ -1-2i & 0 & -7 \\ 2-i & 7 & 0 \end{bmatrix}$

Solution:

i. Let $A = \begin{bmatrix} 1 & 2 & -5 \\ 2 & -3 & 4 \\ -5 & 4 & 9 \end{bmatrix}$

$$\therefore A^T = \begin{bmatrix} 1 & 2 & -5 \\ 2 & -3 & 4 \\ -5 & 4 & 9 \end{bmatrix}$$

$$\therefore A^T = A, \text{ i.e., } A = A^T$$

$\therefore A$ is a symmetric matrix.

ii.

$$\text{Let } A = \begin{bmatrix} 2 & 5 & 1 \\ -5 & 4 & 6 \\ -1 & -6 & 3 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 2 & -5 & -1 \\ 5 & 4 & -6 \\ 1 & 6 & 3 \end{bmatrix}$$

$$\therefore A^T = - \begin{bmatrix} -2 & 5 & 1 \\ -5 & -4 & 6 \\ -1 & -6 & -3 \end{bmatrix}$$

$$\therefore A \neq A^T, \text{ i.e., } A \neq -A^T$$

$\therefore A$ is neither a symmetric nor skew-symmetric matrix.

iii.

$$\text{Let } A = \begin{bmatrix} 0 & 1+2i & i-2 \\ -1-2i & 0 & -7 \\ 2-i & 7 & 0 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 0 & -1-2i & 2-i \\ 1+2i & 0 & 7 \\ i-2 & -7 & 0 \end{bmatrix}$$

$$\therefore A^T = - \begin{bmatrix} 0 & 1+2i & i-2 \\ -1-2i & 0 & -7 \\ 2-i & 7 & 0 \end{bmatrix}$$

$$\therefore A^T = -A, \text{ i.e., } A = -A^T$$

$\therefore A$ is a skew-symmetric matrix.

Question 10.

Construct the matrix $A = [a_{ij}]_{3 \times 3}$, where $a_{ij} = i - j$. State whether A is symmetric or skew-symmetric.

Solution:

$$A = [a_{ij}]_{3 \times 3}$$

$$\therefore A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Given, $a_{ij} = i - j$

$$a_{11} = 1 - 1 = 0, a_{12} = 1 - 2 = -1, a_{13} = 1 - 3 = -2,$$

$$a_{21} = 2 - 1 = 1, a_{22} = 2 - 2 = 0, a_{23} = 2 - 3 = -1,$$

$$a_{31} = 3 - 1 = 2, a_{32} = 3 - 2 = 1, a_{33} = 3 - 3 = 0$$

$$\therefore A = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$\therefore A^T = -A, \text{ i.e., } A = -A^T$$

$\therefore A$ is a skew-symmetric matrix.

Ex 4.5

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-1-chapter-4-determinants-and-matrices/>

Question 1.

If $A = \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 \\ 2 & 2 \\ 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 3 \\ -1 & 4 \\ -2 & 1 \end{bmatrix}$

Show that

i. $A+B=B+A$

ii. $(A+B)+C=A+(B+C)$

Solution:

$$\text{i. } A+B = \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 2 & 2 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & -3+2 \\ 5+2 & -4+2 \\ -6+0 & 1+3 \end{bmatrix}$$

$$\therefore A+B = \begin{bmatrix} 1 & -1 \\ 7 & -2 \\ -6 & 4 \end{bmatrix} \quad \dots(i)$$

$$B+A = \begin{bmatrix} -1 & 2 \\ 2 & 2 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1+2 & 2-3 \\ 2+5 & 2-4 \\ 0-6 & 3+1 \end{bmatrix}$$

$$\therefore B+A = \begin{bmatrix} 1 & -1 \\ 7 & -2 \\ -6 & 4 \end{bmatrix} \quad \dots(ii)$$

From (i) and (ii), we get

$$A + B = B + A$$

$$\begin{aligned}\text{ii. } (A + B) + C &= \left\{ \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 2 & 2 \\ 0 & 3 \end{bmatrix} \right\} + \begin{bmatrix} 4 & 3 \\ -1 & 4 \\ -2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2-1 & -3+2 \\ 5+2 & -4+2 \\ -6+0 & 1+3 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -1 & 4 \\ -2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -1 \\ 7 & -2 \\ -6 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -1 & 4 \\ -2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1+4 & -1+3 \\ 7-1 & -2+4 \\ -6-2 & 4+1 \end{bmatrix}\end{aligned}$$

$$\therefore (A + B) + C = \begin{bmatrix} 5 & 2 \\ 6 & 2 \\ -8 & 5 \end{bmatrix} \quad \dots(\text{i})$$

$$\begin{aligned}A + (B + C) &= \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} + \left\{ \begin{bmatrix} -1 & 2 \\ 2 & 2 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -1 & 4 \\ -2 & 1 \end{bmatrix} \right\} \\ &= \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} + \begin{bmatrix} -1+4 & 2+3 \\ 2-1 & 2+4 \\ 0-2 & 3+1 \end{bmatrix}\end{aligned}$$

$$\begin{aligned} &= \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 5 \\ 1 & 6 \\ -2 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 2+3 & -3+5 \\ 5+1 & -4+6 \\ -6-2 & 1+4 \end{bmatrix} \\ &= \begin{bmatrix} 5 & 2 \\ 6 & 2 \\ -8 & 5 \end{bmatrix} \quad \dots(ii) \end{aligned}$$

$$(A + B) + C = A + (B + C)$$

Question 2.

If $A = \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -3 \\ 4 & -7 \end{bmatrix}$ then find the matrix $A - 2B + 6I$, where I is the unit matrix of order 2.

Solution:

$$\begin{aligned} A - 2B + 6I &= \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix} - 2 \begin{bmatrix} 1 & -3 \\ 4 & -7 \end{bmatrix} + 6 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 8 & -14 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} &= \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 8 & -14 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} \\ &= \begin{bmatrix} 1-2+6 & -2+6+0 \\ 5-8+0 & 3+14+6 \end{bmatrix} \\ &= \begin{bmatrix} 5 & 4 \\ -3 & 23 \end{bmatrix} \end{aligned}$$

Question 3.

If $A = \begin{bmatrix} 1 & 2 & -3 \\ -3 & 7 & -8 \\ 0 & -6 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 9 & -1 & 2 \\ -4 & 2 & 5 \\ 4 & 0 & -3 \end{bmatrix}$ then find the matrix C such that $A + B + C$ is a zero matrix.

Solution:

$A + B + C$ is a zero matrix.

$$\therefore A + B + C = O$$

$$C = -(A + B)$$

$$= - \left\{ \begin{bmatrix} 1 & 2 & -3 \\ -3 & 7 & -8 \\ 0 & -6 & 1 \end{bmatrix} + \begin{bmatrix} 9 & -1 & 2 \\ -4 & 2 & 5 \\ 4 & 0 & -3 \end{bmatrix} \right\}$$

$$= - \begin{bmatrix} 1+9 & 2-1 & -3+2 \\ -3-4 & 7+2 & -8+5 \\ 0+4 & -6+0 & 1-3 \end{bmatrix}$$

$$= - \begin{bmatrix} 10 & 1 & -1 \\ -7 & 9 & -3 \\ 4 & -6 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -1 & 1 \\ 7 & -9 & 3 \\ -4 & 6 & 2 \end{bmatrix}$$

Question 4.

If $A = \begin{bmatrix} 1 & -2 \\ 3 & -5 \\ -6 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -2 \\ 4 & 2 \\ 1 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 4 \\ -1 & -4 \\ -3 & 6 \end{bmatrix}$, find the matrix X such that $3A -$

$$4B + 5X = C.$$

Solution:

$$3A - 4B + 5X = C$$

$$\therefore 5X = C + 4B - 3A$$

$$= \begin{bmatrix} 2 & 4 \\ -1 & -4 \\ -3 & 6 \end{bmatrix} + 4 \begin{bmatrix} -1 & -2 \\ 4 & 2 \\ 1 & 5 \end{bmatrix} - 3 \begin{bmatrix} 1 & -2 \\ 3 & -5 \\ -6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 \\ -1 & -4 \\ -3 & 6 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ 16 & 8 \\ 4 & 20 \end{bmatrix} - \begin{bmatrix} 3 & -6 \\ 9 & -15 \\ -18 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2-4-3 & 4-8+6 \\ -1+16-9 & -4+8+15 \\ -3+4+18 & 6+20-0 \end{bmatrix}$$

$$\therefore 5X = \begin{bmatrix} -5 & 2 \\ 6 & 19 \\ 19 & 26 \end{bmatrix}$$

$$\therefore X = \frac{1}{5} \begin{bmatrix} -5 & 2 \\ 6 & 19 \\ 19 & 26 \end{bmatrix} = \begin{bmatrix} -1 & \frac{2}{5} \\ \frac{6}{5} & \frac{19}{5} \\ \frac{19}{5} & \frac{26}{5} \end{bmatrix}$$

Question 5.

Solve the following equations for X and Y, if $3X - Y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $X - 3Y = \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}$

Solution:

Given equations are

$$= \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \dots\dots\dots (i)$$

$$\text{and } X - 3Y = \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix} \dots\dots\dots (ii)$$

By (i) $\times 3 -$ (ii) we get

$$\begin{aligned} 8X &= 3 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 3-0 & -3+1 \\ -3-0 & 3+1 \end{bmatrix} \end{aligned}$$

$$\therefore 8X = \begin{bmatrix} 3 & -2 \\ -3 & 4 \end{bmatrix}$$

$$\therefore X = \frac{1}{8} \begin{bmatrix} 3 & -2 \\ -3 & 4 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} \frac{3}{8} & \frac{-2}{8} \\ \frac{-3}{8} & \frac{4}{8} \end{bmatrix} = \begin{bmatrix} \frac{3}{8} & \frac{-1}{4} \\ \frac{-3}{8} & \frac{1}{2} \end{bmatrix}$$

By (i) $-$ (ii) $\times 3$, we get

$$8Y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} - 3 \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}$$

$$\begin{aligned}8Y &= \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} - 3 \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix} \\&= \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -3 \\ 0 & -3 \end{bmatrix} \\&= \begin{bmatrix} 1-0 & -1+3 \\ -1-0 & 1+3 \end{bmatrix} \\ \therefore 8Y &= \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} \\ \therefore Y &= \frac{1}{8} \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{8} & \frac{2}{8} \\ -\frac{1}{8} & \frac{4}{8} \end{bmatrix} = \begin{bmatrix} \frac{1}{8} & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{2} \end{bmatrix}\end{aligned}$$

Question 6.

Find the matrices A and B, if $2A - B = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & -7 \end{bmatrix}$

Solution:

Given equations are

$$2A - B = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} \dots\dots\dots (i)$$

$$\text{and } A - 2B = \begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & -7 \end{bmatrix} \dots\dots\dots (ii)$$

By (i) - (ii) $\times 2$, we get

$$\begin{aligned}3B &= \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} - 2 \begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & -7 \end{bmatrix} \\&= \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 4 & 16 \\ -4 & 2 & -14 \end{bmatrix}\end{aligned}$$

$$= \begin{bmatrix} 6-6 & -6-4 & 0-16 \\ -4+4 & 2-2 & 1+14 \end{bmatrix}$$

$$\therefore 3B = \begin{bmatrix} 0 & -10 & -16 \\ 0 & 0 & 15 \end{bmatrix}$$

$$\therefore B = \frac{1}{3} \begin{bmatrix} 0 & -10 & -16 \\ 0 & 0 & 15 \end{bmatrix}$$

$$\therefore B = \begin{bmatrix} 0 & \frac{-10}{3} & \frac{-16}{3} \\ 0 & 0 & 5 \end{bmatrix}$$

By (i) $\times 2$ - (ii), we get

$$\begin{aligned} 3A &= 2 \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & -7 \end{bmatrix} \\ &= \begin{bmatrix} 12 & -12 & 0 \\ -8 & 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & -7 \end{bmatrix} \\ &= \begin{bmatrix} 12-3 & -12-2 & 0-8 \\ -8+2 & 4-1 & 2+7 \end{bmatrix} \end{aligned}$$

$$\therefore 3A = \begin{bmatrix} 9 & -14 & -8 \\ -6 & 3 & 9 \end{bmatrix}$$

$$\therefore A = \frac{1}{3} \begin{bmatrix} 9 & -14 & -8 \\ -6 & 3 & 9 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 3 & \frac{-14}{3} & \frac{-8}{3} \\ -2 & 1 & 3 \end{bmatrix}$$

Question 7.

Simplify $\cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$

Solution:

$$\begin{aligned} & \cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ -\cos \theta \sin \theta & \cos^2 \theta \end{bmatrix} + \begin{bmatrix} \sin^2 \theta & -\cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \theta + \sin^2 \theta & \cos \theta \sin \theta - \cos \theta \sin \theta \\ -\cos \theta \sin \theta + \cos \theta \sin \theta & \cos^2 \theta + \sin^2 \theta \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Question 8.

If $A = \begin{bmatrix} 1 & 2i \\ -3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2i & 1 \\ 2 & -3 \end{bmatrix}$ where $i = \sqrt{-1}$, find $A + B$ and $A - B$. Show that $A + B$ is singular. Is $A - B$ singular? Justify your answer.

Solution:

$$\begin{aligned} A + B &= \begin{bmatrix} i & 2i \\ -3 & 2 \end{bmatrix} + \begin{bmatrix} 2i & i \\ 2 & -3 \end{bmatrix} \\ &= \begin{bmatrix} i + 2i & 2i + i \\ -3 + 2 & 2 - 3 \end{bmatrix} = \begin{bmatrix} 3i & 3i \\ -1 & -1 \end{bmatrix} \\ \therefore |A+B| &= \begin{vmatrix} 3i & 3i \\ -1 & -1 \end{vmatrix} \\ &= 3i(-1) - (-1)3i = 0 \\ \therefore A + B &\text{ is a singular matrix.} \\ A - B &= \begin{bmatrix} i & 2i \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 2i & i \\ 2 & -3 \end{bmatrix} \\ &= \begin{bmatrix} i - 2i & 2i - i \\ -3 - 2 & 2 + 3 \end{bmatrix} = \begin{bmatrix} -i & i \\ -5 & 5 \end{bmatrix} \\ \therefore |A-B| &= \begin{vmatrix} -i & i \\ -5 & 5 \end{vmatrix} = (-i)5 - (-5)i = 0 \\ \therefore A - B &\text{ is also a singular matrix.} \end{aligned}$$

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Question 9.

Find x and y, if $\begin{bmatrix} 2x+y & -1 & 1 \\ 3 & 4y & 4 \end{bmatrix} + \begin{bmatrix} -1 & 6 & 4 \\ 3 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 5 \\ 6 & 18 & 7 \end{bmatrix}$

Solution:

$$\begin{bmatrix} 2x+y & -1 & 1 \\ 3 & 4y & 4 \end{bmatrix} + \begin{bmatrix} -1 & 6 & 4 \\ 3 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 5 \\ 6 & 18 & 7 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2x+y-1 & -1+6 & 1+4 \\ 3+3 & 4y+0 & 4+3 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 5 \\ 6 & 18 & 7 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2x+y-1 & 5 & 5 \\ 6 & 4y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 5 \\ 6 & 18 & 7 \end{bmatrix}$$

$\therefore$ By equality of matrices, we get

$$2x + y - 1 = 3 \text{ and } 4y = 18$$

$$\therefore 2x + y = 4 \text{ and } y = \frac{18}{4} = \frac{9}{2}$$

$$\therefore 2x + \frac{9}{2} = 4$$

$$\therefore 2x = 4 - \frac{9}{2}$$

$$\therefore 2x = \frac{1}{2} =$$

$$\therefore x = -\frac{1}{4} = \text{ and } y = \frac{9}{2} =$$

Question 10.

If $\begin{bmatrix} 2a+b & 3a-b \\ c+2d & 2c-d \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$, find a, b, c and d.

Solution:

$$\begin{bmatrix} 2a+b & 3a-b \\ c+2d & 2c-d \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$$

∴ By equality of matrices, we get

$$2a + b = 2 \dots(i)$$

$$3a - b = 3 \dots(ii)$$

$$c + 2d = 4 \dots(iii)$$

$$2c - d = -1 \dots(iv)$$

Adding (i) and (ii), we get

$$5a = 5$$

$$\therefore a = 1$$

Substituting $a = 1$ in (i), we get

$$2(1) + b = 2$$

$$\therefore b = 0$$

By (iii) + (iv) $\times 2$, we get

$$5c = 2$$

$$\therefore c = \frac{2}{5}$$

Substituting $c = \frac{2}{5}$ in (iii), we get

$$\frac{2}{5} + 2d = 4$$

$$\therefore 2d = 4 - \frac{2}{5}$$

$$\therefore 2d = \frac{18}{5}$$

$$\therefore d = \frac{9}{5}$$

[Note: Answer given in the textbook is $d = \frac{3}{5}$.

However, as per our calculation it is $d = \frac{9}{5}$.]

Question 11.

There are two book shops owned by Suresh and Ganesh. Their sales (in Rupees) for books in

three subjects – Physics, Chemistry and Mathematics for two months, July and August 2017 are given by two matrices A and B.

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July Sales (in Rupees):

	Physics	Chemistry	Mathematics	
$A =$	5600	6750	8500	Suresh
	6650	7055	8905	Ganesh

August Sales (in Rupees):

	Physics	Chemistry	Mathematics	
$B =$	6650	7055	8905	Suresh
	7000	7500	10200	Ganesh

- i. Find the increase in sales in Rupees from July to August 2017.
- ii. If both book shops got 10% profit in the month of August 2017, find the profit for each bookseller in each subject in that month.

Solution:

- i. Increase in sales in rupees from July to August 2017

For Suresh:

Increase in sales for Physics books

$$= 6650 - 5600 = ₹ 1050$$

Increase in sales for Chemistry books

$$= 7055 - 6750 = ₹ 305$$

Increase in sales for Mathematics books

$$= 8905 - 8500 = ₹ 405$$

For Ganesh:

Increase in sales for Physics books

$$= 7000 - 6650 = ₹ 350$$

Increase in sales for Chemistry books

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$$= 7500 - 7055 = ₹ 445$$

Increase in sales for Mathematics books

$$= 10200 - 8905 = ₹ 1295$$

[Note: Answers given in the textbook are 1760, 2090. However, as per our calculation they are 1050, 305, 405, 350, 445, 1295.]

ii. Both book shops got 10% profit in the month of August 2017.

For Suresh:

$$\text{Profit for Physics books} = \frac{6650 \times 10}{100} = ₹ 665$$

$$\text{Profit for Chemistry books} = \frac{7055 \times 10}{100} = ₹ 705.50$$

$$\text{Profit for Mathematics books} = \frac{8905 \times 10}{100} = ₹ 890.50$$

For Ganesh:

$$\text{Profit for Physics books} = \frac{7000 \times 10}{100} = ₹ 700$$

$$\text{Profit for Chemistry books} = \frac{7500 \times 10}{100} = ₹ 750$$

$$\text{Profit for Mathematics books} = \frac{10200 \times 10}{100} = ₹ 1020$$

[Note: Answers given in the textbook for Suresh's profit in Chemistry and Mathematics books are ₹ 675 and ₹ 850 respectively. However, as per our calculation profit amounts are ₹ 705.50 and ₹ 890.50 respectively.]

Ex 4.6

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Question 1.

Evaluate:

i. $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & -4 & 3 \end{bmatrix}$

ii. $\begin{bmatrix} 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$

Solution:

i. $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & -4 & 3 \end{bmatrix} = \begin{bmatrix} 3(2) & 3(-4) & 3(3) \\ 2(2) & 2(-4) & 2(3) \\ 1(2) & 1(-4) & 1(3) \end{bmatrix}$
 $= \begin{bmatrix} 6 & -12 & 9 \\ 4 & -8 & 6 \\ 2 & -4 & 3 \end{bmatrix}$

$$\text{ii. } \begin{bmatrix} 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$$

$$= [2(4) - 1(3) + 3(1)]$$

$$= [8 - 3 + 3] = [8]$$

Question 2.

$$\text{If } A = \begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix} \text{ } B = \begin{bmatrix} 4 & 1 \\ 3 & -2 \end{bmatrix}, \text{ show that } AB \neq BA.$$

Solution:

$$\begin{aligned} AB &= \begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 3 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 4-9 & 1-6 \\ 16+6 & 4-4 \end{bmatrix} \\ &= \begin{bmatrix} -5 & 7 \\ 22 & 0 \end{bmatrix} \quad \dots(i) \end{aligned}$$

$$\begin{aligned} BA &= \begin{bmatrix} 4 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 4+4 & -12+2 \\ 3-8 & -9-4 \end{bmatrix} \\ &= \begin{bmatrix} 8 & -10 \\ -5 & -13 \end{bmatrix} \quad \dots(ii) \end{aligned}$$

From (i) and (ii), we get

$$AB \neq BA$$

Question 3.

If $A = \begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & -3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ state whether $AB = BA$? Justify your answer.

Solution:

$$\begin{aligned} AB &= \begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & -3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \\ 3 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -2+3+1 & -1+0+2 & -4+2+1 \\ 4+9+0 & 2+0+0 & 8+6+0 \\ 2-9+1 & 1+0+2 & 4-6+1 \end{bmatrix} \\ \therefore AB &= \begin{bmatrix} 2 & 1 & -1 \\ 13 & 2 & 14 \\ -6 & 3 & -1 \end{bmatrix} \quad \dots(i) \end{aligned}$$

$$\begin{aligned} BA &= \begin{bmatrix} 2 & 1 & 4 \\ 3 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & -3 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -2+2+4 & 2+3-12 & 2+0+4 \\ -3+0+2 & 3+0-6 & 3+0+2 \\ -1+4+1 & 1+6-3 & 1+0+1 \end{bmatrix} \\ \therefore BA &= \begin{bmatrix} 4 & -7 & 6 \\ -1 & -3 & 5 \\ 4 & 4 & 2 \end{bmatrix} \quad \dots(ii) \end{aligned}$$

From (i) and (ii), we get

$AB \neq BA$

Question 4.

Show that $AB = BA$, where

$$\text{i. } A = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix}$$

$$\text{ii. } A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}, B = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Solution:

$$\begin{aligned} \text{i. } AB &= \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix} \\ &= \begin{bmatrix} -2+6-3 & -6+6-0 & 2-3+1 \\ -1+4-3 & -3+4-0 & 1-2+1 \\ -6+18-12 & -18+18+0 & 6-9+4 \end{bmatrix} \end{aligned}$$

$$\therefore AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \dots(\text{i})$$

$$\begin{aligned} BA &= \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} \\ &= \begin{bmatrix} -2-3+6 & 3+6-9 & -1-3+4 \\ -4-2+6 & 6+4-9 & -2-2+4 \\ -6+0+6 & 9+0-9 & -3+0+4 \end{bmatrix} \end{aligned}$$

$$\therefore BA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \dots(\text{ii})$$

From (i) and (ii), we get

$$AB = BA$$

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$$\begin{aligned}\text{ii. } AB &= \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \theta + \sin^2 \theta & -\sin \theta \cos \theta + \sin \theta \cos \theta \\ -\sin \theta \cos \theta + \sin \theta \cos \theta & \sin^2 \theta + \cos^2 \theta \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \dots(\text{i})\end{aligned}$$

$$\begin{aligned}BA &= \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \theta + \sin^2 \theta & \cos \theta \sin \theta - \sin \theta \cos \theta \\ \sin \theta \cos \theta - \sin \theta \cos \theta & \sin^2 \theta + \cos^2 \theta \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \dots(\text{ii})\end{aligned}$$

From (i) and (ii), we get

$$AB = BA$$

[Note: The question has been modified.]

Question 5.

If $A = \begin{bmatrix} 4 & 8 \\ -2 & -4 \end{bmatrix}$, prove that $A^2 = 0$.

Solution:

$$A^2 = A.A$$

$$\begin{aligned} &= \begin{bmatrix} 4 & 8 \\ -2 & -4 \end{bmatrix} \begin{bmatrix} 4 & 8 \\ -2 & -4 \end{bmatrix} \\ &= \begin{bmatrix} 16 - 16 & 32 - 32 \\ -8 + 8 & -16 + 16 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \end{aligned}$$

Question 6.

Verify $A(BC) = (AB)C$ in each of the following cases:

i. $A = \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix}$

ii. $A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ -2 & 3 \\ 4 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 \\ -2 & 0 \\ 4 & -3 \end{bmatrix}$

Solution:

i. $BC = \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ 2 & 0 & -2 \end{bmatrix}$

$$\begin{aligned} &= \begin{bmatrix} 6-4 & 4-0 & -2+4 \\ -3+2 & -2+0 & 1-2 \\ 0+6 & 0+0 & 0-6 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 4 & 2 \\ -1 & -2 & -1 \\ 6 & 0 & -6 \end{bmatrix} \end{aligned}$$

$$\begin{aligned}\therefore A(BC) &= \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 & 4 & 2 \\ -1 & -2 & -1 \\ 6 & 0 & -6 \end{bmatrix} \\ &= \begin{bmatrix} 2-0+6 & 4-0+0 & 2-0-6 \\ 4-3+0 & 8-6+0 & 4-3-0 \\ 0-4+30 & 0-8+0 & 0-4-30 \end{bmatrix}\end{aligned}$$

$$\therefore A(BC) = \begin{bmatrix} 8 & 4 & -4 \\ 1 & 2 & 1 \\ 26 & -8 & -34 \end{bmatrix} \quad \dots(i)$$

$$\begin{aligned}AB &= \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ 0 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 2+0+0 & -2+0+3 \\ 4-3+0 & -4+3+0 \\ 0-4+0 & 0+4+15 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ -4 & 19 \end{bmatrix}\end{aligned}$$

$$\begin{aligned}\therefore (AB)C &= \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ -4 & 19 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ 2 & 0 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 6+2 & 4+0 & -2-2 \\ 3-2 & 2-0 & -1+2 \\ -12+38 & -8+0 & 4-38 \end{bmatrix}\end{aligned}$$

$$\therefore (AB)C = \begin{bmatrix} 8 & 4 & -4 \\ 1 & 2 & 1 \\ 26 & -8 & -34 \end{bmatrix} \quad \dots(ii)$$

$$\therefore A(BC) = \begin{bmatrix} 8 & 4 & -4 \\ 1 & 2 & 1 \\ 26 & -8 & -34 \end{bmatrix} \quad \dots(i)$$

$$\begin{aligned} AB &= \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ 0 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 2+0+0 & -2+0+3 \\ 4-3+0 & -4+3+0 \\ 0-4+0 & 0+4+15 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ -4 & 19 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \therefore (AB)C &= \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ -4 & 19 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ 2 & 0 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 6+2 & 4+0 & -2-2 \\ 3-2 & 2-0 & -1+2 \\ -12+38 & -8+0 & 4-38 \end{bmatrix} \end{aligned}$$

$$\therefore (AB)C = \begin{bmatrix} 8 & 4 & -4 \\ 1 & 2 & 1 \\ 26 & -8 & -34 \end{bmatrix} \quad \dots(ii)$$

From (i) and (ii), we get

$$A(BC) = (AB)C.$$

$$\begin{aligned}\text{ii. } BC &= \begin{bmatrix} 2 & -2 \\ 3 & 3 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 6-2 & 2-6 \\ 9+3 & 3+9 \\ -3+1 & -1+3 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 12 & 12 \\ -2 & 2 \end{bmatrix}\end{aligned}$$

$$\begin{aligned}\therefore A(BC) &= \begin{bmatrix} 2 & 4 & 3 \\ -1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 4 & -4 \\ 12 & 12 \\ -2 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 8+48-6 & -8+48+6 \\ -4+36-4 & 4+36+4 \end{bmatrix}\end{aligned}$$

$$\therefore A(BC) = \begin{bmatrix} 50 & 46 \\ 28 & 44 \end{bmatrix} \quad \dots(\text{i})$$

$$\begin{aligned}AB &= \begin{bmatrix} 2 & 4 & 3 \\ -1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 3 & 3 \\ -1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 4+12-3 & -4+12+3 \\ -2+9-2 & 2+9+2 \end{bmatrix}\end{aligned}$$

$$\therefore AB = \begin{bmatrix} 13 & 11 \\ 5 & 13 \end{bmatrix}$$

$$\therefore (AB)C = \begin{bmatrix} 13 & 11 \\ 5 & 13 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 39+11 & 13+33 \\ 15+13 & 5+39 \end{bmatrix}$$

$$\therefore (AB)C = \begin{bmatrix} 50 & 46 \\ 28 & 44 \end{bmatrix} \quad \dots(\text{ii})$$

From (i) and (ii), we get
 $A(BC) = (AB)C$

Question 7.

Verify that $A(B + C) = AB + AC$ in each of the following matrices:

i. $A = \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix}$

ii. $A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ -2 & 3 \\ 4 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 \\ -2 & 0 \\ 4 & -3 \end{bmatrix}$

Solution:

$$\begin{aligned} \text{i. } A(B + C) &= \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \left\{ \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix} \right\} \\ &= \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -1+4 & 1+1 \\ 3+2 & -2-1 \end{bmatrix} \\ &= \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix} \\ &= \begin{bmatrix} 12-10 & 8+6 \\ 6+15 & 4-9 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 14 \\ 21 & -5 \end{bmatrix} \quad \dots(i) \end{aligned}$$

$$\begin{aligned} AB + AC &= \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned}
 &= \begin{bmatrix} -4-6 & 4+4 \\ -2+9 & 2-6 \end{bmatrix} + \begin{bmatrix} 16-4 & 4+2 \\ 8+6 & 2-3 \end{bmatrix} \\
 &= \begin{bmatrix} -10 & 8 \\ 7 & -4 \end{bmatrix} + \begin{bmatrix} 12 & 6 \\ 14 & -1 \end{bmatrix} \\
 &= \begin{bmatrix} -10+12 & 8+6 \\ 7+14 & -4-1 \end{bmatrix} \\
 &= \begin{bmatrix} 2 & 14 \\ 21 & -5 \end{bmatrix} \quad \dots(ii)
 \end{aligned}$$

From (i) and (ii), we get

$$A(B + C) = AB + AC.$$

[Note: The question has been modified.]

$$\begin{aligned}
 \text{ii. } A(B + C) &= \begin{bmatrix} 1 & -1 & 3 \\ 2 & 3 & 2 \end{bmatrix} \left\{ \begin{bmatrix} 1 & 0 \\ -2 & 3 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ -2 & 0 \\ 4 & -3 \end{bmatrix} \right\} \\
 &= \begin{bmatrix} 1 & -1 & 3 \\ 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1+1 & 0+2 \\ -2-2 & 3+0 \\ 4+4 & 3-3 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & -1 & 3 \\ 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ -4 & 3 \\ 8 & 0 \end{bmatrix} \\
 &= \begin{bmatrix} 2+4+24 & 2-3+0 \\ 4-12+16 & 4+9+0 \end{bmatrix} \\
 &= \begin{bmatrix} 30 & -1 \\ 8 & 13 \end{bmatrix} \quad \dots(i)
 \end{aligned}$$

$AB + AC$

$$\begin{aligned} &= \begin{bmatrix} 1 & -1 & 3 \\ 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 3 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 3 \\ 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -2 & 0 \\ 4 & -3 \end{bmatrix} \\ &= \begin{bmatrix} 1+2+12 & 0-3+9 \\ 2-6+8 & 0+9+6 \end{bmatrix} + \begin{bmatrix} 1+2+12 & 2+0-9 \\ 2-6+8 & 4+0-6 \end{bmatrix} \\ &= \begin{bmatrix} 15 & 6 \\ 4 & 15 \end{bmatrix} + \begin{bmatrix} 15 & -7 \\ 4 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 15+15 & 6-7 \\ 4+4 & 15-2 \end{bmatrix} \\ &= \begin{bmatrix} 30 & -1 \\ 8 & 13 \end{bmatrix} \quad \dots(ii) \end{aligned}$$

Question 8.

If $A = \begin{bmatrix} 1 & -2 \\ 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 \\ 3 & 7 \end{bmatrix}$, find $AB - 2I$, where I is unit matrix of order 2.

Solution:

$$\begin{aligned} AB - 2I &= \begin{bmatrix} 1 & -2 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 3 & 7 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 3-6 & -1-14 \\ 15+18 & -5+42 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \\ &= \begin{bmatrix} -3 & -15 \\ 33 & 37 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} &= \begin{bmatrix} -3-2 & -15-0 \\ 33-0 & 37-2 \end{bmatrix} \\ &= \begin{bmatrix} -5 & -15 \\ 33 & 35 \end{bmatrix} \end{aligned}$$

Question 9.

If $A = \begin{bmatrix} 4 & 3 & 2 \\ -1 & 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 1 & -2 \end{bmatrix}$, show that matrix AB is non singular.

Solution:

im

$\therefore$ AB is non-singular matrix.

Question 10.

If $A =$, find the product $(A + I)(A - I)$.

Solution:

$$\begin{aligned} A + I &= \begin{bmatrix} 1 & 2 & 0 \\ 5 & 4 & 2 \\ 0 & 7 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1+1 & 2+0 & 0+0 \\ 5+0 & 4+1 & 2+0 \\ 0+0 & 7+0 & -3+1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 0 \\ 5 & 5 & 2 \\ 0 & 7 & -2 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} A - I &= \begin{bmatrix} 1 & 2 & 0 \\ 5 & 4 & 2 \\ 0 & 7 & -3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1-1 & 2-0 & 0-0 \\ 5-0 & 4-1 & 2-0 \\ 0-0 & 7-0 & -3-1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 \\ 5 & 3 & 2 \\ 0 & 7 & -4 \end{bmatrix} \end{aligned}$$

$$\therefore (A + I)(A - I)$$

$$\begin{aligned} &= \begin{bmatrix} 2 & 2 & 0 \\ 5 & 5 & 2 \\ 0 & 7 & -2 \end{bmatrix} \begin{bmatrix} 0 & 2 & 0 \\ 5 & 3 & 2 \\ 0 & 7 & -4 \end{bmatrix} \\ &= \begin{bmatrix} 0+10+0 & 4+6+0 & 0+4-0 \\ 0+25+0 & 10+15+14 & 0+10-8 \\ 0+35+0 & 0+21-14 & 0+14+8 \end{bmatrix} \\ &= \begin{bmatrix} 10 & 10 & 4 \\ 25 & 39 & 2 \\ 35 & 7 & 22 \end{bmatrix} \end{aligned}$$

[Note : Answer given in the textbook is $\begin{bmatrix} 9 & 6 & 4 \\ 15 & 32 & -2 \\ 35 & -7 & 29 \end{bmatrix}$

However, as per our calculation it is $\begin{bmatrix} 10 & 10 & 4 \\ 25 & 39 & 2 \\ 35 & 7 & 22 \end{bmatrix}$.]

Question 11.

If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, find α , if $A^2 = B$.

Solution:

$$A^2 = B$$

$$\therefore \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \alpha^2 + 0 & 0 + 0 \\ \alpha + 1 & 0 + 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$\therefore$ By equality of matrices, we get

$$\alpha^2 = 1 \text{ and } \alpha + 1 = 2$$

$$\therefore \alpha = \pm 1 \text{ and } \alpha = 1$$

$$\therefore \alpha = 1$$

Question 12.

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, show that $A^2 - 4A$ is scalar matrix.

Solution:

$$A^2 - 4A = A.A - 4A$$

$$\begin{aligned} &= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 4 \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 9-4 & 8-8 & 8-8 \\ 8-8 & 9-4 & 8-8 \\ 8-8 & 8-8 & 9-4 \end{bmatrix} \\ &= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}, \text{ which is a scalar matrix.} \end{aligned}$$

Question 13.

If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find k so that $A^2 - 8A - kI = O$, where I is a unit matrix and O is a null matrix of order 2.

Solution:

$$A^2 - 8A - kI = O$$

$$\therefore A.A - 8A - kI = O$$

$$\therefore \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} - 8 \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} - k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1+0 & 0+0 \\ -1-7 & 0+49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} - \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} - \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1-8-k & 0-0-0 \\ -8+8-0 & 49-56-k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore$ by equality of matrices, we get

$$1 - 8 - k = 0$$

$$\therefore k = -7$$

Question 14.

If $A = \begin{bmatrix} 8 & 4 \\ 10 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -4 \\ 10 & -8 \end{bmatrix}$, show that $(A+B)^2 = A^2 + AB + B^2$.

Solution:

We have to prove that $(A + B)^2 = A^2 + AB + B^2$,
i.e., to prove $A^2 + AB + BA + B^2 = A^2 + AB + B^2$,
i.e., to prove $BA = 0$.

$$BA = \begin{bmatrix} 5 & -4 \\ 10 & -8 \end{bmatrix} \begin{bmatrix} 8 & 4 \\ 10 & 5 \end{bmatrix} \\ \begin{bmatrix} 40 - 40 & 20 - 20 \\ 80 - 80 & 40 - 40 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Question 15.

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, prove that $A^2 - 5A + 7I = 0$, where I is unit matrix of order 2.

Solution:

$$A^2 - 5A + 7I = 0 = A.A - 5A + 7I = 0 \\ = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\ = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\ = \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} \\ = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Question 16.

If $A = \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$, show that $(A + B)(A - B) = A^2 - B^2$.

Solution:

We have to prove that $(A + B)(A - B) = A^2 - B^2$,

i.e., to prove $A^2 - AB + BA - B^2 = A^2 - B^2$,

i.e., to prove $-AB + BA = 0$,

i.e., to prove $AB - BA$.

$$\begin{aligned} AB &= \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 6-4 & 3+8 \\ -8-3 & -4+6 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 11 \\ -11 & 2 \end{bmatrix} \quad \dots(i) \end{aligned}$$

$$\begin{aligned} BA &= \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 6-4 & 8+3 \\ -3-8 & -4+6 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 11 \\ -11 & 2 \end{bmatrix} \quad \dots(ii) \end{aligned}$$

From (i) and (ii), we get $AB = BA$

Question 17.

If $A = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & a \\ -1 & b \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, find the values of a and b .

Solution:

Given, $(A + B)^2 = A^2 + B^2$

$$\therefore A^2 + AB + BA + B^2 = A^2 + B^2$$

$$\therefore AB + BA = 0$$

$$\therefore AB = -BA$$

$$\therefore \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 2 & a \\ -1 & b \end{bmatrix} = - \begin{bmatrix} 2 & a \\ -1 & b \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2-2 & a+2b \\ -2+2 & -a-2b \end{bmatrix} = - \begin{bmatrix} 2-a & 4-2a \\ -1-b & -2-2b \end{bmatrix}$$

$$\therefore \begin{bmatrix} 0 & a+2b \\ 0 & -a-2b \end{bmatrix} = \begin{bmatrix} -2+a & -4+2a \\ 1+b & 2+2b \end{bmatrix}$$

$\therefore$ by equality of matrices, we get

$$-2 + a = 0 \text{ and } 1 + b = 0$$

$$a = 2 \text{ and } b = -1$$

[Note: The question has been modified.]

Question 18.

Find matrix X such that $AX = B$,

$$\text{where } A = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$$

Solution:

$$\text{Let } X = \begin{bmatrix} a \\ b \end{bmatrix}$$

But $AX = B$

$$\therefore \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} a - 2b \\ -2a + b \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$$

By equality of matrices, we get

$$a - 2b = -3 \dots (i)$$

$$-2a + b = -1 \dots (ii)$$

By (i) $\times 2$ + (ii), we get

$$-3b = -7$$

$$\therefore b = \frac{7}{3}$$

Substituting $b = \frac{7}{3}$ in (i), we get

$$a - 2\left(\frac{7}{3}\right) = -3$$

$$\therefore a = -3 + \frac{14}{3} = \frac{5}{3}$$

$$\therefore X = \begin{bmatrix} \frac{5}{3} \\ \frac{7}{3} \end{bmatrix}$$

Question 19.

Find k , if $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $A^2 = kA - 2I$

Solution:

$$A^2 = kA - 2I$$

$$\therefore AA + 2I = kA$$

$$\therefore \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = k \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix}$$

$$\therefore \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix}$$

$\therefore$ By equality of matrices, we get

$$3k = 3$$

$$\therefore k = 1$$

Question 20.

Find x , if $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = 0$

Solution:

$$\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1-4+9 \\ 4-10+18 \\ 3-4+15 \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 12 \\ 14 \end{bmatrix} = 0$$

$$\therefore [6 + 12x + 14] = [0]$$

$\therefore$ By equality of matrices, we get

$$\therefore 12x + 20 = 0$$

$$\therefore 12x = -20$$

$$\therefore x = \frac{-5}{3}$$

Question 21.

Find x and y, if $\left\{ 4 \begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -3 & 4 \\ 2 & 1 & 1 \end{bmatrix} \right\} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

Solution:

$$\left\{ 4 \begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -3 & 4 \\ 2 & 1 & 1 \end{bmatrix} \right\} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\therefore \left\{ \begin{bmatrix} 8 & -4 & 12 \\ 4 & 0 & 8 \end{bmatrix} - \begin{bmatrix} 3 & -3 & 4 \\ 2 & 1 & 1 \end{bmatrix} \right\} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\therefore \begin{bmatrix} 8-3 & -4+3 & 12-4 \\ 4-2 & 0-1 & 8-1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\therefore \begin{bmatrix} 5 & -1 & 8 \\ 2 & -1 & 7 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\therefore \begin{bmatrix} 10+1+8 \\ 4+1+7 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\therefore \begin{bmatrix} 19 \\ 12 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$\therefore$ By equality of matrices, we get

$x = 19$ and $y = 12$

Question 22.

Find x, y, z if

$$\left\{ 3 \begin{bmatrix} 2 & 0 \\ 0 & 2 \\ 2 & 2 \end{bmatrix} - 4 \begin{bmatrix} 1 & 1 \\ -1 & 2 \\ 3 & 1 \end{bmatrix} \right\} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} x-3 \\ y-1 \\ 2z \end{bmatrix}$$

Solution:

$$\left\{ 3 \begin{bmatrix} 2 & 0 \\ 0 & 2 \\ 2 & 2 \end{bmatrix} - 4 \begin{bmatrix} 1 & 1 \\ -1 & 2 \\ 3 & 1 \end{bmatrix} \right\} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} x-3 \\ y-1 \\ 2z \end{bmatrix}$$

$$\therefore \left\{ \begin{bmatrix} 6 & 0 \\ 0 & 6 \\ 6 & 6 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ -4 & 8 \\ 12 & 4 \end{bmatrix} \right\} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} x-3 \\ y-1 \\ 2z \end{bmatrix}$$

$$\therefore \begin{bmatrix} 6-4 & 0-4 \\ 0+4 & 6-8 \\ 6-12 & 6-4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} x-3 \\ y-1 \\ 2z \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2 & -4 \\ 4 & -2 \\ -6 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} x-3 \\ y-1 \\ 2z \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2-8 \\ 4-4 \\ -6+4 \end{bmatrix} = \begin{bmatrix} x-3 \\ y-1 \\ 2z \end{bmatrix}$$

$$\therefore \begin{bmatrix} -6 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} x-3 \\ y-1 \\ 2z \end{bmatrix}$$

$\therefore$ By equality of matrices, we get

$$x-3 = -6, y-1 = 0, 2z = -2$$

$$\therefore x = -3, y = 1, z = -1$$

Question 23.

If $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$ show that $A^2 = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$

Solution:

$$A^2 = A.A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$\begin{aligned} &= \begin{bmatrix} \cos^2 \alpha - \sin^2 \alpha & \cos \alpha \sin \alpha + \cos \alpha \sin \alpha \\ -\cos \alpha \sin \alpha - \cos \alpha \sin \alpha & -\sin^2 \alpha + \cos^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \alpha - \sin^2 \alpha & 2 \sin \alpha \cos \alpha \\ -2 \sin \alpha \cos \alpha & \cos^2 \alpha - \sin^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix} \end{aligned}$$

Question 24.

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 4 \\ 2 & -1 \end{bmatrix}$

show that $AB \neq BA$, but $|AB| = |A| \cdot |B|$.

Solution:

$$\begin{aligned} AB &= \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ 2 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 0+4 & 4-2 \\ 0+10 & 12-5 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 2 \\ 10 & 7 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} BA &= \begin{bmatrix} 0 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 0+12 & 0+20 \\ 2-3 & 4-5 \end{bmatrix} \\ &= \begin{bmatrix} 12 & 20 \\ -1 & -1 \end{bmatrix} \neq AB \end{aligned}$$

$$\text{Now, } |AB| = \begin{vmatrix} 4 & 2 \\ 10 & 7 \end{vmatrix} = 28 - 20 = 8$$

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix} = 5 - 6 = -1$$

$$|B| = \begin{vmatrix} 0 & 4 \\ 2 & -1 \end{vmatrix} = 0 - 8 = -8$$

$$\therefore |A| \cdot |B| = (-1) \cdot (-8) = 8 = |AB|$$

$$\therefore AB \neq BA, \text{ but } |AB| = |A| \cdot |B|$$

Question 25.

Jay and Ram are two friends in a class. Jay wanted to buy 4 pens and 8 notebooks, Ram wanted to buy 5 pens and 12 notebooks. Both of them went to a shop. The price of a pen and a notebook which they have selected was 6 and ₹ 10. Using matrix multiplication, find the amount required from each one of them.

Solution:

Let A be the matrix of pens and notebooks and B be the matrix of prices of one pen and one notebook.

$$\therefore A = \begin{matrix} & \text{Pens} & \text{Notebooks} \\ \begin{bmatrix} 4 & 8 \\ 5 & 12 \end{bmatrix} & \begin{matrix} \text{Jay} \\ \text{Ram} \end{matrix} \end{matrix}$$

$$\text{and } B = \begin{matrix} \begin{bmatrix} 6 \\ 10 \end{bmatrix} & \begin{matrix} \text{Pen} \\ \text{Notebook} \end{matrix} \end{matrix}$$

The total amount required for each one of them is obtained by matrix AB.

$$\therefore AB = \begin{bmatrix} 4 & 8 \\ 5 & 12 \end{bmatrix} \begin{bmatrix} 6 \\ 10 \end{bmatrix}$$

$$\begin{aligned} \therefore AB &= \begin{bmatrix} 4 & 8 \\ 5 & 12 \end{bmatrix} \begin{bmatrix} 6 \\ 10 \end{bmatrix} \\ &= \begin{bmatrix} 24+80 \\ 30+120 \end{bmatrix} \\ &= \begin{bmatrix} 104 \\ 150 \end{bmatrix} \end{aligned}$$

$\therefore$ Jay needs ₹ 104 and Ram needs ₹ 150.

Ex 4.7

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-1-chapter-4-determinants-and-matrices/>

Question 1.

Construct a matrix $A = [a_{ij}]_{3 \times 2}$ whose elements a_{ij} are given by

i. $a_{ij} = \frac{(i-j)^2}{5-i}$

ii. $a_{ij} = i - 3j$

iii. $a_{ij} = \frac{(i+j)^3}{5}$

Solution:

$$A = [a_{ij}]_{3 \times 2}$$

$$\therefore A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

i. $a_{ij} = \frac{(i-j)^2}{5-i}$

$$\therefore a_{11} = \frac{(1-1)^2}{5-1} = 0, a_{12} = \frac{(1-2)^2}{5-1} = \frac{(-1)^2}{4} = \frac{1}{4},$$

$$a_{21} = \frac{(2-1)^2}{5-2} = \frac{1}{3}, a_{22} = \frac{(2-2)^2}{5-2} = 0,$$

$$a_{31} = \frac{(3-1)^2}{5-3} = \frac{2^2}{2} = 2, a_{32} = \frac{(3-2)^2}{5-3} = \frac{1}{2}$$

$$\therefore A = \begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{3} & 0 \\ 2 & \frac{1}{2} \end{bmatrix}$$

[Note: Answer given in the textbook is $A = \begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{2} & 0 \\ 2 & \frac{1}{2} \end{bmatrix}$

However, as per our calculation it is $\begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{3} & 0 \\ 2 & \frac{1}{2} \end{bmatrix}$].

ii. $a_{ij} = i - 3j$

$\therefore a_{11} = 1 - 3(1) = 1 - 3 = -2,$

$a_{12} = 1 - 3(2) = 1 - 6 = -5,$

$a_{21} = 2 - 3(1) = 2 - 3 = -1,$

$a_{22} = 2 - 3(2) = 2 - 6 = -4$

$a_{31} = 3 - 3(1) = 3 - 3 = 0,$

$a_{32} = 3 - 3(2) = 3 - 6 = -3$

$\therefore A = \begin{bmatrix} -2 & -5 \\ -1 & -4 \\ 0 & -3 \end{bmatrix}$

iii. $a_{ij} = \frac{(i+j)^3}{5}$

$\therefore a_{11} = \frac{(1+1)^3}{5} = \frac{2^3}{5} = \frac{8}{5}, a_{12} = \frac{(1+2)^3}{5} = \frac{3^3}{5} = \frac{27}{5}$

$a_{21} = \frac{(2+1)^3}{5} = \frac{3^3}{5} = \frac{27}{5}, a_{22} = \frac{(2+2)^3}{5} = \frac{4^3}{5} = \frac{64}{5}$

$a_{31} = \frac{(3+1)^3}{5} = \frac{4^3}{5} = \frac{64}{5}, a_{32} = \frac{(3+2)^3}{5} = \frac{5^3}{5} = \frac{125}{5}$

$\therefore A = \begin{bmatrix} \frac{8}{5} & \frac{27}{5} \\ \frac{27}{5} & \frac{64}{5} \\ \frac{64}{5} & \frac{125}{5} \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 8 & 27 \\ 27 & 64 \\ 64 & 125 \end{bmatrix}$

Question 2.

Classify the following matrices as a row, a column, a square, a diagonal, a scalar, a unit, an upper triangular, a lower triangular, a symmetric or a skew-symmetric matrix.

i.
$$\begin{bmatrix} 3 & -2 & 4 \\ 0 & 0 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 3 & -2 & 4 \\ 0 & 0 & -5 \\ 0 & 0 & 0 \end{bmatrix}$

As every element below the diagonal is zero in matrix A.

$\therefore$ A is an upper triangular matrix.

ii.
$$\begin{bmatrix} 0 & 4 & 7 \\ -4 & 0 & -3 \\ -7 & 3 & 0 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 0 & 4 & 7 \\ -4 & 0 & -3 \\ -7 & 3 & 0 \end{bmatrix}$

$\therefore A^T = \begin{bmatrix} 0 & -4 & -7 \\ 4 & 0 & 3 \\ 7 & -3 & 0 \end{bmatrix}$

$\therefore A^T = - \begin{bmatrix} 0 & 4 & 7 \\ -4 & 0 & -3 \\ -7 & 3 & 0 \end{bmatrix}$

$$\therefore A^T = -A, \text{ i.e., } A = -A^T$$

$\therefore A$ is a skew-symmetric matrix.

Example 10. Find the rank of

iii. $\begin{bmatrix} 5 \\ 4 \\ -3 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 5 \\ 4 \\ -3 \end{bmatrix}$

$\therefore$ As matrix A has only one column.

$\therefore A$ is a column matrix.

iv. $[9 \quad \sqrt{2} \quad -3]$

Solution:

Let $A = [9 \quad \sqrt{2} \quad -3]$

As matrix A has only one row.

$\therefore A$ is a row matrix.

v. $\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

As matrix A has all its non-diagonal elements zero and diagonal elements same.

$\therefore$ A is a scalar matrix.

vi. $\begin{bmatrix} 2 & 0 & 0 \\ 3 & -1 & 0 \\ -7 & 3 & 1 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 2 & 0 & 0 \\ 3 & -1 & 0 \\ -7 & 3 & 1 \end{bmatrix}$

As every element above the diagonal is zero in matrix A.

$\therefore$ A is a lower triangular matrix.

vii. $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$

As matrix A has all its non-diagonal elements zero.

$\therefore$ A is a diagonal matrix.

viii.
$$\begin{bmatrix} 10 & -15 & 27 \\ -15 & 0 & \sqrt{34} \\ 27 & \sqrt{34} & \frac{5}{3} \end{bmatrix}$$

Solution:

$$\text{Let } A = \begin{bmatrix} 10 & -15 & 27 \\ -15 & 0 & \sqrt{34} \\ 27 & \sqrt{34} & \frac{5}{3} \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 10 & -15 & 27 \\ -15 & 0 & \sqrt{34} \\ 27 & \sqrt{34} & \frac{5}{3} \end{bmatrix}$$

$$\therefore A^T = A, \text{ i/e., } A = A^T$$

$\therefore A$ is a symmetric matrix.

ix.
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In matrix A , all the non-diagonal elements are zero and diagonal elements are one.

$\therefore A$ is a unit (identity) matrix.

x. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$\therefore A^T = A$, i/e., $A = A^T$

$\therefore A$ is a symmetric matrix.

$\therefore A$ is a symmetric matrix.

Question 3.

Which of the following matrices are singular or non-singular?

i. $\begin{bmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \\ \mathbf{p} & \mathbf{q} & \mathbf{r} \\ \mathbf{2a - p} & \mathbf{2b - q} & \mathbf{2c - r} \end{bmatrix}$

ii. $\begin{bmatrix} 5 & 0 & 5 \\ 1 & 99 & 100 \\ 6 & 99 & 105 \end{bmatrix}$

iii. $\begin{bmatrix} 3 & 5 & 7 \\ -2 & 1 & 4 \\ 3 & 2 & 5 \end{bmatrix}$

iv. $\begin{bmatrix} 7 & 5 \\ -4 & 7 \end{bmatrix}$

Solution:

$$\text{Let } A = \begin{bmatrix} a & b & c \\ p & q & r \\ 2a-p & 2b-q & 2c-r \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} a & b & c \\ p & q & r \\ 2a-p & 2b-q & 2c-r \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 + R_2$, we get

$$|A| = \begin{vmatrix} a & b & c \\ p & q & r \\ 2a & 2b & 2c \end{vmatrix}$$

Taking 2 common from R_3 , we get

$$|A| = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ a & b & c \end{vmatrix}$$

$$= 2(0) \quad \dots [\because R_1 \text{ and } R_3 \text{ are identical}]$$

$$= 0$$

$\therefore$ A is a singular matrix.

$$\text{ii. Let } A = \begin{bmatrix} 5 & 0 & 5 \\ 1 & 99 & 100 \\ 6 & 99 & 105 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 5 & 0 & 5 \\ 1 & 99 & 100 \\ 6 & 99 & 105 \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 + C_1$

$$|A| = \begin{vmatrix} 5 & 5 & 5 \\ 1 & 100 & 100 \\ 6 & 105 & 105 \end{vmatrix}$$

$= 0 \dots [\because C_2 \text{ and } C_3 \text{ are identical}]$

$\therefore A$ is a singular matrix.

iii. Let $A = \begin{bmatrix} 3 & 5 & 7 \\ -2 & 1 & 4 \\ 3 & 2 & 5 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 3 & 5 & 7 \\ -2 & 1 & 4 \\ 3 & 2 & 5 \end{vmatrix}$$

$$= 3(5 - 8) - 5(-10 - 12) + 7(-4 - 3)$$

$$= -9 + 110 - 49 = 52 \neq 0$$

$\therefore A$ is a non-singular matrix.

iv. Let $A = \begin{bmatrix} 7 & 5 \\ -4 & 7 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 7 & 5 \\ -4 & 7 \end{vmatrix} = 49 + 20 = 69 \neq 0$$

Question 4.

Find k, if the following matrices are singular.

i. $\begin{bmatrix} 7 & 3 \\ -2 & k \end{bmatrix}$

ii. $\begin{bmatrix} 4 & 3 & 1 \\ 7 & k & 1 \\ 10 & 9 & 1 \end{bmatrix}$

iii. $\begin{bmatrix} k-1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & -2 & 4 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 7 & 3 \\ -2 & k \end{bmatrix}$

Since A is a singular matrix,

$$|A|=0$$

$$\therefore \begin{vmatrix} 7 & 3 \\ -2 & k \end{vmatrix} = 0$$

$$\therefore 7k + 6 = 0$$

$$\therefore 7k = -6$$

$$k = -6/7$$

ii. Let $A = \begin{bmatrix} 4 & 3 & 1 \\ 7 & k & 1 \\ 10 & 9 & 1 \end{bmatrix}$

Since A is a singular matrix,

$$|A| = 0$$

$$\therefore \begin{vmatrix} 4 & 3 & 1 \\ 7 & k & 1 \\ 10 & 9 & 1 \end{vmatrix} = 0$$

$$\therefore 4(k - 9) - 3(7 - 10) + 1(63 - 10k) = 0$$

$$\therefore 4k - 36 + 9 + 63 - 10k = 0$$

$$\therefore -6k + 36 = 0$$

$$\therefore 6k = 36$$

$$\therefore k = 6$$

iii. Let $A = \begin{bmatrix} k-1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & -2 & 4 \end{bmatrix}$

Since A is a singular matrix

$$|A| = 0$$

$$\therefore \begin{vmatrix} k-1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & -2 & 4 \end{vmatrix}$$

$$\therefore (k-1)(4+4) - 2(12-2) + 3(-6-1) = 0$$

$$\therefore 8k-8-20-21=0$$

$$\therefore 8k = 49$$

$$\therefore k = 49/8$$

Question 5.

If $A = \begin{bmatrix} 5 & 1 & -1 \\ 3 & 2 & 0 \end{bmatrix}$, find $(A^T)^T$.

Solution:

$$A = \begin{bmatrix} 5 & 1 & -1 \\ 3 & 2 & 0 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 5 & 3 \\ 1 & 2 \\ -1 & 0 \end{bmatrix}$$

$$\therefore (A^T)^T = \begin{bmatrix} 5 & 1 & -1 \\ 3 & 2 & 0 \end{bmatrix}$$

Question 6.

If $A =$, find $(A^T)^T$.

Solution:

$$A = \begin{bmatrix} 7 & 3 & 1 \\ -2 & -4 & 1 \\ 5 & 9 & 1 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 7 & -2 & 5 \\ 3 & -4 & 9 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\therefore (A^T)^T = \begin{bmatrix} 7 & 3 & 1 \\ -2 & -4 & 1 \\ 5 & 9 & 1 \end{bmatrix}$$

Question 7.

Find a, b, c, if $\begin{bmatrix} 1 & \frac{3}{5} & a \\ b & -5 & -7 \\ -4 & c & 0 \end{bmatrix}$ is a symmetric matrix.

Solution:

$$\text{Let } A = \begin{bmatrix} 1 & \frac{3}{5} & a \\ b & -5 & -7 \\ -4 & c & 0 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 1 & b & -4 \\ \frac{3}{5} & -5 & c \\ a & -7 & 0 \end{bmatrix}$$

Since A is a symmetric matrix,

$$A = A^T$$

$$\therefore \begin{bmatrix} 1 & \frac{3}{5} & a \\ b & -5 & -7 \\ -4 & c & 0 \end{bmatrix} = \begin{bmatrix} 1 & b & -4 \\ \frac{3}{5} & -5 & c \\ a & -7 & 0 \end{bmatrix}$$

$\therefore$ By equality of matrices, we get

$$a = -4, b = \frac{3}{5}, c = -7$$

Question 8.

Find x, y, z, if is a symmetric matrix.

Solution:

$$\text{Let } A = \begin{bmatrix} 0 & -5i & x \\ y & 0 & z \\ \frac{3}{2} & -\sqrt{2} & 0 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 0 & y & \frac{3}{2} \\ -5i & 0 & -\sqrt{2} \\ x & z & 0 \end{bmatrix}$$

Since A is a skew-symmetric matrix,

$$A = -A^T$$

$$\therefore \begin{bmatrix} 0 & -5i & x \\ y & 0 & z \\ \frac{3}{2} & -\sqrt{2} & 0 \end{bmatrix} = - \begin{bmatrix} 0 & y & \frac{3}{2} \\ -5i & 0 & -\sqrt{2} \\ x & z & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 0 & -5i & x \\ y & 0 & z \\ \frac{3}{2} & -\sqrt{2} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -y & \frac{-3}{2} \\ 5i & 0 & \sqrt{2} \\ -x & -z & 0 \end{bmatrix}$$

$\therefore$ By equality of matrices, we get

$$x = \frac{-3}{2}, y = 5i, z = \sqrt{2}$$

Question 9.

For each of the following matrices, using its transpose, state whether it is symmetric, skew-symmetric or neither.

symmetric or neither.

i. $\begin{bmatrix} 1 & 2 & -5 \\ 2 & -3 & 4 \\ -5 & 4 & 9 \end{bmatrix}$

ii. $\begin{bmatrix} 2 & 5 & 1 \\ -5 & 4 & 6 \\ -1 & -6 & 3 \end{bmatrix}$

iii. $\begin{bmatrix} 0 & 1+2i & i-2 \\ -1-2i & 0 & -7 \\ 2-i & 7 & 0 \end{bmatrix}$

Solution:

i. Let $A = \begin{bmatrix} 1 & 2 & -5 \\ 2 & -3 & 4 \\ -5 & 4 & 9 \end{bmatrix}$

$$\therefore A^T = \begin{bmatrix} 1 & 2 & -5 \\ 2 & -3 & 4 \\ -5 & 4 & 9 \end{bmatrix}$$

$$\therefore A^T = A, \text{ i.e., } A = A^T$$

$\therefore A$ is a symmetric matrix.

ii.

$$\text{Let } A = \begin{bmatrix} 2 & 5 & 1 \\ -5 & 4 & 6 \\ -1 & -6 & 3 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 2 & -5 & -1 \\ 5 & 4 & -6 \\ 1 & 6 & 3 \end{bmatrix}$$

$$\therefore A^T = - \begin{bmatrix} -2 & 5 & 1 \\ -5 & -4 & 6 \\ -1 & -6 & -3 \end{bmatrix}$$

$$\therefore A \neq A^T, \text{ i.e., } A \neq -A^T$$

$\therefore A$ is neither a symmetric nor skew-symmetric matrix.

iii.

$$\text{Let } A = \begin{bmatrix} 0 & 1+2i & i-2 \\ -1-2i & 0 & -7 \\ 2-i & 7 & 0 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 0 & -1-2i & 2-i \\ 1+2i & 0 & 7 \\ i-2 & -7 & 0 \end{bmatrix}$$

$$\therefore A^T = - \begin{bmatrix} 0 & 1+2i & i-2 \\ -1-2i & 0 & -7 \\ 2-i & 7 & 0 \end{bmatrix}$$

$$\therefore A^T = -A, \text{ i.e., } A = -A^T$$

$\therefore A$ is a skew-symmetric matrix.

Question 10.

Construct the matrix $A = [a_{ij}]_{3 \times 3}$, where $a_{ij} = i - j$. State whether A is symmetric or skew-symmetric.

Solution:

$$A = [a_{ij}]_{3 \times 3}$$

$$\therefore A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Given, $a_{ij} = i - j$

$$a_{11} = 1 - 1 = 0, a_{12} = 1 - 2 = -1, a_{13} = 1 - 3 = -2,$$

$$a_{21} = 2 - 1 = 1, a_{22} = 2 - 2 = 0, a_{23} = 2 - 3 = -1,$$

$$a_{31} = 3 - 1 = 2, a_{32} = 3 - 2 = 1, a_{33} = 3 - 3 = 0$$

$$\therefore A = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$\therefore A^T = -A, \text{ i.e., } A = -A^T$$

$\therefore A$ is a skew-symmetric matrix.



Maharashtra Board Solutions

Class 11 Arts & Science Maths

(Part 1)

- Chapter 1- Angle and its Measurement
- Chapter 2- Trigonometry – I
- Chapter 3- Trigonometry – II
- Chapter 4- Determinants and Matrices
- Chapter 5- Straight Line
- Chapter 6- Circle
- Chapter 7- Conic Sections
- Chapter 8- Measures of Dispersion
- Chapter 9- Probability

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The Maharashtra State Board of Secondary and Higher Secondary Education or MSBSHSE (Marathi: महाराष्ट्र राज्य माध्यमिक आणि उच्च माध्यमिक शिक्षण मंडळ), is an **autonomous and statutory body established in 1965**. The board was amended in the year 1977 under the provisions of the Maharashtra Act No. 41 of 1965.

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