1. $\lim_{n\to\infty} \dfrac{1^{99}+2^{99}+\cdots+n^{99}}{n^{100}}$ equals to

   (A) $\frac{9}{100}$ (B) $\frac{1}{100}$ (C) $\frac{1}{99}$ (D) $\frac{1}{101}$

2. If $\frac{1}{a}+\frac{1}{c}+\frac{1}{a-b}+\frac{1}{c-b}=0$, and $b \neq a+c$, then which one of the following is true?

   (A) a, b, c are in GP (B) a, b, c are in AP
   (C) a, b, c are in HP (D) None of these

3. If a, b, c are distinct positive rational numbers and they are in AP, then the roots of the equation $ax^2+bx+c=0$ are

   (A) imaginary (B) rational and equal
   (C) irrational (D) rational and unequal

4. If $ab = 2a+3b, a>0, b>0$, then the minimum value of ab is

   (A) 18 (B) 36 (C) 24 (D) $\frac{1}{4}$

5. The value of $\displaystyle\int_{-\infty}^{0} e^{x+e^x}\,dx$ is

   (A) 0 (B) $e-1$ (C) e (D) 1

6. If $|a|<1$ and $|b|<1$, then the sum of the series

   $$1+(1+a)b+(1+a+a^2)b^2+\cdots$$

   is

   (A) $\dfrac{1}{(1-a)(1-b)}$ (B) $\dfrac{1}{(1-a)(1-ab)}$
   (C) $\dfrac{1}{(1-b)(1-ab)}$ (D) $\dfrac{1}{(1-a)(1-b)(1-ab)}$

7. Suppose P is the set of different terms obtained in the expansion of $(a+b+\cdots+z)^2$, Q is the set of different terms obtained in the expansion of $(a+b+2)^2$ and S is the set of elements in $P \backslash Q$. Then the cardinality of S is

   (A) 348 (B) 354 (C) 351 (D) 345

8. Let

$$f(x) = \begin{cases} \dfrac{x-4}{|x-4|} + a, & \text{for } x < 4 \\ a+b, & \text{for } x = 4 \\ \dfrac{x-4}{|x-4|} + b, & \text{for } x > 4. \end{cases}$$

   Then $f(x)$ is continuous at $x = 4$, when

   (A) $a = 0$ and $b = 0$ (B) $a = 1$ and $b = 1$
   (C) $a = -1$ and $b = 1$ (D) $a = 1$ and $b = -1$

9. Consider the following matrix

$$M = \begin{bmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{bmatrix}$$

   The determinant of M is

   (A) -1 (B) 1 (C) 0 (D) $\frac{1}{2}$

10. If $\log_2(5.2^x + 1)$, $\log_4(2^{1-x} + 1)$ and 1 are in AP, then x equals to

    (A) $\log_2 5$ (B) $1 - \log_2 5$
    (C) $\log_5 2$ (D) None of these

11. Let $f(x)$ be a quadratic polynomial such that $f(-2)+f(3)=0$. If one of the roots of $f(x)=0$ is -1, then the sum of the roots of $f(x)=0$ is equal to

(A) $\frac{11}{3}$ (B) $\frac{7}{3}$ (C) $\frac{13}{3}$ (D) $\frac{14}{3}$

12. Let $f(x)$ be a continuous function in $[0,\pi]$. If $f(\pi)=2$ and $\int_0^\pi \big(f(x)+f''(x)\big)\sin x\,dx=5$, then $f(0)$ equals

(A) 0 (B) 1 (C) 2 (D) 3

13. Let x and y be distinct integers, where $1\le x\le 25$ and $1\le y\le 25$. Then the probability that $x+y$ is divisible by 5 is

(A) $\frac{125}{625}$ (B) $\frac{120}{625}$ (C) $\frac{125}{600}$ (D) $\frac{120}{600}$

14. If $Z=\log_2\log_2\log_4 256+\log_{\sqrt{2}}2$, then Z is equal to

(A) 2 (B) 3 (C) 5 (D) 7

15. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour is returned to the bag. If another ball is drawn at random from the bag, then what is the probability that this drawn ball is red?

(A) $\frac{1}{5}$ (B) $\frac{3}{4}$ (C) $\frac{3}{10}$ (D) $\frac{2}{5}$

16. A fair coin is tossed 100 times. The probability of getting odd number of tails is

(A) $\frac{1}{100}$ (B) $\frac{1}{50}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

17. Let $f(x) = \dfrac{9^x}{9^x + 3}$, then $f\left(\dfrac{1}{1996}\right) + f\left(\dfrac{2}{1996}\right) + \cdots + f\left(\dfrac{1995}{1996}\right)$ equals

(A) 996.5 (B) 997.5 (C) 998.5 (D) 999.5

18. Two six-faced dice are tossed. What is the probability that the total score is a prime number?

(A) $\dfrac{1}{3}$ (B) $\dfrac{5}{12}$ (C) $\dfrac{4}{9}$ (D) $\dfrac{7}{18}$

19. If the area bounded by $y = kx^2$ and $x = ky^2,\ k > 0$, is 1, then the value of k is

(A) 1 (B) $\dfrac{1}{\sqrt{3}}$ (C) $\dfrac{1}{3}$ (D) $\dfrac{2}{3}$

20. Consider the equation $-x^2 + \alpha x - \beta = 0,\ \alpha > 0, \beta > 0$. For what values of α and β, this equation has distinct roots?

(A) 5 and 4 (B) 1 and 1
(C) 3 and 4 (D) 6 and 10

21. The last digit in 7^{300} is

(A) 1 (B) 3 (C) 7 (D) 9

22. The value of $\dfrac{2}{3!} + \dfrac{4}{5!} + \dfrac{6}{7!} + \cdots$ is equal to

(A) e^{-1} (B) e^{-2} (C) $1 - \dfrac{1}{e}$ (D) e

23. Consider the following system of equations

$$\begin{bmatrix} -5 & -6 & 3 \\ -3 & 6 & p \\ 11 & -6 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ -1 \end{bmatrix}.$$

For what value of p, this system of equations does not have a unique solution?

(A) 1 (B) 0 (C) -1 (D) 2

24. Consider a matrix $P = \begin{bmatrix} 0 & p \\ 0 & 0 \end{bmatrix}$, $p \in \mathbb{R}$ and I is a 2×2 identity matrix, then $I + P + P^2 + \cdots + P^{21}$ equals

(A) $\begin{bmatrix} 0 & p \\ 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & p \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & p^{21} \\ 0 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & p^{20} \\ 0 & 1 \end{bmatrix}$

25. A hydraulic cylinder manufacturing company has decided to outsource the surface hardening process of piston rods to one of the five shortlisted heat treatment companies. Each company has been given 100 piston rods for surface hardening, and any rod with unsatisfactory surface hardness will be rejected. The number of rejected rods is less than 10 for each company. If the median rejection is 5 and the mode is 6, what is the maximum number of piston rods that could be rejected for a single heat treatment company?

(A) 5 (B) 6 (C) 7 (D) 8

26. A random variable X follows a Poisson distribution with parameter $\lambda > 0$. The probability that X takes the value 2 is equal to the probability that X takes the value 3. The value of λ is

(A) 1.5 (B) 2 (C) 3 (D) 6

27. Starting with $n = 1$, a formula

$$U_n = a + (n-1)b + \frac{(n-1)(n-2)c}{2}$$

generates the sequence $4, 11, 24, 43, \ldots$ Assuming it has 101 terms, the median of the data generated is

(A) 7704 (B) 7410 (C) 7854 (D) 8004

28. If the expression $9\tan(\theta + 30) + 6\tan(\theta - 30)$ can be written as $15\tan\theta + p\tan^2\theta + \frac{17}{1-q\tan^2\theta}$, then p and q are

(A) 12 and 13 (B) 5 and 12
(C) 6 and 13 (D) 5 and 13

29. The sum of the rational terms in the expansion of $\left(\sqrt{2} + \sqrt[5]{3}\right)^{10}$ is

(A) 32 (B) 41 (C) 18 (D) 82

30. A bag contains 9 red, 4 white and 3 blue marbles. One marble is picked up at random. Find the probability that the marble will be red or blue.

(A) $\frac{7}{16}$ (B) $\frac{4}{5}$ (C) $\frac{3}{4}$ (D) $\frac{13}{16}$