AJ-1553

M.A./M.Sc. (Final)

Term End Examination, 2021-22

MATHEMATICS

FUZZY SETS AND THEIR APPLICATIONS

Time : Three hours] **[Maximum Marks : 100**

Note: *Answer any five questions. All questions carry equal marks.*

1. (a) Order the Fuzzy sets defined by the following membership function grades. (assuming $x = 0$) by the inclusive relation.

$$\tilde{A}(x) = \frac{1}{1+10x}, \tilde{B}(x) = \left(\frac{1}{1+10x}\right)^{1/2}, \tilde{C}(x) = \left(\frac{1}{1+10x}\right)^{2}$$

(b) Prove that Fuzzy set $\tilde{A}$ on R is convex if and only if $\tilde{A}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\tilde{A}(x_1), \tilde{A}(x_2)\}$ for all $x_{1,} x_2 \in R$ and $\lambda \in [0, 1]$.

2. (a) If f is a decreasing generator, then a function g defined by $g(a) = f(0) - f(a)$ For any $a \in [0,1]$ is an increasing generator with $g(1) = f(0)$ and its pseudo-inverse $g^{(-1)}(a)$ is given by $g^{(-1)}(a) = f^{(-1)}(f(o) - a)$ for any $a \in R$.

(b) If $< i\ , u, c >$ is a dual triple that satisfies the law of excluded middle and the law of contradiction. Then prove that $< i, u, c, >$ does not satisfy the distributive laws.

3. (a) If $\tilde{A}$ and $\tilde{B}$ are two fuzzy sets defined on universal set X whose membership functions are given by:

$$\tilde{A} = \frac{\cdot 5}{-1} + \frac{1}{0} + \frac{\cdot 5}{1} + \frac{\cdot 3}{2}$$

$$\tilde{B} = \frac{\cdot 5}{2} + \frac{1}{3} + \frac{\cdot 5}{4} + \frac{\cdot 3}{5}$$

If a function $f : x \times x \to X$ be defined for all $x_1 x_2 \in X$ by $f(x_1, x_2) = x_1 \cdot x_2$ ·Find $f(\tilde{A}, \tilde{B})$.

(b) Prove that the standard Fuzzy intersection is the only idempotent t-norm. Define symmetric fuzzy relation with example.

4. (a) If $\tilde{R}$ is symmetric, then prove that each power of $\tilde{R}$ is symmetric.

(b) What do you mean by transitive closure. Find the transitive max-min closure of Fuzzy relations :

$$\tilde{R} = \begin{bmatrix} 0\cdot 8 & 0\cdot 6 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0\cdot 2 & 0\cdot 4 & 0 & 0 \\ 0 & 0 & 0\cdot 9 & 0\cdot 8 \end{bmatrix}$$

5. Solve the following fuzzy relations equation using max-min composition

$$\tilde{p}\ o \begin{bmatrix} 0\cdot 9 & 0\cdot 6 & 1 \\ 0\cdot 8 & 0\cdot 8 & 0\cdot 5 \\ 0\cdot 6 & 0\cdot 4 & 0\cdot 6 \end{bmatrix} = [0\cdot 6 \quad 0\cdot 6 \quad 0\cdot 5]$$

[P.T.O.]

6. Consider the ''if then'' rules

(i) if X is $\widetilde{A}_1$, then Y is $\widetilde{B}_1$ (ii) If X is $\widetilde{A}_2$, then Y is $\widetilde{B}_2$

Where $\widetilde{A}_J$ and $\widetilde{B}_j$ are Fuzzy sets such that

$$\widetilde{A}_1 = \frac{1}{x_1} + \frac{0\cdot 9}{x_2} + \frac{0\cdot 1}{x_3}$$

$$\widetilde{A}_2 = \frac{0\cdot 9}{x_1} + \frac{1}{x_2} + \frac{0\cdot 2}{x_3}$$

$$\widetilde{B}_1 = \frac{1}{y_1} + \frac{0\cdot 2}{y_2} \text{ and } \widetilde{B}_2 = \frac{0\cdot 2}{y_1} + \frac{0\cdot 9}{y_2}$$

Given that fact: X is $\widetilde{A}$' where

$$\widetilde{A}' = \frac{0.8}{x_1} + \frac{0.9}{x_2} + \frac{0.1}{x_3}$$

Using the method of interpolation, calculate the conclusion $\widetilde{B}$'.

7. (a) If a given finite body of evidence (A, m) is nested then for all $A, B, \in (P(x)$, prove that:

(i) bel $(\widetilde{A} \cap \widetilde{B}) =$ min { bel $(\widetilde{A})$, bel $(\widetilde{B})$}

(ii) pl $(\widetilde{A} \cup \widetilde{B}) =$ max {pl $(\widetilde{A})$, pl $(\widetilde{B})$}

(b) If a Fuzzy set F is defined on N by:

$$F = \frac{0\cdot 4}{1} + \frac{0\cdot 7}{2} + \frac{1}{3} + \frac{0\cdot 8}{4} + \frac{0\cdot 5}{5} \text{ and } A(x) = 0 \text{ for all } x \notin \{1, 2, 3, 4, 5\}.$$

Find nec $(\widetilde{A})$ and pos $(\widetilde{A})$ induced by F for all $A \in P(\{1, 2, 3, 4, 5\})$.

8. (a) Write an essay on fuzzy quantifiers.

(b) Write short note on Fuzzy implication rule.

9. (a) Discuss the methods of defuzzifications.

(b) Write short notes on:

(i) Fuzzy rule base (ii) Fuzzy inference engine.

10. (a) Solve the following Fuzzy linear programming problem :

Max $z = 5x_1 + 4x_2$

Under $(4, 2, 1)\ x_1 + (5, 3, 1)\ x_2 \le (24, 5, 8)$

$(4, 1, 2)\ x_1 + (1, 0\cdot 5, 1)\ x_2 \le (12, 6, 3)$

$x, x_2 \ge 0$

(b) If each individual of four decision maker has a total preference ordering $P_i\ (i \in N)$ on a set of alternatives.

$X = \{a, b, c, d\}$ as

$P_1 = \{a, b, d, c\}$

$P_2 = \{a, c, b, d\}$

$P_3 = \{b, a, c, d\}$

$P_4 = \{a, d, b, c\}$

Find the fuzzy preference relation. Also find the α - cuts of the fuzzy relation.