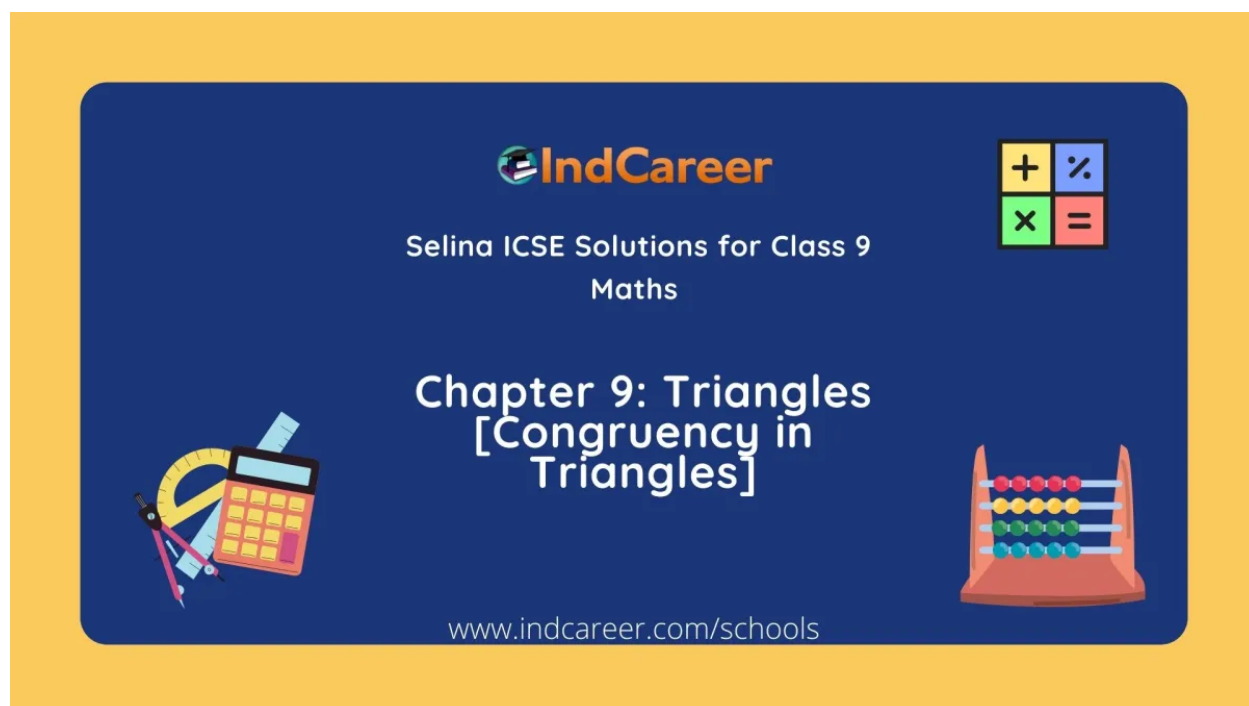


Selina Class 9 ICSE Solutions Mathematics :

Chapter 9- Triangles [Congruency in Triangles]



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Selina Class 9 ICSE Solutions Mathematics :

Chapter 9- Triangles [Congruency in Triangles]

Class 9: Maths Chapter 9 solutions. Complete Class 9 Maths Chapter 9 Notes.

Selina Class 9 ICSE Solutions Mathematics : Chapter 9- Triangles [Congruency in Triangles]

Selina 9th Maths Chapter 9, Class 9 Maths Chapter 9 solutions

Exercise 9(A)

Solution 1:

<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangles-congruency-in-triangles/>

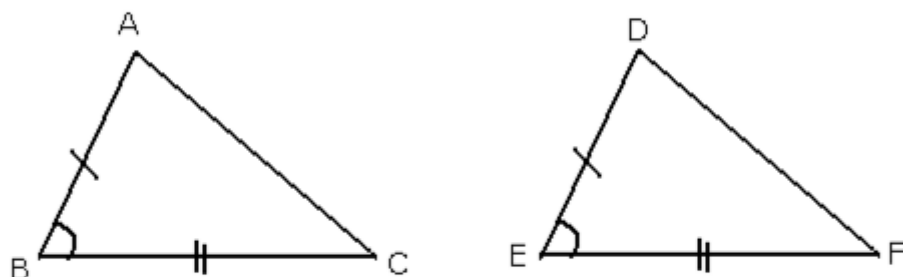
(a)

In $\triangle ABC$ and $\triangle DEF$

$AB = DE$ [Given]

$\angle B = \angle E$ [Given]

$BC = EF$ [Given]



By Side-Angle-Side criterion of congruency, the triangles $\triangle ABC$ and $\triangle DEF$ are congruent to each other.

$\therefore \triangle ABC \cong \triangle DEF$

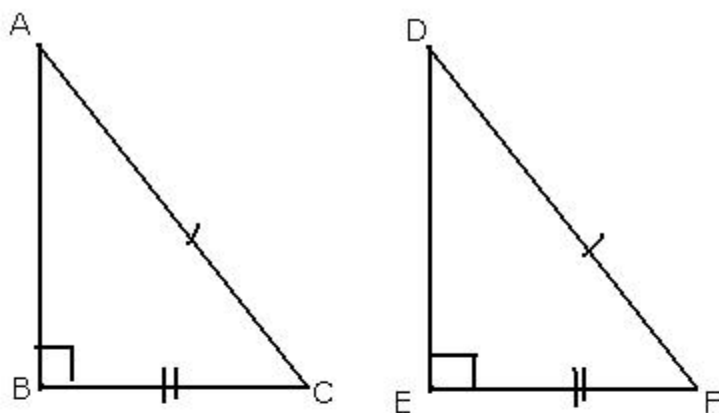
(b)

In $\triangle ABC$ and $\triangle DEF$

$\angle B = \angle E = 90^\circ$

Hyp. $AC = \text{Hyp. } DF$

$BC = EF$



By Right Angle-Hypotenuse-Side criterion of congruency, the triangles $\triangle ABC$ and $\triangle DEF$ are congruent to each other.
 $\therefore \triangle ABC \cong \triangle DEF$

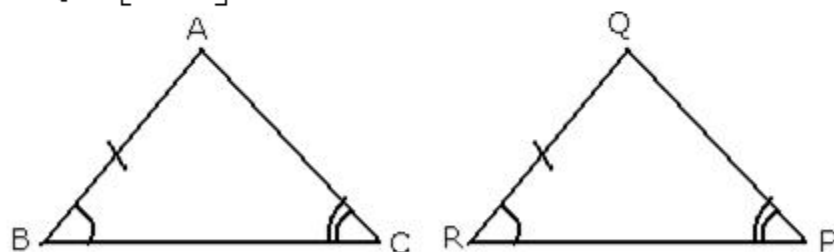
(c)

In $\triangle ABC$ and $\triangle QRP$

$\angle B = \angle R$ [Given]

$\angle C = \angle P$ [Given]

$AB = QR$ [Given]



By Angle-Angle-Side criterion of congruency, the triangles $\triangle ABC$ and $\triangle QRP$ are congruent to each other.
 $\therefore \triangle ABC \cong \triangle QRP$

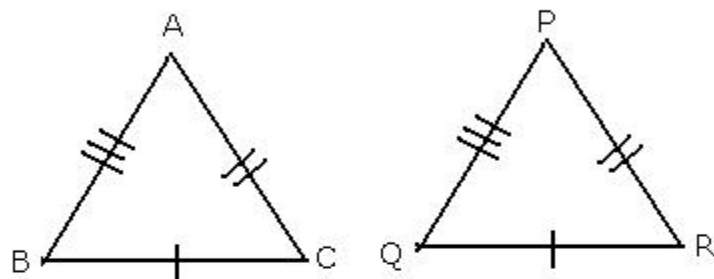
(d)

In $\triangle ABC$ and $\triangle PQR$

$AB=PQ$ [Given]

$AC=PR$ [Given]

$BC=QR$ [Given]



By Side-Side-Side criterion of congruency, the triangles $\triangle ABC$ and $\triangle PQR$ are congruent to each other.
 $\therefore \triangle ABC \cong \triangle PQR$

(e)

In $\triangle PQR$

$\angle R=40^\circ, \angle Q=50^\circ$

$\angle P + \angle Q + \angle R = 180^\circ$ [Sum of all the angles in a triangle = 180°]

$\Rightarrow \angle P + 50^\circ + 40^\circ = 180^\circ$

$\Rightarrow \angle P + 90^\circ = 180^\circ$

$\Rightarrow \angle P = 180^\circ - 90^\circ$

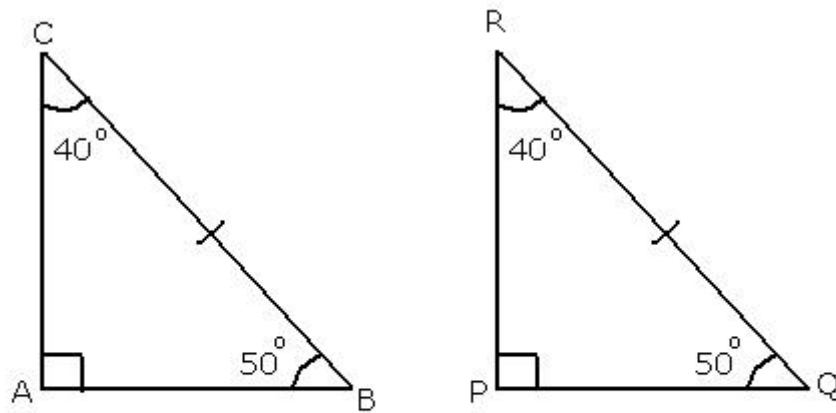
$\Rightarrow \angle P = 90^\circ$

In $\triangle ABC$ and $\triangle PQR$

$\angle A = \angle P$

$\angle C = \angle R$

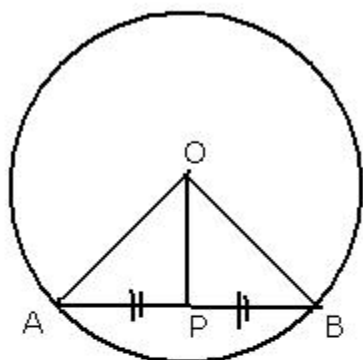
$BC = QR$



By Angle-Angle-Side criterion of congruency, the triangles $\triangle ABC$ and $\triangle PQR$ are congruent to each other.
 $\therefore \triangle ABC \cong \triangle PQR$

Solution 2:

Given: In the figure, O is centre of the circle,
and AB is chord. P is a point on AB such that $AP = PB$.
We need to prove that, $OP \perp AB$



Construction: Join OA and OB

Proof:

In $\triangle OAP$ and $\triangle OBP$

$OA = OB$ [radii of the same circle]

$OP = OP$ [common]

$AP = PB$ [given]

$\therefore$ By Side-Side-Side criterion of congruency,
 $\triangle OAP \cong \triangle OBP$

The corresponding parts of the congruent
triangles are congruent.

$\therefore \angle OPA = \angle OPB$ [by c.p.c.t.]

But $\angle OPA + \angle OPB = 180^\circ$ [linear pair]

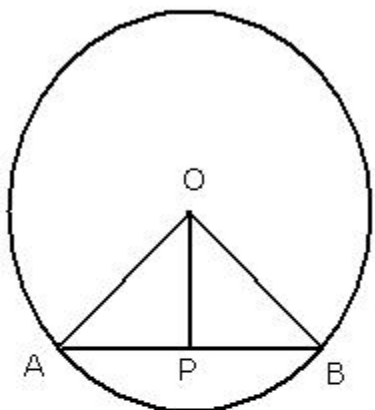
$\therefore \angle OPA = \angle OPB = 90^\circ$

Hence $OP \perp AB$.

Solution 3:

<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangles-congruency-in-triangles/>

Given: In the figure, O is centre of the circle,
and AB is chord. P is a point on AB such that $AP = PB$.
We need to prove that, $AP = BP$



Construction: Join OA and OB

Proof:

In right triangles $\triangle OAP$ and $\triangle OBP$

Hypotenuse $OA = OB$ [radii of the same circle]

Side $OP = OP$ [common]

$\therefore$ By Right angle-Hypotenuse-Side criterion of congruency,

$\triangle OAP \cong \triangle OBP$

The corresponding parts of the congruent
triangles are congruent.

$\therefore AP = BP$ [by c.p.c.t.]

Hence proved.

Solution 4:

<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangles-congruency-in-triangles/>

Given: A $\triangle ABC$ in which D is the mid-point of BC.

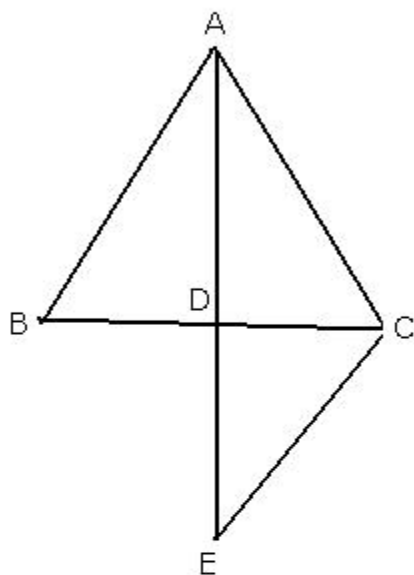
AD is produced to E so that $DE = AD$

We need to prove that

(i) $\triangle ABD \cong \triangle ECD$

(ii) $AB = EC$

(iii) $AB \parallel EC$



(i) In $\triangle ABD$ and $\triangle ECD$

$BD = DC$ [D is the midpoint of BC]

$\angle ADB = \angle CDE$ [vertically opposite angles]

$AD = DE$ [Given]

$\therefore$ By Side-Angle-Side criterion of congruence, we have,

$\triangle ABD \cong \triangle ECD$

(ii) The corresponding parts of the congruent triangles are congruent.

$\therefore AB = EC$ [c.p.c.t.]

(iii) Also, $\angle DAB = \angle DEC$ [c.p.c.t.]

$AB \parallel EC$ [$\angle DAB$ and $\angle DEC$ are alternate angles]

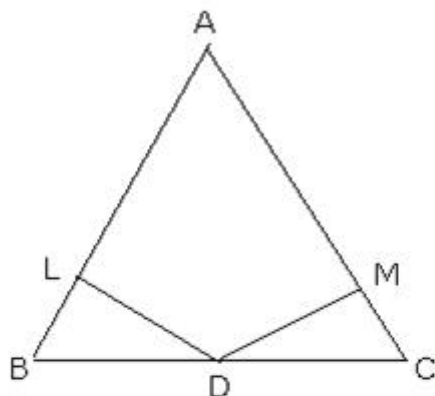
<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangles-congruency-in-triangles/>

Solution 5:

(i) Given: A $\triangle ABC$ in which $\angle B = \angle C$.

DL is the perpendicular from D to AB

DM is the perpendicular from D to AC



We need to prove that

$$DL = DM$$

Proof:

In $\triangle DLB$ and $\triangle DMC$

$$\angle DLB = \angle DMC = 90^\circ \quad [DL \perp AB \text{ and } DM \perp AC]$$

$$\angle B = \angle C \quad [\text{Given}]$$

$$BD = DC \quad [D \text{ is the midpoint of } BC]$$

$\therefore$ By Angle-Angle-Side criterion of congruence,
 $\triangle DLB \cong \triangle DMC$

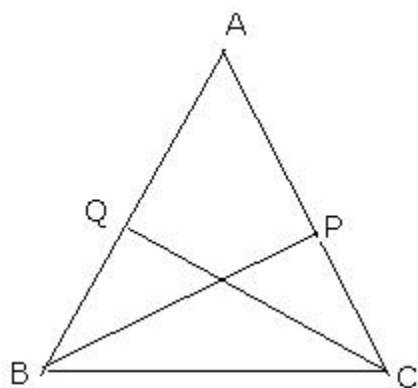
The corresponding parts of the congruent triangles are congruent.

$$\therefore DL = DM \quad [\text{c.p.c.t}]$$

(ii) Given: A $\triangle ABC$ in which $\angle B = \angle C$.

BP is the perpendicular from D to AC

CQ is the perpendicular from C to AB



We need to prove that

$$BP = CQ$$

Proof:

In $\triangle BPC$ and $\triangle CQB$

$$\angle B = \angle C \quad [\text{Given}]$$

$$\angle BPC = \angle CQB = 90^\circ \quad [BP \perp AC \text{ and } CQ \perp AB]$$

$$BC = BC \quad [\text{Common}]$$

$\therefore$ By Angle-Angle-Side criterion of congruence,
 $\triangle BPC \cong \triangle CQB$

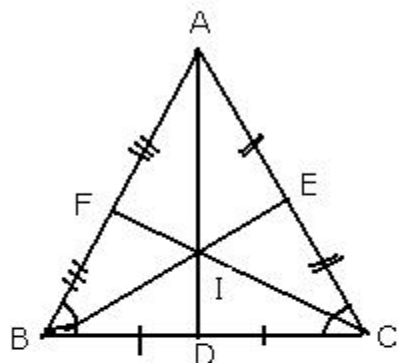
The corresponding parts of the congruent triangles are congruent.

$$\therefore BP = CQ \quad [\text{c.p.c.t}]$$

Solution 6:

<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangles-congruency-in-triangles/>

Given: A $\triangle ABC$ in which AD is the perpendicular bisector of BC
BE is the perpendicular bisector of CA
CF is the perpendicular bisector of AB
AD, BE and CF meet at I



We need to prove that

$$IA = IB = IC$$

Proof:

In $\triangle BID$ and $\triangle CID$

$$BD = DC \quad [\text{Given}]$$

$$\angle BDI = \angle CDI = 90^\circ \quad [AD \text{ is the perpendicular bisector of } BC]$$

$$BC = BC \quad [\text{Common}]$$

$\therefore$ By Side-Angle-Side criterion of congruence,

$$\triangle BID \cong \triangle CID$$

The corresponding parts of the congruent triangles are congruent.

$$\therefore IB = IC \quad [\text{c.p.c.t}]$$

Similarly, in $\triangle CIE$ and $\triangle AIE$

$$CE = AE \quad [\text{Given}]$$

$$\angle CEI = \angle AEI = 90^\circ \quad [AD \text{ is the perpendicular bisector of } BC]$$

$$IE = IE \quad [\text{Common}]$$

$\therefore$ By Side-Angle-Side criterion of congruence,

$$\triangle CIE \cong \triangle AIE$$

The corresponding parts of the congruent triangles are congruent.

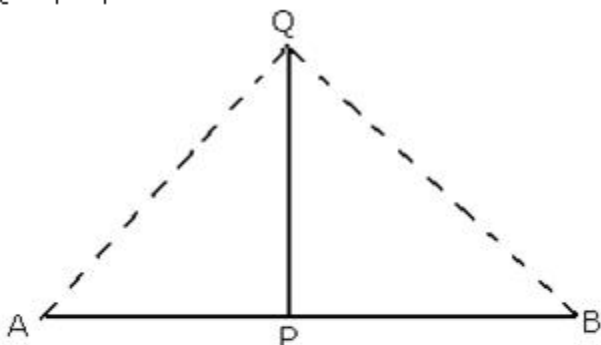
$$\therefore IC = IA \quad [\text{c.p.c.t}]$$

Thus, $IA = IB = IC$

Solution 7:

<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangles-congruency-in-triangles/>

Given: A $\triangle ABC$ in which AB is bisected at P
 PQ is perpendicular to AB



We need to prove that

$$QA = QB$$

Proof:

In $\triangle APQ$ and $\triangle BPQ$

$$AP = PB \quad [P \text{ is the mid-point of } AB]$$

$$\angle APQ = \angle BPQ = 90^\circ \quad [PQ \text{ is perpendicular to } AB]$$

$$PQ = PQ \quad [\text{Common}]$$

$\therefore$ By Side-Angle-Side criterion of congruence,

$$\triangle APQ \cong \triangle BPQ$$

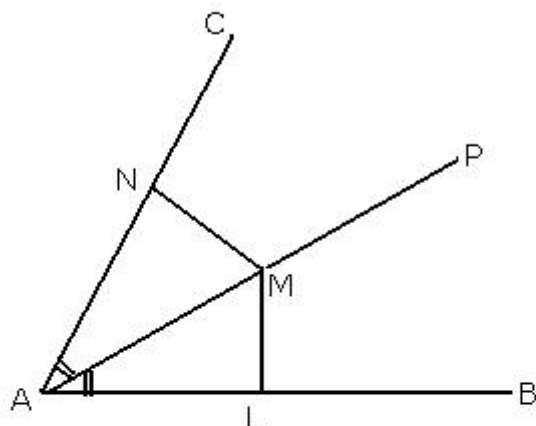
The corresponding parts of the congruent triangles are congruent.

$$\therefore QA = QB \quad [\text{c.p.c.t}]$$

Solution 8:

<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangles-congruency-in-triangles/>

From M, draw ML such that ML is perpendicular to AB and MN is perpendicular to AC



In $\triangle ALM$ and $\triangle ANM$

$$\angle LAM = \angle MAN \quad [\because AP \text{ is the bisector of } \angle BAC]$$

$$\angle ALM = \angle ANM = 90^\circ \quad [\because ML \perp AB, MN \perp AC]$$

$$AM = AM \quad [\text{Common}]$$

$\therefore$ By Angle-Angle-Side criterion of congruence,
 $\triangle ALM \cong \triangle ANM$

The corresponding parts of the congruent triangles are congruent.

$$\therefore ML = MN \quad [\text{c.p.c.t}]$$

Hence proved.

Solution 9:

<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangles-congruency-in-triangles/>

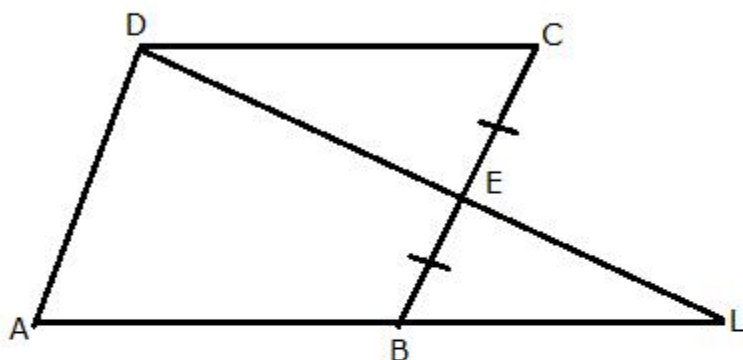
Given: ABCD is a parallelogram in which E is the mid-point of BC.

We need to prove that

(i) $\triangle DCE \cong \triangle LBE$

(ii) $AB = BL$

(iii) $AL = 2DC$



(i) In $\triangle DCE$ and $\triangle LBE$

$\angle DCE = \angle EBL$ [DC $\parallel$ AB, alternate angles]

$CE = EB$ [E is the midpoint of BC]

$\angle DEC = \angle LEB$ [vertically opposite angles]

$\therefore$ By Angle-Side-Angle criterion of congruence, we have,

$\triangle DCE \cong \triangle LBE$

The corresponding parts of the congruent triangles are congruent.

$\therefore DC = LB$ [c.p.c.t] ... (1)

(ii) $DC = AB$ [opposite sides of a parallelogram] ... (2)

From (1) and (2), $AB = BL$... (3)

(iii) $AL = AB + BL$... (4)

From (3) and (4), $AL = AB + AB$

$\Rightarrow AL = 2AB$

$\Rightarrow AL = 2DC$ [from (2)]

Solution 10:

<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangle-s-congruency-in-triangles/>

Given: In the figure $AB = DB$, $AC = DC$, $\angle ABD = 58^\circ$,
 $\angle DBC = (2x - 4)^\circ$, $\angle ACB = (y + 15)^\circ$ and $\angle DCB = 63^\circ$
We need to find the values of x and y .

In $\triangle ABC$ and $\triangle DBC$

$$AB = DB \quad [\text{given}]$$

$$AC = DC \quad [\text{given}]$$

$$BC = BC \quad [\text{common}]$$

$\therefore$ By Side-Side-Side criterion of congruence, we have,

$$\triangle ABC \cong \triangle DBC$$

The corresponding parts of the congruent triangles are congruent.

$$\therefore \angle ACB = \angle DCB \quad [\text{c.p.c.t}]$$

$$\Rightarrow y^\circ + 15^\circ = 63^\circ$$

$$\Rightarrow y^\circ = 63^\circ - 15^\circ$$

$$\Rightarrow y^\circ = 48^\circ$$

$$\text{and } \angle ABC = \angle DBC \quad [\text{c.p.c.t}]$$

$$\text{But, } \angle DBC = (2x - 4)^\circ$$

$$\text{We have } \angle ABC + \angle DBC = \angle ABD$$

$$\Rightarrow (2x - 4)^\circ + (2x - 4)^\circ = 58^\circ$$

$$\Rightarrow 4x - 8^\circ = 58^\circ$$

$$\Rightarrow 4x = 58^\circ + 8^\circ$$

$$\Rightarrow 4x = 66^\circ$$

$$\Rightarrow x = \frac{66^\circ}{4}$$

$$\Rightarrow x = 16.5^\circ$$

Thus the values of x and y are:

$$x = 16.5^\circ \text{ and } y = 48^\circ$$

Solution 11:

<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangles-congruency-in-triangles/>

In the given figure $AB \parallel FD$,

$$\Rightarrow \angle ABC = \angle FDC$$

Also $AC \parallel GE$,

$$\Rightarrow \angle ACB = \angle GEB$$

Consider the two triangles $\triangle GBE$ and $\triangle FDC$

$$\angle B = \angle D$$

$$\angle C = \angle E$$

Also given that

$$BD = CE$$

$$\Rightarrow BD + DE = CE + DE$$

$$\Rightarrow BE = DC$$

$\therefore$ By Angle - Side - Angle criterion of congruence

$$\triangle GBE \cong \triangle FDC$$

$$\therefore \frac{GB}{FD} = \frac{BE}{DC} = \frac{GE}{FC}$$

But $BE = DC$

$$\Rightarrow \frac{BE}{DC} = \frac{BE}{BE} = 1$$

$$\therefore \frac{GB}{FD} = \frac{BE}{DC} = 1$$

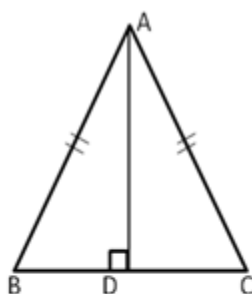
$$\Rightarrow GB = FD$$

$$\therefore \frac{GE}{FC} = \frac{BE}{DC} = 1$$

$$\Rightarrow GE = FC$$

Solution 12:

<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangles-congruency-in-triangles/>



In $\triangle ADB$ and $\triangle ADC$,

$AB = AC$ (Since $\triangle ABC$ is an isosceles triangle)

$AD = AD$ (common side)

$\angle ADB = \angle ADC$ (Since AD is the altitude so each is 90°)

$\Rightarrow \triangle ADB \cong \triangle ADC$ (RHS congruence criterion)

$BD = DC$ (cpct)

$\Rightarrow AD$ is the median.

Solution 13:

In $\triangle DLB$ and $\triangle DMC$,

$BL = CM$ (given)

$\angle DLB = \angle DMC$ (Both are 90°)

$\angle BDL = \angle CDM$ (vertically opposite angles)

$\therefore \triangle DLB \cong \triangle DMC$ (AAS congruence criterion)

$BD = CD$ (cpct)

Hence, AD is the median of $\triangle ABC$.

Solution 14:

(i) In $\triangle ADB$ and $\triangle ADC$,
 $\angle ADB = \angle ADC$ (Since AD is perpendicular to BC)
 $AB = AC$ (given)
 $AD = AD$ (common side)
 $\therefore \triangle ADB \cong \triangle ADC$ (RHS congruence criterion)
 $\Rightarrow BD = CD$ (cpct)
(ii) In $\triangle EFB$ and $\triangle EDB$,
 $\angle EFB = \angle EDB$ (both are 90°)
 $EB = EB$ (common side)
 $\angle FBE = \angle DBE$ (given)
 $\therefore \triangle EFB \cong \triangle EDB$ (AAS congruence criterion)
 $\Rightarrow EF = ED$ (cpct)
that is, $ED = EF$.

Solution 15:

In $\triangle ABC$ and $\triangle EFD$,
 $AB \parallel EF \Rightarrow \angle ABC = \angle EFD$ (alternate angles)
 $AC = ED$ (given)
 $\angle ACB = \angle EDF$ (given)
 $\therefore \triangle ABC \cong \triangle EFD$ (AAS congruence criterion)
 $\Rightarrow AB = FE$ (cpct)
and $BC = DF$ (cpct)
 $\Rightarrow BD + DC = CF + DC$ (B - D - C - F)
 $\Rightarrow BD = CF$

Solution 16:

<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangles-congruency-in-triangles/>

In $\triangle ABC$ and $\triangle EFD$,
 $AB \parallel EF \Rightarrow \angle ABC = \angle EFD$ (alternate angles)
 $AC = ED$ (given)
 $\angle ACB = \angle EDF$ (given)
 $\therefore \triangle ABC \cong \triangle EFD$ (AAS congruence criterion)
 $\Rightarrow AB = FE$ (cpct)
and $BC = DF$ (cpct)
 $\Rightarrow BD + DC = CF + DC$ (B - D - C - F)
 $\Rightarrow BD = CF$

Solution 17:

In $\triangle ADE$ and $\triangle DBF$,
 $DF \parallel AE \Rightarrow \angle EAD = \angle FDB$ (corresponding angles)
 $DE \parallel BF \Rightarrow \angle ADE = \angle DBF$ (corresponding angles)
 $AD = DB$ (given)
 $\therefore \triangle ADE \cong \triangle DBF$ (ASA congruence criterion)
 $\Rightarrow DE = BF$ (cpct)

Exercise 9(B)**Solution 1:**

<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangles-congruency-in-triangles/>

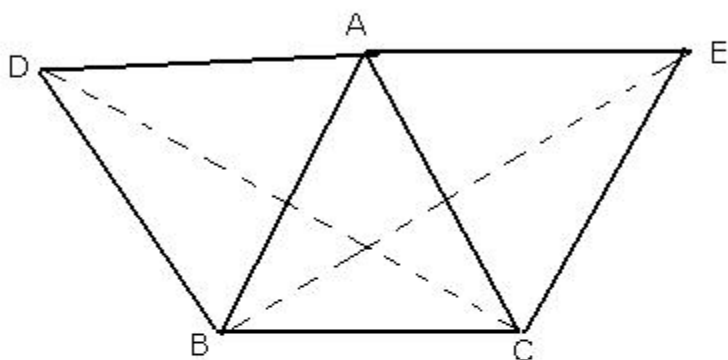
Given: $\triangle ABD$ is an equilateral triangle

$\triangle ACE$ is an equilateral triangle

We need to prove that

(i) $\angle CAD = \angle BAE$

(ii) $CD = BE$



Proof:

(i)

$\triangle ABD$ is equilateral

$\therefore$ Each angle = 60°

$\Rightarrow \angle BAD = 60^\circ$... (1)

Similarly,

$\triangle ACE$ is equilateral

$\therefore$ Each angle = 60°

$\Rightarrow \angle CAE = 60^\circ$... (2)

$\Rightarrow \angle BAD = \angle CAE$ [from (1) and (2)] ... (3)

Adding $\angle BAC$ to both sides, we have

$$\angle BAD + \angle BAC = \angle CAE + \angle BAC$$

$$\Rightarrow \angle CAD = \angle BAE \quad \dots(4)$$

(ii)

In $\triangle CAD$ and $\triangle BAE$

$$AC = AE \quad [\triangle ACE \text{ is equilateral}]$$

$$\angle CAD = \angle BAE \quad [\text{from (4)}]$$

$$AD = AB \quad [\triangle ABD \text{ is equilateral}]$$

$\therefore$ By Side-Angle-Side criterion of congruency,

$$\triangle CAD \cong \triangle BAE$$

The corresponding parts of the congruent triangles are congruent.

$$\therefore CD = BE \quad [\text{by c.p.c.t}]$$

Hence proved.

Solution 2:

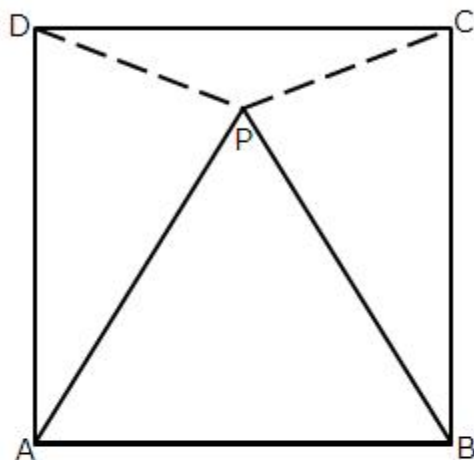
<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangles-congruency-in-triangles/>

Given: ABCD is a square and $\triangle APB$ is an equilateral triangle.

We need to

(i) Prove that, $\triangle APD \cong \triangle BPC$

(ii) To find angles of $\triangle DPC$



(a)

(i) Proof:

$AP = PB = AB$ [$\triangle APB$ is an equilateral triangle]

Also, we have,

$$\angle PBA = \angle PAB = \angle APB = 60^\circ \quad \dots(1)$$

Since ABCD is a square, we have

$$\angle A = \angle B = \angle C = \angle D = 90^\circ \quad \dots(2)$$

$$\text{Since } \angle DAP = \angle A - \angle PAB \quad \dots(3)$$

$$\Rightarrow \angle DAP = 90^\circ - 60^\circ$$

$$\Rightarrow \angle DAP = 30^\circ \quad [\text{from (1) and (2)}] \quad \dots(4)$$

Similarly $\angle CBP = \angle B - \angle PBA$

$$\Rightarrow \angle CBP = 90^\circ - 60^\circ$$

$$\Rightarrow \angle CBP = 30^\circ \quad [\text{from (1) and (2)}] \quad \dots(5)$$

$$\Rightarrow \angle DAP = \angle CBP \quad [\text{from (4) and (5)}] \quad \dots(6)$$

In $\triangle APD$ and $\triangle BPC$

$$AD = BC \quad [\text{Sides of square } ABCD]$$

$$\angle DAP = \angle CBP \quad [\text{from (6)}]$$

$$AP = BP \quad [\text{Sides of equilateral } \triangle APB]$$

$\therefore$ By Side-Angle-Side criterion of congruence, we have,

$$\triangle APD \cong \triangle BPC$$

(ii)

$$AP = PB = AB \quad [\triangle APB \text{ is an equilateral triangle}] \quad \dots(7)$$

$$AB = BC = CD = DA \quad [\text{Sides of square } ABCD] \quad \dots(8)$$

From (7) and (8), we have

$$AP = DA \text{ and } PB = BC \quad \dots(9)$$

In $\triangle APD$,

$$AP = DA \quad [\text{from (9)}]$$

$$\therefore \angle ADP = \angle APD \quad \left[\begin{array}{l} \text{Angles opposite to} \\ \text{equal sides are equal} \end{array} \right] \quad \dots(10)$$

$$\angle ADP + \angle APD + \angle DAP = 180^\circ \quad \left[\begin{array}{l} \text{Sum of angles of} \\ \text{a triangle} = 180^\circ \end{array} \right]$$

$$\Rightarrow \angle ADP + \angle ADP + 30^\circ = 180^\circ \quad \left[\begin{array}{l} \text{from (3), } \angle DAP = 30^\circ \\ \text{from (10), } \angle ADP = \angle APD \end{array} \right]$$

$$\Rightarrow \angle ADP + \angle ADP = 180^\circ - 30^\circ$$

$$\Rightarrow 2\angle ADP = 150^\circ$$

$$\Rightarrow \angle ADP = \frac{150^\circ}{2}$$

$$\Rightarrow \angle ADP = 75^\circ$$

We have $\angle PDC = \angle D - \angle ADP$

$$\Rightarrow \angle PDC = 90^\circ - 75^\circ$$

$$\Rightarrow \angle PDC = 15^\circ \quad \dots(11)$$

In $\triangle BPC$,

$$PB=BC \quad [\text{from (9)}]$$

$$\therefore \angle PCB = \angle BPC \quad \left[\begin{array}{l} \text{Angles opposite to} \\ \text{equal sides are equal} \end{array} \right] \quad \dots(12)$$

$$\angle PCB + \angle BPC + \angle CBP = 180^\circ \quad \left[\begin{array}{l} \text{Sum of angles of} \\ \text{a triangle} = 180^\circ \end{array} \right]$$

$$\Rightarrow \angle PCB + \angle PCB + 30^\circ = 180^\circ \quad \left[\begin{array}{l} \text{from (5), } \angle CBP = 30^\circ \\ \text{from (12), } \angle PCB = \angle BPC \end{array} \right]$$

$$\Rightarrow 2\angle PCB = 180^\circ - 30^\circ$$

$$\Rightarrow \angle PCB = \frac{150^\circ}{2}$$

$$\Rightarrow \angle PCB = 75^\circ$$

We have $\angle PCD = \angle C - \angle PCB$

$$\Rightarrow \angle PCD = 90^\circ - 75^\circ$$

$$\Rightarrow \angle PCD = 15^\circ \quad \dots(13)$$

In $\triangle DPC$,

$$\angle PDC = 15^\circ$$

$$\angle PCD = 15^\circ$$

$$\angle PCD + \angle PDC + \angle DPC = 180^\circ \quad \left[\begin{array}{l} \text{Sum of angles of} \\ \text{a triangle} = 180^\circ \end{array} \right]$$

$$\Rightarrow 15^\circ + 15^\circ + \angle DPC = 180^\circ$$

$$\Rightarrow \angle DPC = 180^\circ - 30^\circ$$

$$\Rightarrow \angle DPC = 150^\circ$$

$\therefore$ Angles of $\triangle DPC$, are: $15^\circ, 150^\circ, 15^\circ$

(b)

(i) Proof: In $\triangle APB$

$$AP=PB=AB \quad [\triangle APB \text{ is an equilateral triangle}]$$

Also, we have,

$$\angle PBA = \angle PAB = \angle APB = 60^\circ \quad \dots(1)$$

Since ABCD is a square, we have

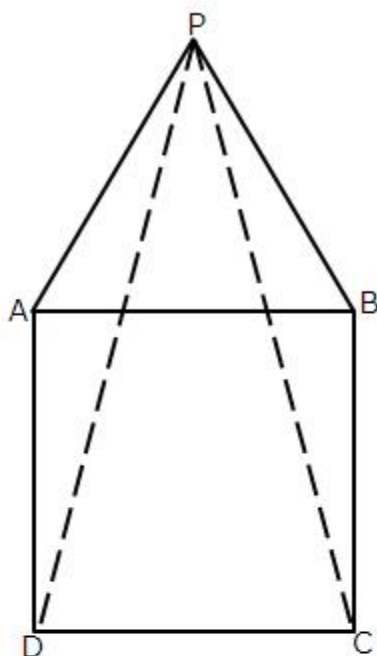
$$\angle A = \angle B = \angle C = \angle D = 90^\circ \quad \dots(2)$$

$$\text{Since } \angle DAP = \angle A + \angle PAB \quad \dots(3)$$

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$$\Rightarrow \angle DAP = 90^\circ + 60^\circ$$

$$\Rightarrow \angle DAP = 150^\circ \quad [\text{from (1) and (2)}] \quad \dots(4)$$



Similarly $\angle CBP = \angle B + \angle PBA$

$$\Rightarrow \angle CBP = 90^\circ + 60^\circ$$

$$\Rightarrow \angle CBP = 150^\circ \quad [\text{from (1) and (2)}] \quad \dots(5)$$

$$\Rightarrow \angle DAP = \angle CBP \quad [\text{from (4) and (5)}] \quad \dots(6)$$

In $\triangle APD$ and $\triangle BPC$

$$AD = BC \quad [\text{Sides of square } ABCD]$$

$$\angle DAP = \angle CBP \quad [\text{from (6)}]$$

$$AP = BP \quad [\text{Sides of equilateral } \triangle APB]$$

$\therefore$ By Side-Angle-Side criterion of congruence, we have,

$$\triangle APD \cong \triangle BPC$$

(ii)

$$AP = PB = AB \quad [\triangle APB \text{ is an equilateral triangle}] \quad \dots(7)$$

$$AB = BC = CD = DA \quad [\text{Sides of square } ABCD] \quad \dots(8)$$

From (7) and (8), we have

$$AP=DA \text{ and } PB=BC \quad \dots(9)$$

In $\triangle APD$,

$$AP=DA \quad [\text{from (9)}]$$

$$\therefore \angle ADP = \angle APD \quad \left[\begin{array}{l} \text{Angles opposite to} \\ \text{equal sides are equal} \end{array} \right] \quad \dots(10)$$

$$\angle ADP + \angle APD + \angle DAP = 180^\circ \quad \left[\begin{array}{l} \text{Sum of angles of} \\ \text{a triangle} = 180^\circ \end{array} \right]$$

$$\Rightarrow \angle ADP + \angle ADP + 150^\circ = 180^\circ \quad \left[\begin{array}{l} \text{from (3), } \angle DAP = 150^\circ \\ \text{from (10), } \angle ADP = \angle APD \end{array} \right]$$

$$\Rightarrow \angle ADP + \angle ADP = 180^\circ - 150^\circ$$

$$\Rightarrow 2\angle ADP = 30^\circ$$

$$\Rightarrow \angle ADP = \frac{30^\circ}{2}$$

$$\Rightarrow \angle ADP = 15^\circ$$

We have $\angle PDC = \angle D - \angle ADP$

$$\Rightarrow \angle PDC = 90^\circ - 15^\circ$$

$$\Rightarrow \angle PDC = 75^\circ \quad \dots(11)$$

In $\triangle BPC$,

$$PB=BC \quad [\text{from (9)}]$$

$$\therefore \angle PCB = \angle BPC \quad \left[\begin{array}{l} \text{Angles opposite to} \\ \text{equal sides are equal} \end{array} \right] \quad \dots(12)$$

$$\angle PCB + \angle BPC + \angle CBP = 180^\circ \quad \left[\begin{array}{l} \text{Sum of angles of} \\ \text{a triangle} = 180^\circ \end{array} \right]$$

$$\Rightarrow \angle PCB + \angle PCB + 150^\circ = 180^\circ \quad \left[\begin{array}{l} \text{from (5), } \angle CBP = 150^\circ \\ \text{from (12), } \angle PCB = \angle BPC \end{array} \right]$$

$$\Rightarrow 2\angle PCB = 180^\circ - 150^\circ$$

$$\Rightarrow \angle PCB = \frac{30^\circ}{2}$$

$$\Rightarrow \angle PCB = 15^\circ$$

We have $\angle PCD = \angle C - \angle PCB$

$$\Rightarrow \angle PCD = 90^\circ - 15^\circ$$

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$$\Rightarrow \angle PCD = 75^\circ \quad \dots(13)$$

In $\triangle DPC$,

$$\angle PDC = 75^\circ$$

$$\angle PCD = 75^\circ$$

$$\angle PCD + \angle PDC + \angle DPC = 180^\circ \left[\begin{array}{l} \text{Sum of angles of} \\ \text{a triangle} = 180^\circ \end{array} \right]$$

$$\Rightarrow 75^\circ + 75^\circ + \angle DPC = 180^\circ$$

$$\Rightarrow \angle DPC = 180^\circ - 150^\circ$$

$$\Rightarrow \angle DPC = 30^\circ$$

$\therefore$ Angles of $\triangle DPC$, are: $75^\circ, 30^\circ, 75^\circ$

Solution 3:

Given: A $\triangle ABC$ is right angled at B .

$ABPQ$ and $ACRS$ are squares

We need to prove that

(i) $\triangle ACQ \cong \triangle ASB$

(ii) $CQ = BS$

Proof:

(i)

$$\angle QAB = 90^\circ \quad [ABPQ \text{ is a square}] \quad \dots(1)$$

$$\angle SAC = 90^\circ \quad [ACRS \text{ is a square}] \quad \dots(2)$$

From (1) and (2), we have

$$\angle QAB = \angle SAC \quad \dots(3)$$

Adding $\angle BAC$ to both sides of (3), we have

$$\angle QAB + \angle BAC = \angle SAC + \angle BAC$$

$$\Rightarrow \angle QAC = \angle SAB \quad \dots(4)$$

In $\triangle ACQ$ and $\triangle ASB$,

$$QA = QB \quad [\text{sides of a square } ABPQ]$$

$$\angle QAC = \angle SAB \quad [\text{from (4)}]$$

$$AC = AS \quad [\text{sides of a square } ACRS]$$

$\therefore$ By Angle-Angle-Side criterion of congruence,

$$\triangle ACQ \cong \triangle ASB$$

(ii)

The corresponding parts of the congruent triangles are congruent.

$$\therefore CQ = BS \quad [c.p.c.t.]$$

Solution 4:

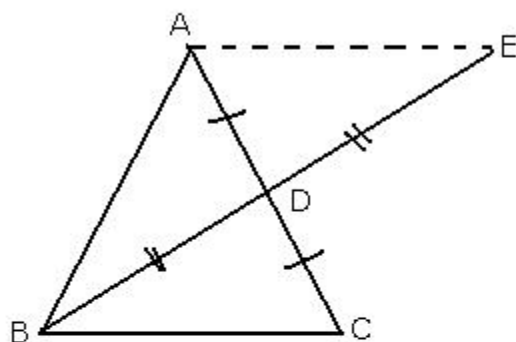
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Given: A $\triangle ABC$ in which BD is the median to AC .

BD is produced to E such that $BD = DE$.

We need to prove that $AE \parallel BC$.

Construction: Join AE



Proof:

$AD = DC$ [BD is median to AC] ... (1)

In $\triangle BDC$ and $\triangle ADE$

$BD = DE$ [Given]

$\angle BDC = \angle ADE = 90^\circ$ [vertically opposite angles]

$AD = DC$ [from (1)]

$\therefore$ By Side-Angle-Side criterion of congruence,

$\triangle BDC \cong \triangle ADE$

The corresponding parts of the congruent triangles are congruent.

$\therefore \angle EAD = \angle BCD$ [c.p.c.t]

But these are alternate angles and AC is the transversal

Thus, $AE \parallel BC$

Solution 5:

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Given: A $\triangle PQR$ in which QX is the bisector of $\angle Q$
and RX is the bisector of $\angle R$.

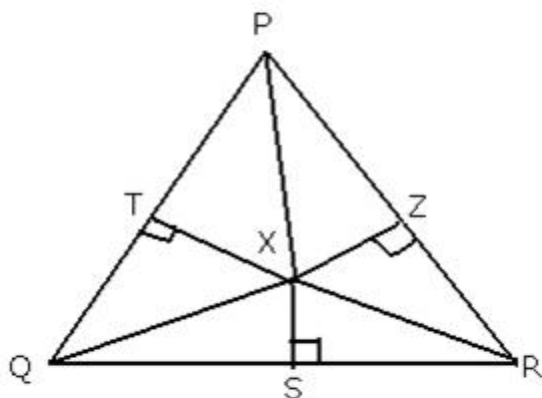
$XS \perp QR$ and $XT \perp PQ$.

We need to prove that

(i) $\triangle XTQ \cong \triangle XSQ$

(ii) PX bisects $\angle P$

Construction: Draw $XZ \perp PR$ and join PX .



Proof:

(i)

In $\triangle XTQ$ and $\triangle XSQ$

$\angle QTX = \angle QSX = 90^\circ$ [$XS \perp QR$ and $XT \perp PQ$]

$\angle TQX = \angle SQX$ [QX is bisector of $\angle Q$]

$QX = QX$ [Common]

$\therefore$ By Angle-Angle-Side criterion of congruence,

$\triangle XTQ \cong \triangle XSQ$

...(1)

(ii)

The corresponding parts of the congruent triangles are congruent.

$\therefore XT = XS$ [c.p.c.t.]

In $\triangle XSR$ and $\triangle XZR$

$\angle XSR = \angle XZR = 90^\circ$ [$XS \perp QR$ and $\angle XSR = 90^\circ$]

$\angle SRX = \angle ZRX$ [RX is bisector of $\angle R$]

$$RX = RX \quad [\text{Common}]$$

$\therefore$ By Angle-Angle-Side criterion of congruence,

$$\triangle XSR \cong \triangle XZR$$

The corresponding parts of the congruent triangles are congruent.

$$\therefore XS = XZ \quad [\text{c.p.c.t}] \quad \dots(2)$$

From (1) and (2)

$$XT = XZ \quad \dots(3)$$

In $\triangle XTP$ and $\triangle XZP$

$$\angle XTP = \angle XZP = 90^\circ \quad [\text{Given}]$$

$$\text{Hyp. } XP = \text{Hyp. } XP \quad [\text{Common}]$$

$$XT = XZ \quad [\text{from (3)}]$$

$\therefore$ By Right angle-Hypotenuse-Side criterion of congruence,

$$\triangle XTP \cong \triangle XZP$$

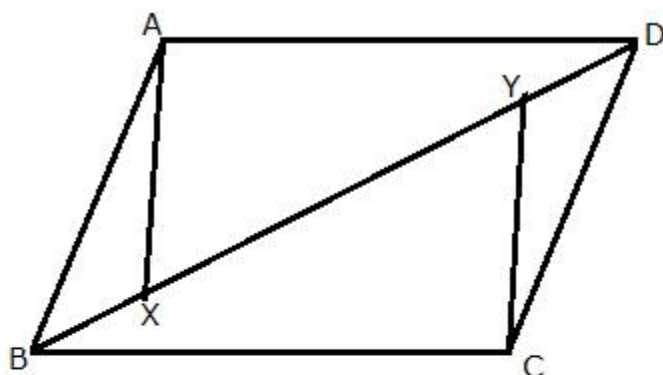
The corresponding parts of the congruent triangles are congruent.

$$\therefore \angle XPT = \angle XPZ \quad [\text{c.p.c.t}]$$

$\therefore PX$ bisects $\angle P$

Solution 6:

ABCD is a parallelogram in which $\angle A$ and $\angle C$ are obtuse.



Points X and Y are taken on the diagonal BD such that $\angle XAD = \angle YCB = 90^\circ$.

We need to prove that $XA = YC$

Proof:

In $\triangle XAD$ and $\triangle YCB$

$$\angle XAD = \angle YCB = 90^\circ \quad [\text{Given}]$$

$$AD = BC \quad [\text{Opposite sides of a parallelogram}]$$

$$\angle ADX = \angle CBY \quad [\text{Alternate angles}]$$

$\therefore$ By Angle-Side-Angle criterion of congruence,

$$\triangle XAD \cong \triangle YCB$$

The corresponding parts of the congruent triangles are congruent.

$$\therefore XA = YC \quad [\text{c.p.c.t}]$$

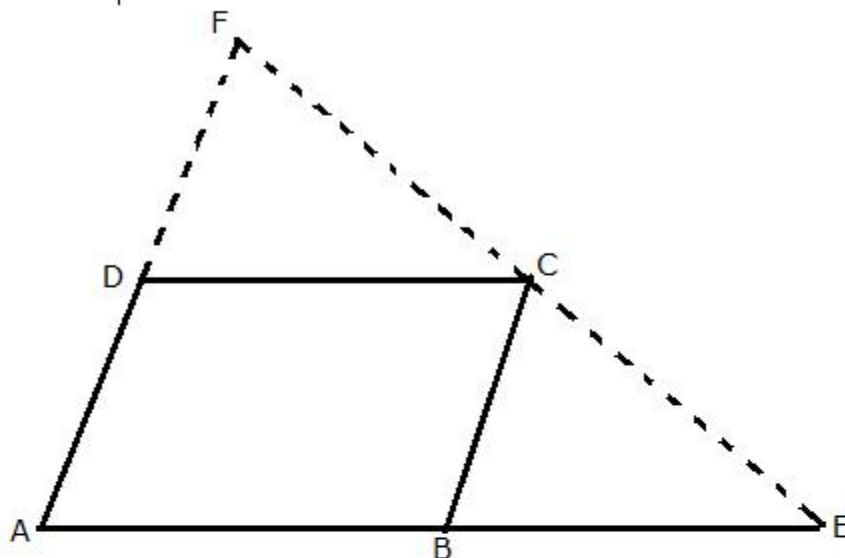
Hence proved.

Solution 7:

<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangles-congruency-in-triangles/>

ABCD is a parallelogram. The sides AB and AD are produced to E and F respectively, such that $AB = BE$ and $AD = DF$.

We need to prove that $\triangle BEC \cong \triangle DCF$



Proof:

$$AB = DC \quad \left[\begin{array}{l} \text{Opposite sides of a} \\ \text{parallelogram} \end{array} \right] \quad \dots(1)$$

$$AB = BE \quad [\text{Given}] \quad \dots(2)$$

From (1) and (2), we have

$$BE = DC \quad \dots(3)$$

$$AD = BC \quad \left[\begin{array}{l} \text{Opposite sides of a} \\ \text{parallelogram} \end{array} \right] \quad \dots(4)$$

$$AD = DF \quad [\text{Given}] \quad \dots(5)$$

From (4) and (5), we have

$$BC = DF \quad \dots(6)$$

Since $AD \parallel BC$, the corresponding angles are equal.

$$\therefore \angle DAB = \angle CBE \quad \dots(7)$$

Since $AB \parallel DC$, the corresponding angles are equal.

$$\therefore \angle DAB = \angle FDC \quad \dots(8)$$

From (7) and (8), we have

$$\angle CBE = \angle FDC \quad \dots(9)$$

In $\triangle BEC$ and $\triangle DCF$

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In $\triangle BEC$ and $\triangle DCF$

$$BE = DC \quad [\text{from (3)}]$$

$$\angle CBE = \angle FDC \quad [\text{from (9)}]$$

$$BC = DF \quad [\text{from (6)}]$$

$\therefore$ By Side-Angle-Side criterion of congruence,

$$\triangle BEC \cong \triangle DCF$$

Hence proved.

Solution 8:

<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangles-congruency-in-triangles/>

Since, $BC=QR$, we have

$$BD=QS \text{ and } DC=SR \left[\begin{array}{l} D \text{ is the midpoint of } BC \text{ and} \\ S \text{ is the midpoint of } QR \end{array} \right]$$

In $\triangle ABD$ and $\triangle PQS$

$$AB = PQ \quad \dots(1)$$

$$AD = PS \quad \dots(2)$$

$$BD = QS \quad \dots(3)$$

Thus, by Side-Side-Side criterion of congruence,
we have $\triangle ABD \cong \triangle PQS$

Similarly, in $\triangle ADC$ and $\triangle PSR$

$$AD = PS \quad \dots(4)$$

$$AC = PR \quad \dots(5)$$

$$DC = SR \quad \dots(6)$$

Thus, by Side-Side-Side criterion of congruence,
we have $\triangle ADC \cong \triangle PSR$

We have

$$\begin{aligned} BC &= BD + DC \quad [D \text{ is the midpoint of } BC] \\ &= QS + SR \quad [\text{from (3) and (6)}] \\ &= QR \quad [S \text{ is the midpoint of } QR] \quad \dots(7) \end{aligned}$$

Now consider the triangles $\triangle ABC$ and $\triangle PQR$

$$AB = PQ \quad [\text{from (1)}]$$

$$BC = QR \quad [\text{from (7)}]$$

$$AC = PR \quad [\text{from (5)}]$$

$\therefore$ By Side-Side-Side criterion of congruence, we
have $\triangle ABC \cong \triangle PQR$

Hence proved.

Solution 9:

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In the figure, AP and BQ are equal and parallel to each other. $\therefore AP=BQ$ and $AP \parallel BQ$.

We need to prove that

(i) $\triangle AOP \cong \triangle BOQ$

(ii) AB and PQ bisect each other

(i) $\because AP \parallel BQ$

$\therefore \angle APO = \angle BQO$ [Alternate angles] ... (1)

and $\angle PAO = \angle QBO$ [Alternate angles] ... (2)

Now in $\triangle AOP$ and $\triangle BOQ$,

$\angle APO = \angle BQO$ [from (1)]

$AP = BQ$ [given]

$\angle PAO = \angle QBO$ [from (2)]

$\therefore$ By Angle-Side-Angle criterion of congruence, we have

$\triangle AOP \cong \triangle BOQ$

(ii)

The corresponding parts of the congruent triangles are congruent.

$\therefore OP = OQ$ [c.p.c.t.]

$OA = OB$ [c.p.c.t.]

Hence AB and PQ bisect each other.

Solution 10:

<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangle-s-congruency-in-triangles/>

Given:

In the figure, $OA=OC$, $AB=BC$

We need to prove that,

(i) $\angle AOB = 90^\circ$

(ii) $\triangle AOD \cong \triangle COD$

(iii) $AD = CD$

(i) In $\triangle ABO$ and $\triangle CBO$,

$AB=BC$ [given]

$AO=CO$ [given]

$OB=OB$ [common]

$\therefore$ By Side-Side-Side criterion of congruence, we have

$\triangle ABO \cong \triangle CBO$

The corresponding parts of the congruent triangles are congruent.

$\therefore \angle ABO = \angle CBO$ [c.p.c.t.]

$\Rightarrow \angle ABD = \angle CBD$

and $\angle AOB = \angle COB$ [c.p.c.t.]

We have

$\angle AOB + \angle COB = 180^\circ$ [Linear pair]

$\Rightarrow \angle AOB = \angle COB = 90^\circ$ and $AC \perp BD$

(ii) In $\triangle AOD$ and $\triangle COD$,

$OD=OD$ [common]

$\angle AOD = \angle COD$ [each $= 90^\circ$]

$AO=CO$ [given]

$\therefore$ By Side-Angle-Side criterion of congruence, we have

$\triangle AOD \cong \triangle COD$

(iii)

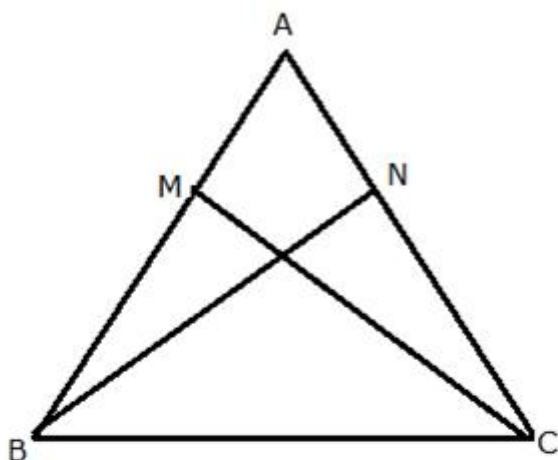
The corresponding parts of the congruent triangles are congruent.

$\therefore AD=CD$ [c.p.c.t.]

Hence proved.

Solution 11:

In $\triangle ABC$, $AB = AC$. M and N are points on AB and AC such that $BM = CN$. BN and CM are joined.



(i) In $\triangle AMC$ and $\triangle ANB$

$$AB = AC \quad [\text{Given}] \quad \dots(1)$$

$$BM = CN \quad [\text{Given}] \quad \dots(2)$$

Subtracting (2) from (1), we have

$$AB - BM = AC - CN$$

$$\Rightarrow AM = AN \quad \dots(3)$$

(ii) Consider the triangles $\triangle AMC$ and $\triangle ANB$

$$AC = AB \quad [\text{given}]$$

$$\angle A = \angle A \quad [\text{common}]$$

$$AM = AN \quad [\text{from (3)}]$$

$\therefore$ By Side-Angle-Side criterion of congruence, we have $\triangle AMC \cong \triangle ANB$

(iii)

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The corresponding parts of the congruent triangles are congruent.

$$\therefore CM = BN \quad [\text{c.p.c.t}] \quad \dots(4)$$

(iv) Consider the triangles $\triangle BMC$ and $\triangle CNB$

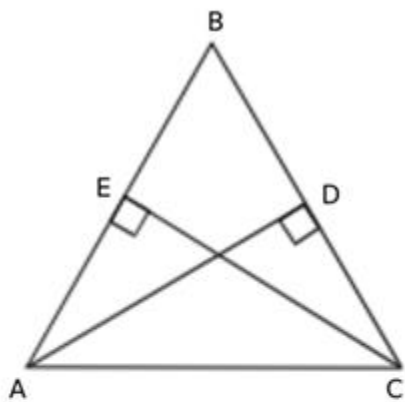
$$BM = CN \quad [\text{given}]$$

$$BC = BC \quad [\text{common}]$$

$$CM = BN \quad [\text{from (4)}]$$

$\therefore$ By Side-Side-Side criterion of congruence, we have $\triangle BMC \cong \triangle CNB$

Solution 12:



In $\triangle ABD$ and $\triangle CBE$,

$$AB = BC \quad (\text{given})$$

$$\angle ADB = \angle CEB = 90^\circ$$

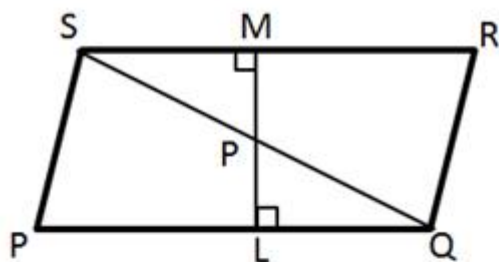
$$\angle B = \angle B \quad (\text{common angle})$$

$$\therefore \triangle ABD \cong \triangle CBE \quad (\text{by SAS congruence})$$

$$\Rightarrow AD = CE \quad (\text{cpct})$$

Solution 13:

<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangles-congruency-in-triangles/>



Given : $PL = RM$

To prove : $SP = PQ$ and $MP = PL$

Proof :

Since SR and PQ are opposite sides of a parallelogram,

$$PQ = SR \quad \dots(1)$$

$$\text{Also, } PL = RM \quad \dots(2)$$

Subtracting (2) from (1),

$$PQ - PL = SR - RM$$

$$\Rightarrow LQ = SM \quad \dots(3)$$

Now, in $\triangle SMP$ and $\triangle QLP$,

$$\angle MSP = \angle PQL \quad (\text{alternate interior angles})$$

$$\angle SMP = \angle PLQ \quad (\text{alternate interior angles})$$

$$SM = LQ \quad [\text{From (3)}]$$

$$\therefore \triangle SMP \cong \triangle QLP \quad (\text{by ASA congruence})$$

$$\Rightarrow SP = PQ \text{ and } MP = PL \quad (\text{cpct})$$

$$\Rightarrow LM \text{ and } QS \text{ bisect each other.}$$

Solution 14:

<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangles-congruency-in-triangles/>

$\triangle ABC$ is an equilateral triangle.

So, each of its angles equals 60° .

QP is parallel to AC,

$$\Rightarrow \angle PQB = \angle RAQ = 60^\circ$$

In $\triangle QBP$,

$$\angle PBQ = \angle BQP = 60^\circ$$

So, $\angle PBQ + \angle BQP + \angle BPQ = 180^\circ$ (angle sum property)

$$\Rightarrow 60^\circ + 60^\circ + \angle BPQ = 180^\circ$$

$$\Rightarrow \angle BPQ = 60^\circ$$

So, $\triangle BPQ$ is an equilateral triangle.

$$\Rightarrow QP = BP$$

$$\Rightarrow QP = CR \dots (i)$$

Now, $\angle QPM + \angle BPQ = 180^\circ$ (linear pair)

$$\Rightarrow \angle QPM + 60^\circ = 180^\circ$$

$$\Rightarrow \angle QPM = 120^\circ$$

Also, $\angle RCM + \angle ACB = 180^\circ$ (linear pair)

$$\Rightarrow \angle RCM + 60^\circ = 180^\circ$$

$$\Rightarrow \angle RCM = 120^\circ$$

In $\triangle RCM$ and $\triangle QMP$,

$$\angle RCM = \angle QPM \quad (\text{each is } 120^\circ)$$

$$\angle RMC = \angle QMP \quad (\text{vertically opposite angles})$$

$$QP = CR \quad (\text{from (i)})$$

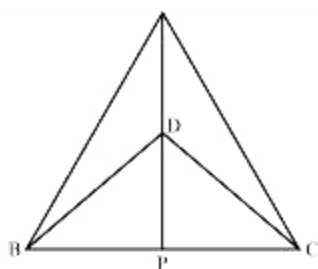
$$\Rightarrow \triangle RCM \cong \triangle QMP \quad (\text{AAS congruence criterion})$$

So, $CM = PM$

$\Rightarrow QR$ bisects PC .

Solution 15:

<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangles-congruency-in-triangles/>



(i) In $\triangle ABD$ and $\triangle ACD$,

$$AB = AC \quad (\text{given})$$

$$BD = CD \quad (\text{given})$$

$$AD = AD \quad (\text{common})$$

$$\Rightarrow \triangle ABD \cong \triangle ACD \quad (\text{by SSS congruence})(\text{proved})$$

$$\Rightarrow \angle BAD = \angle CAD \quad (\text{cpct})$$

$$\Rightarrow \angle BAP = \angle CAP \quad \dots(1)$$

(ii) In $\triangle ABP$ and $\triangle ACP$,

$$AB = AC \quad (\text{given})$$

$$\angle BAP = \angle CAP \quad [\text{from (1)}]$$

$$AP = AP \quad (\text{common})$$

$$\Rightarrow \triangle ABP \cong \triangle ACP \quad (\text{by SAS congruence})(\text{proved})$$

$$\Rightarrow BP = CP \quad (\text{cpct}) \quad \dots(2)$$

(iii) From equation (1),

$$\angle BAP = \angle CAP$$

Hence, AP bisects $\angle A$.

Now, in $\triangle BDP$ and $\triangle CDP$,

$$BD = CD \quad (\text{given})$$

$$DP = DP \quad (\text{common})$$

$$BP = CP \quad [\text{From (2)}]$$

$$\Rightarrow \triangle BDP \cong \triangle CDP \quad (\text{by SSS congruence})$$

$$\Rightarrow \angle BDP = \angle CDP \quad (\text{opct}) \quad \dots(3)$$

Hence, AP bisects $\angle D$.

(iv) We have $\triangle BDP \cong \triangle CDP$

$$\Rightarrow \angle BPD = \angle CPD \quad (\text{opct}) \quad \dots(4)$$

$$\text{Now, } \angle BPD + \angle CPD = 180^\circ \quad (\text{linear pair})$$

$$\Rightarrow \angle BPD + \angle BPD = 180^\circ \quad [\text{From (4)}]$$

$$\Rightarrow 2\angle BPD = 180^\circ$$

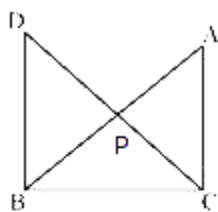
$$\Rightarrow \angle BPD = 90^\circ \quad \dots(5)$$

From equations (2) and (5), we can say that

[From (2)]

AP is perpendicular bisector of BC.

Solution 16:



(i) In $\triangle APC$ and $\triangle BPC$,

$AP = BP$ (P is the mid-point of AB)

$\angle APC = \angle BPC$ (vertically opposite angles)

$CP = DP$ (given)

$\Rightarrow \triangle APC \cong \triangle BPD$ (by SAS congruence)(proved)

$\Rightarrow AC = BD$ (cpct)

And, $\angle ACP = \angle BDP$ (cpct)

(ii) We have $\angle ACP = \angle BDP$

But, $\angle ACP$ and $\angle BDP$ are alternate interior angles

Since alternate angles are equal, we can say that

$DB \parallel AC$

$\Rightarrow \angle DBC + \angle ACB = 180^\circ$ (co-interior angles)

$\Rightarrow \angle DBC + 90^\circ = 180^\circ$

$\Rightarrow \angle DBC = 90^\circ$

Hence, $\angle DBC$ is a right angle.

(iii) Now in $\triangle DBC$ and $\triangle ACB$,

$$DB = AC \quad (\text{already proved})$$

$$\angle DBC = \angle ACB \quad (\text{each } 90^\circ)$$

$$BC = CB \quad (\text{common})$$

$$\Rightarrow \triangle DBC \cong \triangle ACB \quad (\text{by SAS congruence})(\text{proved})$$

$$\Rightarrow AC = BD \quad (\text{cpct})$$

$$\text{And, } \angle ACP = \angle BDP \quad (\text{cpct})$$

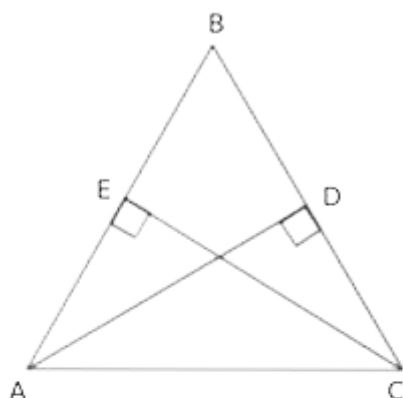
(iv) We have $\triangle DBC \cong \triangle ACB$

$$\Rightarrow DC = AB$$

$$\Rightarrow 2CP = AB$$

$$\Rightarrow CP = \frac{1}{2}AB$$

Solution 17:



In $\triangle ABD$ and $\triangle CBE$,

$$AB = BC \quad (\text{given})$$

$$\angle ADB = \angle CEB = 90^\circ$$

$$\angle B = \angle B \quad (\text{common angle})$$

$$\therefore \triangle ABD \cong \triangle CBE \quad (\text{by SAS congruence})$$

$$\Rightarrow AD = CE \quad (\text{cpct})$$

Solution 18:

<https://www.indcareer.com/schools/selina-class-9-icse-solutions-mathematics-chapter-9-triangles-congruency-in-triangles/>

Since ABCD is a rectangle, $\angle BAD$ is 90° .

Let $\angle BAE = x^\circ$

$\Rightarrow \angle ABE = (90 - x)^\circ \dots$ (angle sum property)

and $\angle DAO = (90 - x)^\circ \dots$ (linear angles)

$\Rightarrow \angle ODA = (90 - x)^\circ \dots$ (angle sum property)

Also, $\angle CDA = 90^\circ$

$\Rightarrow \angle CDF = (90 - x)^\circ \dots$ (linear angles)

In $\triangle ABE$ and $\triangle CDF$,

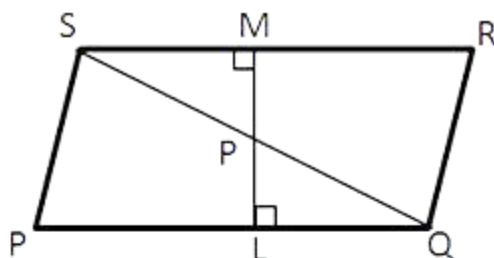
$\angle CDF = \angle ABE$ [each is $(90 - x)^\circ$]

$CD = AB$ (opposite sides of a rectangle are equal)

$\angle CFD = \angle AEB$ (each is 90°)

So, $\triangle ABE \cong \triangle CDF$ (AAS congruence criterion)

Solution 19:



Given : $PL = RM$

To prove: $SP = PQ$ and $MP = PL$

Proof :

Since SR and PQ are opposite sides of a parallelogram,

$$PQ = SR \quad \dots(1)$$

$$\text{Also, } PL = RM \quad \dots(2)$$

Subtracting (2) from (1),

$$PQ - PL = SR - RM$$

$$\Rightarrow LQ = SM \quad \dots(3)$$

Now, in $\triangle SMP$ and $\triangle QLP$,

$$\angle MSP = \angle PQL \quad (\text{alternate interior angles})$$

$$\angle SMP = \angle PLQ \quad (\text{alternate interior angles})$$

$$SM = LQ \quad [\text{From (3)}]$$

$$\therefore \triangle SMP \cong \triangle QLP \quad (\text{by ASA congruence})$$

$$\Rightarrow SP = PQ \text{ and } MP = PL \quad (\text{cpct})$$

$$\Rightarrow LM \text{ and } QS \text{ bisect each other.}$$

Solution 20:

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$\triangle ABC$ is an equilateral triangle.

So, each of its angles equals 60° .

QP is parallel to AC ,

$$\Rightarrow \angle PQB = \angle RAQ = 60^\circ$$

In $\triangle QBP$,

$$\angle PBQ = \angle BQP = 60^\circ$$

So, $\angle PBQ + \angle BQP + \angle BPQ = 180^\circ$ (angle sum property)

$$\Rightarrow 60^\circ + 60^\circ + \angle BPQ = 180^\circ$$

$$\Rightarrow \angle BPQ = 60^\circ$$

So, $\triangle BPQ$ is an equilateral triangle.

$$\Rightarrow QP = BP$$

$$\Rightarrow QP = CR \dots (i)$$

Now, $\angle QPM + \angle BPQ = 180^\circ$ (linear pair)

$$\Rightarrow \angle QPM + 60^\circ = 180^\circ$$

$$\Rightarrow \angle QPM = 120^\circ$$

Also, $\angle RCM + \angle ACB = 180^\circ$ (linear pair)

$$\Rightarrow \angle RCM + 60^\circ = 180^\circ$$

$$\Rightarrow \angle RCM = 120^\circ$$

In $\triangle RCM$ and $\triangle QMP$,

$$\angle RCM = \angle QPM \quad (\text{each is } 120^\circ)$$

$$\angle RMC = \angle QMP \quad (\text{vertically opposite angles})$$

$$QP = CR \quad (\text{from (i)})$$

$$\Rightarrow \triangle RCM \cong \triangle QMP \quad (\text{AAS congruence criterion})$$

So, $CM = PM$

$\Rightarrow QR$ bisects PC .



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- Chapter 2- Compound Interest (Without using formula)
- Chapter 3 - Compound Interest (Using Formula)
- Chapter 4- Expansions (Including Substitution)
- Chapter 5- Factorisation
- Chapter 6 - Simultaneous (Linear) Equations (Including Problems)
- Chapter 7- Indices (Exponents)
- Chapter 8- Logarithms
- Chapter 9- Triangles [Congruency in Triangles]
- Chapter 10- Isosceles Triangles
- Chapter 11- Inequalities
- Chapter 12- Mid-point and Its Converse [Including Intercept Theorem]
- Chapter 13- Pythagoras Theorem [Proof and Simple Applications with Converse]
- Chapter 14- Rectilinear Figures [Quadrilaterals: Parallelogram, Rectangle, Rhombus, Square and Trapezium]
- Chapter 15- Construction of Polygons
- Chapter 16- Area Theorems [Proof and Use]

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- Chapter 17- Circle
- Chapter 18- Statistics
- Chapter 19- Mean and Median
- Chapter 20 - Area and Perimeter of Plane Figures
- Chapter 21- Solids [Surface Area and Volume of 3-D Solids]
- Chapter 22- Trigonometrical Ratios
- Chapter 23- Trigonometrical Ratios of Standard Angles
- Chapter 24- Solution of Right Triangles
- Chapter 25- Complementary Angles
- Chapter 26- Co-ordinate Geometry
- Chapter 27- Graphical Solution (Solution of Simultaneous Linear Equations, Graphically)
- Chapter 28- Distance Formula

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