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H-5342

M.Sc. (Semester-IV) Examination, 2022

MATHEMATICS

(Partial Differential Equation and Mechanics - II)

Time Allowed : Three Hours

Maximum Marks : 70

Minimum Passing Marks : 25

Note : This question paper is divided into four sections. All sections are **compulsory**. Attempt questions as per given instructions in each section.

SECTION-A

(Objective Type Questions)

Note : Attempt **any ten** questions. Each question carries **1** mark. [1x10=10]

1. (A) Fill in the blanks :

(i) The equation $\hat{u}(y) = \frac{1}{(2\pi)^{n/2}} \int_{R^n} e^{-ix\cdot y} . u(x) dx$, $(y \in R^n)$ is called _____.

(Fourier Transform / Laplace Transform / Legendre Transform)

(ii) Generalised Co-ordinates is also known as _____. (Generalised potential / Cyclic co-ordinates / Degree of freedom)

(iii) The solid figure formed by the revolution of an arc with given surface area with maximum volume is _____. (catenary / cone / cycloid)

(iv) $\delta \int_{t_1}^{t_2} L.dt = 0$ is called _____. (Hamiton canonical equation / principle of least action / Hamilton principle)

(v) $\Delta \int_{t_1}^{t_2} \sum p_k \dot{q}_k dt = 0$ is called _____ .

(Hamilton Principle / Principle of least action / Hamilton Canonical equation)

(B) Choose the correct alternative :

(vi) A bead sliding on a circular wire is an example of :

(a) Holonomic

(b) Non-holonomic

(c) Rheonomic

(d) None of the above

(vii) The molecules moving inside gas container is an example of :

(a) Holonomic

(b) Non-holonomic

(c) Sclereonomic

(d) None of the above

(viii) q_k is said to be cyclic if :

(a) $\frac{\partial L}{\partial \dot{q}_k} = 0$

(b) $\frac{\partial L}{\partial q_k} = 0$

(c) $\frac{\partial L}{\partial p_k} = 0$

(d) $\frac{\partial L}{\partial \dot{p}_k} = 0$

(ix) Which of the following is true for Poisson bracket?

(a) They follow distributive law

(b) They follow commutative law

(c) Both (a) and (b)

(d) None of the above

(x) Out of the two statements :

(i) Lagrange's brackets are invariant under canonical transformation.

(ii) Poisson brackets are invariant under canonical transformation.

Which are of the following is / are true?

(a) (i) is true, (ii) is false

(b) (i) is false, (ii) is true

(c) (i) and (ii) both are true

(d) (i) and (ii) both are false

(xi) Jacobi form of least action is :

(a) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j$

(b) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = 0$

(c) $\Delta\omega = 0$

(d) $\Delta\int\sqrt{2[H - V(q)]}\, dp = 0$

(xii) "If the dynamical variables u and v are constants of motion, then their poisson brackets are also constants of motion." This is known as :

(a) Lagrange's equation of motion

(b) Jacobi - Poisson theorem

(c) Donkin's theorem

(d) Poisson's identity

SECTION-B

(Very Short Answer Type Questions)

Note : Attempt **any five** questions. Each question carries **02** marks. (Maximum word limit is **25-30** words). [5x2=10]

2. (i) Write properties of Laplace transform.

(ii) Write Lagrange's equation for conservative and holonomic dynamical system.

(iii) State fundamental lemma of calculus of variations.

(iv) Write Brachistochrone problem.

(v) Write Poisson identity.

(vi) Write properties of Lagrange's bracket.

(vii) Write Hamilton - Canonical equations of motion.

SECTION-C

(Short Answer Type Questions)

Note : Attempt **any five** questions. Each question carries **04** marks. (Maximum word limit is **250** words). [5x4=20]

3. (i) State and prove properties of Fourier Transform.

(ii) Explain Hodograph and Legendre Transforms.

(iii) Derive Routh's equations of motion.

(iv) Derive Lagrange's equation of second kind.

(v) Prove that Lagrange's brackets are invariant under canonical transformation.

(vi) Find the relation between Poisson Bracket and Lagrange's Bracket.

(vii) Write the properties of the Poisson bracket and show that Poisson bracket is distributive.

SECTION-D

(Essay Type Questions)

Note : Attempt **any three** questions. Each question carries **10** marks. (More than **500** words). [3x10=30]

4. (i) Discuss plane and travelling waves.

(ii) Write a short notes on Holonomic, Non-holonomic, Scleronomic and Rheonomic systems.

(iii) Prove that the geodesics of a plane is a straight line.

(iv) State and prove Jacobi-Poisson Theorem.

OR

Derive Euler's dynamical equation for the motion of a rigid body about an axis.

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