Paper code :6405

M.Sc. (Mathematics) Semester-3
JULY-SEP-2021
Mathematics -MCB-2 Advanced Linear Algebra (New-Old Course)

કુલ સમય :– ૩૦ મીનીટ કુલ પ્રશ્નો : ૨૦ વિદ્યાર્થીએ ટીક કરવાના પ્રશ્નો : ૧૮

નોંધ : વિદ્યાર્થીએ માત્ર ૧૮ પ્રશ્નો જ ટીક કરવાના રહેશે. જો વધારે ટીક કરેલ હશે તો પ્રથમ ૧૮ પ્રશ્નો જ ગણતરીમાં લેવામાં આવશે.

1. Consider the vector space $V = \mathbb{R}[x]$, the set of all polynomials in x with real coefficients, over the field $\mathbb{R}$. For any $q(x) \in V$, $q(x) = a_0 + a_1x + \cdots + a_nx_n$ define elements $T \in A(V)$ by $T(q(x)) = \int_1^x q(x)\,dx$. Then

(a) T is left intertitle but not right invertible B.T is right intertitle but not left invertible

$C.T$ is not left intertitle nor right invertible C.T is left intertitle and right invertible.

2. Let V be a finite dimensional vector space over $\mathbb{F}$ and $S, T \in A(V)$. Consider the following two statements

(1) $r(TS) \leq r(T)$

(2) If S is regular then $r(ST) = R(TS) = r(T)$.

(a)Both statement (1) and (2) are true. B.Both statement (1) and (2) are false.

C.Only the statement (1) is true. D.Only the statement (2) is true.

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that the Eigen values of are $1, \sqrt{2}, \sqrt{2}$. Then the maximum number of linearly independent Eigen vectors of T is

(a)3 B.4 C.5 D.6

4. Let $A \in M_2(\mathbb{F})$ and let characteristic polynomial of A be $(x - \lambda_1)(x - \lambda_2)$ with $\lambda_1 \neq \lambda_2$ then the minimal polynomial of A is

(a)$(x - \lambda_1)$ B.$(x - \lambda_2)$ C.$(x - \lambda_1)(x - \lambda_2)$ D.$None\ of\ these.$

5. Let A be a square matrix and its characteristic polynomial is given by $p(t) = (t-1)^3(t-2)^2(t-3)^4(t-4)$. Which of the following is the rank of A?

(a) 7 (b) 8 (c) 9 (d) None of these.

6. Let $B_1 = \{v_1, v_2, \cdots, v_n\}$ and $B_2 = w_1, w_2 \ldots, w_n\}$ be bases of a vector space Vover $\mathbb{F}$. Let $S, T \in A(V)$ and let $A = (a_{ij}) \in M_n(\mathbb{F})$. Suppose that A is the matrix of T with respect to the basis B_1 and A is the matrix of S with respect to the basis B_2. Then

(a)S and T are similar. B.S is triangular. C.T is triangular. D.$S$and$T$ are invertible.

7. Let V be a finite dimensional vector space over $\mathbb{F}$. Let $S, T \in A(V)$ and let S be regular. Then determine the correct option for the statements:

(1) $\lambda \in \mathbb{F}$ is a characteristic root of T if and only if it is a characteristic root of $S^{-1}TS$.

(2) $T$and $S^{-1}TS$ have the same minimal polynomial.

(a)Both statement (1) and (2) are true. B.Both statement (1) and (2) are false.

C.Only the statement (1) is true. D.Only the statement (2) is true.

8. Which of the following subspace W of $\mathbb{R}^2$ is invariant under $T : R^2 \to R^2$ $d$efined as $T(x; y) = (x + y, x + y)$?

(a) $W = \{(x; y) : x + 3y = 0\}$ (b) $W = \{(x; y) : x + y = 0\}$

(c) $W = \{(x; y) : x - y = 0\}$ (d) $W = \{f(x; y) : x - y = 1\}$

9. If A is nilpotent matrix of order $n \times n$. Which of the following is FLASE?

(a) 0 is the Eigen value of A. (b) $I + A$ is invertible. (c) A is singular. (d) None of above.

10. Let $A, B \in M_n(\mathbb{R})$. If A and B are nilpotent and $AB = BA$. Which of the following is TRUE?

(a) AB is nilpotent. (b) $A + B$ is nilpotent. (c) $A - B$ is nilpotent. (d) All above.

11. Let V be definite dimensional vector space. Suppose $T \in A(V)$ and assume that for some positive integer k, we have $T^k(v) = 0$ but $T^{k-1}(v) \neq 0$. Then for the set $S = \{v, Tv, T^2v, \ldots, T^{k-1}v\}$, which of the following is TRUE?

(a) The elements S are linearly independent.

(b) The Linear span W of S is a invariant subspace of Vunder T.

(c) The operator $T|_W$ is nilpotent with index k.

(d) All above.

12.If characteristic polynomial of a 4×4 matrix over $\mathbb{C}$ is $(x-1)^2(x-2)^2$and its minimal polynomial is $(x-1)^2(x-2)$. Then its Jordan canonical form is

(a) $\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&2&0\\0&0&0&2\end{bmatrix}$ $(b)\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&2&0\\0&-6&1&2\end{bmatrix}$ (c)$\begin{bmatrix}1&0&0&0\\1&1&0&0\\0&0&2&0\\0&0&0&2\end{bmatrix}$ (d)$\begin{bmatrix}1&0&0&0\\1&1&0&0\\0&0&2&0\\0&0&1&2\end{bmatrix}$

13.Which of the following statements are FALSE? Here $tr(A)$ is denoted by trance of matrix A of order $n \times n$.

(a)$tr(AB) = tr(A) \cdot tr(B)$

(b) $tr(A+B) = tr(A) + tr(B)$

(c)$tr(\alpha A) = \alpha\, tr(A)$, where $\alpha \in \mathbb{R}$.

(d) $tr(I) = n$ and $tr(\theta) = 0$, where I is $n \times n$ identity matrix and θ is $n \times n$ zero matrix.

14.Let $\bar{v}$ and $\bar{u}$ be two $n \times 1$ column vectors. If $\bar{u}^T$ denotes the transpose of $\bar{u}$, then what is the trace of $\bar{v} \cdot \bar{u}^T$?

(a)$\bar{v}^T \cdot \bar{u}$ b.$\bar{v}^T \cdot \bar{u}^T$ c.$\bar{v} \cdot \bar{u}^T$ d.$\bar{v} \cdot \bar{u}$

15.Let A be an $n \times n$ matrix such that $tr(A^n) = 0$, for all $n \in \mathbb{N}$. Then

(a)A is nilpotent matrix. B.A is zero matrix. C.$A$is identity matrix. D.$A$is invertible.

16.Let $\mathbb{F}$ be a field of characteristic _____ and V be a vector space over $\mathbb{F}$. If $S, T \in A(V)$ such that $ST - TS$ commutes with S, then $ST - TS$is nilpotent.

(a)0 b.1 c.2 d.3

17.Let $\mathbb{F}$ be a field of characteristic 0, V be a vector space over $\mathbb{F}$and $T \in A(V)$.

(1) T is nilpotent then $tr(T^i) = 0$ for all $i \geq 1$.

(2) $tr(T^i) = 0$ for all $i \geq 1$ then T is nilpotent then

(a)Only the statement (1) is true

(b) Only the statement (2) is true

(c)The statements (1) & (2) are true.

(d) The statements (1) & (2) are false.

18.Which of the following statements is false?

(a) If $A = (a_{ij}) \in M_n(\mathbb{R})$ is triangular matrix then determinant of A is product of entries on the diagonal of A.

(b) $Det(AB) = Det(A) \cdot Det(B)$

(c)$Det(A+B) = Det(A) + Det(B)$.

(d) $Det(A^T) = Det(A)$.

19.Let A be matrix with order 2020×2020 such that $A = (a_{ij})$, where $a_{ij} = 1$ for all i, j Then the determinant of A is

(a)1 b.0 c.2019 d.2020

20.Let $A = \begin{bmatrix}a&b\\c&d\end{bmatrix}$be an$2 \times 2$ matrix. Express the Eigenvalues of A in terms of the trace and the determinate of A.

(a) $\frac{\boldsymbol{tr(A) \pm \sqrt{tr(A)^2 - 4\det(A)}}}{\boldsymbol{2}}$

(b) $\frac{\boldsymbol{-tr(A) \pm \sqrt{tr(A)^2 - 4\det(A)}}}{\boldsymbol{2}}$

(c) $\frac{\boldsymbol{tr(A) \pm \sqrt{tr(A)^2 + 4\det(A)}}}{\boldsymbol{2}}$

(d) $\frac{\boldsymbol{-tr(A) - \sqrt{tr(A)^2 - 4\det(A)}}}{\boldsymbol{2}}$