

	On Conjugates :- $\{(a + ib)(a - ib) = a^2 + b^2\}$	
Q.11)	If $x + iy = \sqrt{\frac{a+ib}{c+id}}$ show that $(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$.	
Sol.11)	We have $x + iy = \sqrt{\frac{a+ib}{c+id}}$ (i) Taking conjugate on both sides $\Rightarrow \overline{x + iy} = \overline{\sqrt{\frac{a+ib}{c+id}}}$ $\Rightarrow x - iy = \sqrt{\frac{a-ib}{c-id}} \text{ (ii)}$ Equation (i) x (ii) $\Rightarrow (x + iy)(x - iy) = \sqrt{\frac{a+ib}{c+id}} \times \sqrt{\frac{a-ib}{c-id}}$ $\Rightarrow x^2 + y^2 = \sqrt{\frac{(a+ib)(a-ib)}{(c+id)(c-id)}}$ $\Rightarrow x^2 + y^2 = \sqrt{\frac{a^2+b^2}{c^2+d^2}} \text{ } \{(a + ib)(a - ib) = a^2 + b^2\}$ Squaring $(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2} \text{ (proved)}$	
Q.12)	If $\frac{(a+i)^2}{2a-i} = p + iq$ show that $p^2 + q^2 = \frac{(a^2+1)^2}{4a^2+1}$	
Sol.12)	We have $\frac{(a+i)^2}{2a-i} = p + iq$ (i) Taking conjugate on both sides $\Rightarrow \overline{\frac{(a+i)^2}{2a-i}} = \overline{p + iq}$ $\Rightarrow \frac{(a-i)^2}{2a+i} = p - iq \text{ (ii)}$ Equation (i) x (ii) $\Rightarrow \frac{(a+i)^2}{2a-i} \times \frac{(a-i)^2}{2a+i} = (p + iq)(p - iq)$ $\Rightarrow \frac{(a+i)(a-i)^2}{4a^2+1} = p^2 + q^2 \text{ } \{(a + ib)(a - ib) = a^2 + b^2\}$ Squaring $\frac{(a^2+1)^2}{4a^2+1} = p^2 + q^2 \text{ (proved)}$	
Q.13)	If $(1 + i)(2 + 2i)(1 + 3i) \dots (1 + ni) = x + iy$ show that $2.5.10 \dots (1 + n^2) = x^2 + y^2$.	
Sol.13)	We have $(1 + i)(2 + 2i)(1 + 3i) \dots (1 + ni) = x + iy$ (i) Taking conjugate on both sides $\Rightarrow \overline{(1 + i)(2 + 2i)(1 + 3i) \dots (1 + ni)} = \overline{x + iy}$ $\Rightarrow (1 - i)(2 - 2i)(1 - 3i) \dots (1 - ni) = x - iy \text{ (ii)}$ Equation (i) x (ii) $\Rightarrow (1 + i)(1 - i)(2 + 2i)(2 - 2i)(1 + 3i)(1 - 3i) \dots (1 + ni)(1 - ni)$ $= (x + iy)(x - iy)$ $\Rightarrow (1 + 1)(1 - 4)(1 + 9) \dots (1 + n^2) = x^2 + y^2 \text{ } \{(a + ib)(a - ib) = a^2 + b^2\}$ Squaring $2.5.10 \dots (1 + n^2) = x^2 + y^2 \text{ (proved)}$	

	On Standard Form:-	
Q.14)	Find θ such that $\frac{3+2i \sin \theta}{1-2i \sin \theta}$ is purely real.	
Sol.14)	Let $z = \frac{3+2i \sin \theta}{1-2i \sin \theta}$ Rationalize $\Rightarrow z = \frac{3+2i \sin \theta}{1-2i \sin \theta} \times \frac{1+2i \sin \theta}{1+2i \sin \theta}$ $\Rightarrow z = \frac{3+6i \sin \theta+2i \sin \theta+4i^2 \sin^2 \theta}{1-4i^2 \sin^2 \theta}$ $\Rightarrow z = \frac{3+8i \sin \theta-4 \sin^2 \theta}{1+4 \sin^2 \theta}$ $\Rightarrow z = \frac{(3-4 \sin^2 \theta)}{1+4 \sin^2 \theta} + \frac{8i \sin \theta}{1+4 \sin^2 \theta}$ It is given that z is purely real $\therefore \operatorname{Im}(z) = 0$ $\Rightarrow \frac{8 \sin \theta}{1+4 \sin^2 \theta} = 0$ $\Rightarrow 8 \sin \theta = 0$ $\Rightarrow \sin \theta = 0$ $\Rightarrow \theta = n\pi ; n \in \mathbb{Z} \text{ ans.}$	
Q.15)	If z is complex number such that $ z = 1$ prove that $\left(\frac{z-1}{z+1}\right)$ is purely imaginary. What will be your conclusion of $z = 1$.	
Sol.15)	Let $z = x + iy$ $z = 1$ (given) $\Rightarrow \sqrt{x^2 + y^2} = 1$ $\Rightarrow x^2 + y^2 = 1 \dots\dots\dots (i)$ Now, $\frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1} = \frac{(x-1)+iy}{(x+1)+iy}$ $\Rightarrow \frac{z-1}{z+1} = \frac{[(x-1)+iy][(x+1)-iy]}{[(x+1)+iy][(x+1)-iy]} \dots\dots\dots (\text{rationalize})$ $= \frac{(x^2-1)-iy(x-1)+iy(x+1)-i^2y^2}{(x+1)^2-i^2y^2}$ $= \frac{(x^2-1)+i(-yx+y+yx+y)+y^2}{(x+1)^2+y^2}$ $= \frac{(x^2+y^2-1)+2iy}{(x+1)^2+y^2}$ $= \frac{0+2iy}{(x+1)^2+y^2} \dots\dots\dots (x^2 + y^2 = 1 \text{ from eq. (i)})$ $\Rightarrow \frac{z-1}{z+1} = \frac{2iy}{(x+1)^2+y^2} \text{ which is purely imaginary}$ Now when $z = 1$ Then $\frac{z-1}{z+1} = \frac{1-1}{1+1} = \frac{0}{2} = 0$ which is purely real ans.	
	Natural Number:-	
Q.16)	Find the least +ve integral value of m , if $\left(\frac{1+i}{1-i}\right)^m = 1$	
Sol.16)	We have $\left(\frac{1+i}{1-i}\right)^m = 1$ $\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^m = 1$ $\Rightarrow \left(\frac{1+i^2+2i}{1-i^2}\right)^m = 1$ $\Rightarrow \left(\frac{1-1+2i}{1+1}\right)^m = 1$	

	$\Rightarrow \left(\frac{2i}{2}\right)^m = 1$ $\Rightarrow (i)^m = 1$ $\therefore \text{the least +ve integral value of } m = 4 \text{ ans. (Since } i^4 = 1)$	
Q.17)	Find the smallest +ve integer value of n for which $\frac{(1+i)^n}{(1-i)^{n-2}}$ is a real number.	
Sol.17)	We have $\frac{(1+i)^n}{(1-i)^{n-2}}$ $= \frac{(1+i)^n}{(1+i)^n(1-i)^{-2}}$ $= \left(\frac{1+i}{1-i}\right)^n (1-i)^2$ $= \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^n (1-i)^2$ $= \left(\frac{1+i^2+2i}{1-i^2}\right)^n (1-i^2-2i)$ $= \left(\frac{1-1+2i}{1+1}\right)^n (1-1-2i)$ $= (i)^n(-2i)$ $= -2(i)^{n+1}$ For real number : n should be equal to 1 Since $-2(i)^{1+1} = -2(i)^2 = -2(-1) = 2$ which is a real number $\therefore n = 1$ ans.	
Q.18)	Find the number of non-zero integral solutions if $ 1-i ^x = 2^x$	
Sol.18)	We have $1-i ^x = 2^x$ $\Rightarrow (\sqrt{1+1})^x = 2^x \dots\dots\dots \{ z = \sqrt{a^2+b^2}\}$ $\Rightarrow (\sqrt{2})^x = 2^x$ $\Rightarrow (2)^{\frac{x}{2}} = 2^x$ $\Rightarrow \frac{x}{2} = x$ $\Rightarrow x = 2x$ $\Rightarrow 2x - x = 0$ $\Rightarrow x = 0$ Hence, there is no non-zero integral solution	
Q.19)	If α & β are different complex numbers with $ \beta = 1$. Find $\left \frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right $	
Sol.19)	We have $\left \frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right ^2 = \left(\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right)\left(\frac{\bar{\beta}-\bar{\alpha}}{1-\alpha\bar{\beta}}\right) \dots\dots\dots z ^2 = z\bar{z}$ $= \left(\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right)\left(\frac{\bar{\beta}-\bar{\alpha}}{1-\alpha\bar{\beta}}\right) \dots\dots\dots \{(\bar{\bar{z}}) = z\}$ $= \frac{\beta\bar{\beta}-\beta\bar{\alpha}-\alpha\bar{\beta}+\alpha\bar{\alpha}}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+\alpha\bar{\alpha}\beta\bar{\beta}}$ $= \frac{ \beta ^2-\beta\bar{\alpha}-\alpha\bar{\beta}+ \alpha ^2}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+ \alpha ^2 \beta ^2} \dots\dots\dots z ^2 = z\bar{z}$ $= \frac{1-\beta\bar{\alpha}-\alpha\bar{\beta}+ \alpha ^2}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+ \alpha ^2} \dots\dots\dots \text{given } \beta = 1$ $\Rightarrow \left \frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right ^2 = 1$ $\Rightarrow \left \frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right = 1 \text{ ans.}$	
Q.20)	If z_1 and z_2 are any two complex numbers then show that	

	$Re(z_1 z_2) = Re(z_1)Re(z_2) - Im(z_1)Im(z_2).$	
Sol.20)	Let $z_1 = a + ib$ and $z_2 = c + id$ $\therefore Re(z_1) = a$ and $Re(z_2) = c$ $Im(z_1) = b$ and $Im(z_2) = d$ Now, $z_1 z_2 = (a + ib)(c + id)$ $= ac + iad + ibc + i^2 bd$ $= ac + iad + ibc - bd$ $z_1 z_2 = (ac + bd) + i(ad + bc)$ Here, $Re(z_1 z_2) = ac - bd$ $= Re(z_1)Re(z_2) - Im(z_1)Im(z_2)$ RHS (proved)	
	Quadratic Equation:-	
Q.21)	Solve $2x^2 + 3ix + 2 = 0$	
Sol.21)	Here $a = 2, b = 3i, c = 2$ By quadratic formula, $\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\Rightarrow x = \frac{-3i \pm \sqrt{9i^2 - 4(2)(2)}}{4}$ $\Rightarrow x = \frac{-3i \pm \sqrt{-9 - 16}}{4}$ $\Rightarrow x = \frac{-3i \pm \sqrt{-25}}{4}$ $\Rightarrow x = \frac{-3i \pm 5i}{4}$ $\Rightarrow x = \frac{-3i + 5i}{4}$ and $x = \frac{-3i - 5i}{4}$ $\Rightarrow x = \frac{i}{2}$ and $x = -2i$ ans.	
Q.22)	Solve $2x^2 + (3 + 7i)x - (3 - 9i) = 0.$	
Sol.22)	Here $a = 2, b = -(3 + 7i), c = -(3 - 9i)$ By quadratic formula, $\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\Rightarrow x = \frac{(3+7i) \pm \sqrt{(3+7i)^2 - 4(2)(3-9i)}}{4}$ $\Rightarrow x = \frac{(3+7i) \pm \sqrt{9+49i^2+42i+24-72i}}{4}$ $\Rightarrow x = \frac{3+7i \pm \sqrt{9-49-30i+24}}{4}$ $\Rightarrow x = \frac{3+7i \pm \sqrt{-16-30i}}{4}$ Let $a + ib = \sqrt{-16 - 30i}$ Squaring, $\Rightarrow a^2 + i^2 b^2 + 2iab = -16 - 30i$ $\Rightarrow (a^2 - b^2) + 2iab = -16 - 30i$ $\Rightarrow a^2 - b^2 = -16$ (i) And $2ab = -30$ Now $(a^2 - b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$ $= 256 + 900$ $= 1156$ $\Rightarrow a^2 - b^2 = 34$ (ii) Adding (i) & (ii) $\Rightarrow a^2 = 18$	

	$\Rightarrow a^2 = a$ $\Rightarrow a = \pm 3$ Put $a^2 = 9$ in eq.(ii) $\Rightarrow 9 + b^2 = 34$ $\Rightarrow b^2 = 25$ $\Rightarrow b = \pm 5$ $\therefore \sqrt{-16 - 30i} = \pm(3 - 5i)$ $\therefore$ value of x becomes $\Rightarrow x = \frac{3+7i \pm (3-5i)}{4}$ $\Rightarrow x = \frac{3+7i+(3-5i)}{4}$ and $x = \frac{3+7i-(3-5i)}{4}$ $\Rightarrow x = \frac{6+2i}{4}$ and $x = \frac{12i}{4}$ $\Rightarrow x = \frac{3}{2} + \frac{1}{2}i$ and $x = 3i$ ans.	
Q.23)	If $z = x + iy$ and $w = \frac{1-iz}{z-i}$, If $ w = 1$ show that z is purely real.	
Sol.23)	We have $z = x + iy, w = \frac{1-iz}{z-i}$ and $ w = 1$ $\Rightarrow \left \frac{1-iz}{1-iz} \right = 1$ $\Rightarrow \frac{ 1-iz }{ z-i } = 1 \dots\dots\dots \left\{ \left \frac{z_1}{z_2} \right = \frac{ z_1 }{ z_2 } \right\}$ $\Rightarrow 1-iz = z-i $ $\Rightarrow 1-i(x+iy) = x+iy-i $ $\Rightarrow 1-ix-i^2y = x+i(y-i) $ $\Rightarrow (1+y)-ix = x+i(y-i) $ $\Rightarrow \sqrt{(1+y)^2 - x^2} = \sqrt{x^2 + (y-1)^2}$ Squaring $\Rightarrow 1 + y^2 + 2y + x^2 = x^2 + y^2 + 1 - 2y$ $\Rightarrow 4y = 0$ $\Rightarrow y = 0$ Since $z = x + iy$ $\Rightarrow z = x + i(0)$ $\Rightarrow z = x$ which is purely real (proved)	
Q.24)	Convert into polar form $z = \sin\left(\frac{\pi}{5}\right) + i\left(1 - \cos\frac{\pi}{5}\right)$.	
Sol.24)	We have $z = \sin\left(\frac{\pi}{5}\right) + i\left(1 - \cos\frac{\pi}{5}\right)$ Here $a = \sin\left(\frac{\pi}{5}\right)$ and $b = 1 - \cos\frac{\pi}{5}$ $\Rightarrow r = \sqrt{a^2 + b^2} = \sqrt{\sin^2\frac{\pi}{5} + \left(1 - \cos\frac{\pi}{5}\right)^2}$ $= \sqrt{\sin^2\frac{\pi}{5} + 1 + \cos^2\frac{\pi}{5} - 2\cos\frac{\pi}{5}}$ $= \sqrt{1 + 1 - 2\cos\frac{\pi}{5}}$ $= \sqrt{2 - 2\cos\frac{\pi}{5}}$ $= \sqrt{2} \sqrt{1 - \cos\frac{\pi}{5}}$	

	$= \sqrt{2} \sqrt{2 \sin^2 \left(\frac{\pi}{10} \right)}$ $\Rightarrow r = 2 \sin \left(\frac{\pi}{10} \right)$ Now $\tan \alpha = \left \frac{b}{a} \right = \left \frac{1 - \cos \frac{\pi}{5}}{\sin \frac{\pi}{5}} \right$ $\Rightarrow \tan \alpha = \left \frac{2 \sin^2 \left(\frac{\pi}{10} \right)}{2 \sin \left(\frac{\pi}{10} \right) \cdot \cos \frac{3}{10}} \right $ $\Rightarrow \tan \alpha = \tan \left(\frac{\pi}{10} \right)$ $\Rightarrow \alpha = \frac{\pi}{10}$ Since z is in 1st quadrant $\begin{pmatrix} a + ve \\ b + ve \end{pmatrix}$ $\therefore \theta = \alpha$ $\Rightarrow \theta = \frac{\pi}{10} \text{ (argument is also called AMPLITUDE)}$ Polar form is given by $\Rightarrow z = r [\cos(\theta) + i \sin(\theta)]$ $\Rightarrow z = 2 \sin \left(\frac{\pi}{10} \right) \left[\cos \left(\frac{\pi}{10} \right) + i \sin \left(\frac{\pi}{10} \right) \right] \text{ ans.}$	
Q.25)	Evaluate: $(1 + i)^6 + (1 - i)^3$.	
Sol.25)	$\Rightarrow ((1 + i)^2)^3 + (1 - i)^3$ $\Rightarrow (1 + i^2 + 2i)^3 + (1 - i)^3$ $\Rightarrow (2i)^3 + 1^3 - i^3 - 3i + 3i^2$ $\Rightarrow 8i^3 + 1 + i - 3i - 3$ $\Rightarrow -8i + 1 + i - 3i - 3$ $\Rightarrow -2 - 10i \text{ ans.}$	