

- Measures of Dispersion, Mean, Mode, Median for a Grouped Data and an Ungrouped Data. Average Deviation, Standard Deviation and the coefficient of Variation for a Grouped data and an Ungrouped Data :

(1) The mean for an Ungrouped Data :

$$(i) \bar{x} = \frac{\sum_{i=1}^n x_i}{n} \quad \text{where } x_1, x_2, \dots, x_n \text{ are the } n \text{ observations of given data.}$$

$$(ii) \bar{x} = A + \frac{\sum d_i}{n} \quad \text{where } d_i = x_i - A \quad (\text{short-cut method})$$

- The mean for a Grouped Data :

$$(i) \bar{x} = \frac{\sum f_i x_i}{n} \quad \text{where } n = \sum f_i, f_i = \text{frequency}$$

$$(ii) \bar{x} = A + \frac{\sum f_i d_i}{n} \quad \text{where } d_i = x_i - A, A \text{ is a constant}$$

- The mean for a Continuous Frequency Distribution :

$$(i) \bar{x} = \frac{\sum f_i x_i}{n} \quad \text{where } x_i = \text{central value for } i\text{th class, } n = \sum f_i$$

$$(ii) \bar{x} = A + \frac{\sum f_i d_i}{n} \times c, \quad d_i = \frac{x_i - A}{c}, c = \text{class length (uniform)}$$

- The combined Mean for a Mixed Group :

The number of observations of groups T_1 , T_2 and T_3 are n_1 , n_2 and n_3 respectively. $\bar{x}_1$, $\bar{x}_2$ and $\bar{x}_3$ are the means of groups T_1 , T_2 and T_3 respectively. Then the combined mean of the mixed group is given by $\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2 + n_3 \bar{x}_3}{n_1 + n_2 + n_3}$.

(2) The median :

The median for an ungrouped data : First, arrange the given observations in ascending order :

(i) If n is an odd integer, then the median, $M = \text{Value of } \left(\frac{n+1}{2}\right)\text{th observation.}$

(ii) If n is an even integer, the median $M =$

$$\frac{\text{Value of } \frac{n}{2}\text{th observation} + \text{Value of } \left(\frac{n}{2} + 1\right)\text{th observation}}{2}$$

The median for a discrete frequency distribution : $M = \left(\frac{n+1}{2}\right)\text{th observation, } n = \sum f_i$

The median for continuous frequency distribution : $M = L + \frac{\frac{n}{2} - F}{f} \times c$

where $L =$ The lower boundary point of the median class

$f =$ The frequency of the median class

$F =$ The cumulative frequency of class preceding the median class

$c =$ The class length of the median class

$n =$ Total frequency $= \sum f_i$

(3) Dispersion :

There are three principal measures of dispersion :

(1) Range (2) Average Deviation (3) Standard Deviation

(1) **Range** : $R = \text{The largest observation} - \text{The smallest observation}$

(2) The Average Deviation :

- The average deviation from the mean of an ungrouped data :

$$\delta \bar{x} = \frac{\sum |x_i - \bar{x}|}{n} \quad \text{where } n = \text{the number of observations}$$

$\bar{x}$ is the mean of observations $x_1, x_2, \dots, x_n$

- The average deviation from the mean of a discrete frequency distribution :

$$\delta \bar{x} = \frac{\sum f_i |x_i - \bar{x}|}{n} \quad \text{where } n = \sum f_i \text{ and } \bar{x} \text{ is the mean. } x_i \text{ are observations,}$$

f_i are frequencies of x_i 's.

- The average deviation from the mean of a continuous frequency distribution :

$$\delta \bar{x} = \frac{\sum f_i |x_i - \bar{x}|}{n} \quad \text{where } n = \sum f_i, f_i = \text{frequency of the } i\text{th class}$$

x_i = the central value of the i th class, $\bar{x}$ = the mean of data

- The average deviation from the median for an ungrouped data :

$$\delta M = \frac{\sum |x_i - M|}{n} \quad n = \text{the number of observations, } M = \text{the median}$$

- The average deviation from the median of a discrete frequency distribution :

$$\delta M = \frac{\sum f_i |x_i - M|}{n} \quad n = \sum f_i, f_i \text{ is the frequency of the } i\text{th observation}$$

- The average deviation from the median of a continuous frequency distribution :

$$\delta M = \frac{\sum f_i |x_i - M|}{n} \quad n = \sum f_i, f_i \text{ is the frequency of the } i\text{th class,}$$

x_i is the central value of the i th class

(3) Standard Deviation :

The positive square root of the number, obtained by dividing the sum of the squares of deviations from the mean of observations of given data by the number of observations is called the standard deviation of data. It is denoted by s .

- For an ungrouped data :

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} \quad \text{or} \quad s = \sqrt{\frac{\sum x_i^2}{n} - \bar{x}^2},$$

where n = the number of observations, $\bar{x}$ = the mean

- For a discrete frequency distribution :

$$s = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{n}} \quad \text{or} \quad s = \sqrt{\frac{\sum f_i x_i^2}{n} - \left(\frac{\sum f_i x_i}{n}\right)^2}, \quad \text{where } n = \sum f_i$$

- For a continuous frequency distribution :

$$s = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{n}}$$

x_i = the central value of the i th class, f_i = the frequency of the i th class, $n = \sum f_i$

or

If $x_1, x_2, \dots, x_k$ are the central values of k classes of given continuous frequency distribution and $f_1, f_2, \dots, f_k$ are corresponding frequencies of these classes, then

$$s = \sqrt{\frac{\sum f_i d_i^2}{n} - \left(\frac{\sum f_i d_i}{n}\right)^2} \times c$$

where $d_i = \frac{x_i - A}{c}$, c = the class length, $n = \sum f_i$, A = Assumed mean

(4) Coefficient of Variation :

The square of the standard deviation is called the variance. i.e., s^2 = variance

The quotient obtained when the standard deviation divided by the mean is multiplied by 100, then we get the coefficient of variation in percentage.

$$\text{C.V.} = \frac{s}{\bar{x}} \times 100$$

- Lesser the value of C.V., the data is more consistent.
- Greater the value of C.V., the data is not consistent. i.e. data is more deviated from the measure of central tendency.
- $y = ax + b \Rightarrow \bar{y} = a\bar{x} + b$
 $y = ax + b \Rightarrow s_y = |a| s_x$
- Correct $\sum x_i = n\bar{x} - (\text{the sum of incorrect observations}) + (\text{the sum of correct observations})$
- Correct $\sum x_i^2 = n(s^2 + \bar{x}^2) - (\text{the sum of squares of incorrect observations}) + (\text{the sum of squares of correct observations})$
- Variance of 1, 2, 3, ..., n is $s^2 = \frac{1}{12}(n^2 - 1)$
- Variance of 2, 4, 6, ..., $2n$ is $s^2 = \frac{1}{3}(n^2 - 1)$
- Variance of 1, 3, 5, ..., $2n - 1$ is $s^2 = \frac{1}{3}(n^2 - 1)$
- Coefficient of variation of 1, 2, 3, ..., n is $= \sqrt{\frac{1}{3}\left(\frac{n-1}{n+1}\right)} \times 100$

(5) Mode :

The mode is the value or the number which is most frequently occurring in a set of observations (data). It is denoted by Z.

- In discrete series, mode can be determined by finding the variable which has highest frequency associated with it.
- In continuous frequency distribution, the class which contains highest frequency is called the modal class.

$$\text{Mode} = L + \frac{f - f_1}{2f - f_1 - f_2} \times c$$

L = The lower boundary point of modal class

f_1 = The frequency of the class preceding the modal class

f_2 = The frequency of the class succeeding the modal class

f = frequency of the modal class

c = class length of the modal class.

- In symmetrical frequency distribution mean = median = mode
- In a positive skewed distribution, mode < median < mean
- In a negative skewed distribution, mean < median < mode
- For normal frequency distribution :

$$\text{Mode} = 3 (\text{Median}) - 2 (\text{Mean}) \text{ i.e. } Z = 3M - 2\bar{x}$$

: Multiple Choice Questions :

- (1) $\bar{x}$ is the mean of n observations. $2, 2^2, 2^3, 2^4, \dots, 2^n$ are added to corresponding observations. The new mean is

(A) $\bar{x} + \frac{2^n - 1}{2^n}$ (B) $\bar{x} + \frac{2^n}{2}$ (C) $\bar{x} + 2\frac{(2^n - 1)}{n}$ (D) $\bar{x} + \frac{2^{n+1}}{n}$

Sol. : $\bar{x}$ is the mean of the observations $x_1, x_2, \dots, x_n$.

$$\therefore \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

$2, 2^2, 2^3, 2^4, \dots, 2^n$ are added to the corresponding observations respectively.

New observations are $x_1 + 2, x_2 + 2^2, x_3 + 2^3, \dots, x_n + 2^n$

$$\text{The new mean} = \frac{(x_1 + 2) + (x_2 + 2^2) + (x_3 + 2^3) + \dots + (x_n + 2^n)}{n}$$

$$= \frac{x_1 + x_2 + \dots + x_n}{n} + \frac{2 + 2^2 + 2^3 + \dots + 2^n}{n}$$

$$= \bar{x} + \frac{2[2^n - 1]}{(2 - 1)n} \quad \left(s_n = \frac{a(r^n - 1)}{r - 1} \text{ [Geometric series]} \right)$$

$$\text{The new mean} = \bar{x} + \frac{2[2^n - 1]}{n}$$

Ans. : (C)

- (2) If the mean of the observations $x_1, x_2, \dots, x_n$ is $\bar{x}$, then the mean of the observations $x_1 + 1, x_2 + 3, x_3 + 5, \dots, x_n + (2n - 1)$ is

(A) $\bar{x} + n$ (B) $\bar{x} + \frac{n+1}{2}$ (C) $(2n - 1)\bar{x}$ (D) $\bar{x} + \frac{(n+1)^2}{4}$

Sol. : $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$

$$\text{The new mean} = \frac{(x_1 + 1) + (x_2 + 3) + (x_3 + 5) + \dots + x_n + (2n - 1)}{n}$$

$$= \frac{x_1 + x_2 + \dots + x_n}{n} + \frac{1 + 3 + 5 + \dots + (2n - 1)}{n}$$

$$= \bar{x} + \frac{n^2}{n}$$

$$= \bar{x} + n$$

Ans. : (A)

- (3) The median of the following observations is

3, 9, 5, 3, 12, 10, 18, 4, 7, 19, 21

(A) 8 (B) 9 (C) 10 (D) 7

Sol. : Arrange the given observations in ascending order 3, 3, 4, 5, 7, 9, 10, 12, 18, 19, 21

$$\therefore \text{Median} = \left(\frac{n+1}{2} \right) \text{th observation} = 6\text{th observation} \quad (n = 11)$$

$$\therefore \text{Median} = 9$$

Ans. : (B)

- (4) In a class of 100 students, there are 70 boys. Their average marks in a subject are 75. If the average marks of the whole class is 72, then what are the average marks of the girls ?

[AIEEE : 2002]

- (A) 73 (B) 68 (C) 65 (D) 74

Sol. : Let n_1 = the total number of boys $\bar{x}_1$ = average marks of boys

n_2 = the total number of girls $\bar{x}_2$ = average marks of girls

$$n_1 = 70 \quad n_2 = 30 \quad \bar{x}_1 = 75 \quad \bar{x} = 72 \quad \bar{x}_2 = ?$$

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

$$\therefore 72 \times 100 = 70 \times 75 + 30 \bar{x}_2$$

$$7200 - 5250 = 30 \bar{x}_2$$

$$\frac{1950}{30} = \bar{x}_2$$

$$\therefore \bar{x}_2 = 65$$

Ans. : (C)

- (5) In an experiment with 15 observations about variable x , the following results were available. $\sum x_i^2 = 2830$ and $\sum x_i = 170$. If one observation 20 was found to be incorrect and was replaced by the correct value 30, then the corrected variance is [AIEEE : 2003]

- (A) 188.66 (B) 177.33 (C) 8.33 (D) 78.00

Sol. : Correct $\sum x_i = 170 - 20 + 30 = 180$

$$\text{Correct } \sum x_i^2 = 2830 - (20)^2 + (30)^2 = 2830 - 400 + 900 = 3330$$

$$\begin{aligned} \text{Correct variance} &= \frac{\text{Correct } \sum x_i^2}{n} - \left(\frac{\text{Correct } \sum x_i}{n} \right)^2 \\ &= \frac{3330}{15} - \left(\frac{180}{15} \right)^2 = 222 - 144 = 78 \end{aligned}$$

Ans. : (D)

- (6) The median of a set of 9 distinct observations is 20.5. If each of the last 4 observations, of the set in increasing order is increased by 2, then the median of the new set [AIEEE : 2003]

- (A) Is decreased by 2 (B) Is two times the original median
(C) Remains the same as that of the original (D) Is increased by 2

Sol. : $n = 9$. Median $M = \left(\frac{n+1}{2} \right)$ th observation = 5th observation

Observations in ascending order are $x_1, x_2, x_3, x_4, \boxed{x_5}, x_6, x_7, x_8, x_9$.

Here, the fifth observation is 20.5. The median is x_5 .

Here, last four observations are increased by 2. The increased order of observations is the same. The median is the 5th observation. So there will be no change in the median.

$\therefore$ The median remains the same as that of the original set.

Ans. : (C)

- (7) The standard deviation of 1, 2, 3, 4, 5, 6 and 7 is 2. Then the standard deviation of 12, 23, 34, 45, 56, 67 and 78 is

- (A) 11 (B) 22 (C) 4 (D) 2

Sol. : The standard deviation of 1, 2, 3, 4, 5, 6 and 7 is 2.

If each observation is multiplied by 11 and then 1 is added to each of them, we get

12, 23, 34, 45, 56, 67 and 78.

$$\therefore y_i = 11x_i + 1$$

$$\therefore s_y = 11s_x$$

$$s_y = 11 \times 2 = 22$$

Ans. : (B)

(8) The standard deviation of 1, 2, 3, 4, 5 is

- (A) $\sqrt{3}$ (B) $\frac{3}{5}$ (C) $\frac{2}{5}$ (D) $\sqrt{2}$

Sol. : The standard deviation of 1, 2, 3, ..., n is $\sqrt{\frac{n^2-1}{12}}$

$$\therefore \text{The standard deviation of 1, 2, 3, 4, 5} = \sqrt{\frac{5^2-1}{12}} = \sqrt{2}$$

Ans. : (D)

(9) The variance of the observations 3, 4, 5, 6 is

- (A) 1.25 (B) 2 (C) 2.25 (D) 0.75

$$\begin{array}{l} \text{Sol. : } \bar{x} = \frac{3+4+5+6}{4} \\ \quad = \frac{18}{4} = 4.5 \end{array} \quad \left| \quad \begin{array}{l} \sum x_i^2 = 9 + 16 + 25 + 36 \\ \quad = 86 \end{array} \right.$$

$$\text{Variance} = s^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n} \right)^2 = \frac{86}{4} - \frac{81}{4} = \frac{5}{4} = 1.25$$

$$[\text{Short-cut : Variance of 3, 4, 5, 6} = \text{Variance of 1, 2, 3, 4} = \frac{n^2-1}{12} = \frac{16-1}{12} = \frac{5}{4} = 1.25]$$

Ans. : (A)

(10) The mean of 20 observations is 10. If the incorrect observation 8 is replaced by correct observation 12, then the corrected mean is

- (A) 10 (B) 10.2 (C) 14 (D) 13

$$\begin{aligned} \text{Sol. : Correct } \sum x_i &= n\bar{x} - (\text{incorrect observation}) + (\text{correct observation}) \\ &= 200 - 8 + 12 = 204 \end{aligned}$$

$$\text{Correct Mean} = \frac{\text{Correct } \sum x_i}{20} = \frac{204}{20} = 10.2$$

Ans. : (B)

(11) The mean and the standard deviation of 10 observations are 4 and 3 respectively. If one of the observation 5 is taken incorrect as -5, then the correct mean and the correct standard deviation are

- (A) $\bar{x} = 0$; $s = 5$ (B) $\bar{x} = 5$; $s = 0$ (C) $\bar{x} = 3$; $s = 2.5$ (D) $\bar{x} = 5$; $s = 1.1$

$$\text{Sol. : } n = 10, \quad \bar{x} = 4, \quad s = 3$$

$$\begin{aligned} \text{Correct } \sum x_i &= n\bar{x} - \text{incorrect observation} + \text{correct observation} \\ &= 10(4) - (-5) + 5 = 40 + 10 = 50 \end{aligned}$$

$$\text{Correct } \bar{x} = \frac{\text{Correct } \sum x_i}{n} = \frac{50}{10} = 5$$

$$\begin{aligned} \text{Correct } \sum x_i^2 &= n(s^2 + \bar{x}^2) - (\text{incorrect observation})^2 + (\text{correct observation})^2 \\ &= 10(9 + 16) - (-5)^2 + (5)^2 = 250 \end{aligned}$$

$$\text{Correct } s = \sqrt{\frac{\text{Correct } \sum x_i^2}{n} - \left(\frac{\text{Correct } \sum x_i}{n} \right)^2} = \sqrt{\frac{250}{10} - (5)^2} = 0$$

$$\therefore \bar{x} = 5, s = 0$$

Ans. : (B)

- (12) The mean and variance of five observations are 4.4 and 8.24 respectively. If three observations are 1, 2 and 6, then the remaining two observations are
 (A) 4, 9 (B) 3, 9 (C) 4, 8 (D) 5, 8

Sol. : Suppose a and b are the rest of the observations.

$$\sum x_i = n\bar{x}$$

$$\therefore 1 + 2 + 6 + a + b = 5(4.4)$$

$$\therefore a + b + 9 = 22$$

$$\therefore a + b = 13 \quad (1)$$

$$\sum x_i^2 = 1^2 + 2^2 + 6^2 + a^2 + b^2 = 41 + a^2 + b^2$$

$$s^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$$

$$8.24 + 19.36 = \frac{a^2 + b^2 + 41}{5}$$

$$5 \times 27.6 = a^2 + b^2 + 41$$

$$138 = a^2 + b^2 + 41$$

$$\therefore a^2 + b^2 = 97 \quad (2)$$

$$a^2 + (13 - a)^2 = 97 \quad (\text{From (1)})$$

$$a^2 + a^2 - 26a + 169 = 97$$

$$a^2 - 13a + 36 = 0$$

$$\therefore a = 9 \text{ or } a = 4 \text{ i.e. } b = 4 \text{ or } b = 9 \quad \text{Ans. : (A)}$$

- (13) There are $2n$ observations in a sequence. Out of them, half of the observations are a and the rest of the observations are $-a$. If the standard deviation of the observations is 2, then $|a| = \dots\dots$ [AIEEE : 2004]

(A) 2 (B) $\sqrt{2}$ (C) $\frac{1}{n}$ (D) $\frac{\sqrt{2}}{n}$

Sol. : $\sum x_i = \frac{(a + a + \dots + a \text{ } n \text{ times}) - (a + a + \dots + a \text{ } n \text{ times})}{2n} = \frac{0}{2n} = 0$

$$\begin{aligned} \sum x_i^2 &= a^2 + a^2 + \dots + a^2 (n \text{ times}) + (-a)^2 + (-a)^2 + \dots + (-a)^2 (n \text{ times}) \\ &= na^2 + na^2 = 2na^2 \end{aligned}$$

$$s^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$$

$$\therefore 4 = \frac{2na^2}{2n} - 0. \quad \text{So, } 4 = a^2$$

$$\therefore |a| = 2 \quad \text{Ans. : (A)}$$

- (14) The mean and the median of some frequency distribution are 21 and 22. The mode of the frequency distribution is [AIEEE : 2005]
 (A) 22.0 (B) 20.5 (C) 25.5 (D) 24.0

Sol. : $Z = 3M - 2\bar{x}$ $Z = \text{Mode, } M = \text{Median, } \bar{x} = \text{Mean}$

$$Z = 3(22) - 2(21) = 66 - 42 = 24 \quad \text{Ans. : (D)}$$

- (15) The average deviation from the median for the following data 1, 2, 3, 4, 5, 6, 7, 8, 9 is

(A) $\frac{10}{9}$ (B) 0 (C) 5 (D) $\frac{20}{9}$

Sol. : $M = \left(\frac{n+1}{2}\right)$ th observation = $\left(\frac{9+1}{2}\right)$ th observation = 5th observation = 5

x_i	$ x_i - M $
1	4
2	3
3	2
4	1
5	0
6	1
7	2
8	3
9	4
$\Sigma x_i - M = 20$	

$$\begin{aligned} \text{Average deviation } \delta M &= \frac{\Sigma|x_i - M|}{n} \\ &= \frac{20}{9} \quad \text{Ans. : (D)} \end{aligned}$$

- (16) Suppose $x_1, x_2, x_3, \dots, x_n$ are the n observations such that $\Sigma x_i^2 = 400$ and $\Sigma x_i = 80$. A possible value of n among the following is

[AIEEE : 2005]

(A) 9 (B) 12 (C) 15 (D) 18

Sol. : $s^2 \geq 0$

$$\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2 \geq 0$$

$$\frac{400}{n} - \left(\frac{80}{n}\right)^2 \geq 0$$

$$400n - 6400 \geq 0$$

$$n \geq 16$$

$\therefore$ A possible value of n is 18.

Ans. : (D)

- (17) The standard deviation of n observations $x_1, x_2, \dots, x_n$ is 2. If $\sum_{i=1}^n x_i = 20$ and $\sum_{i=1}^n x_i^2 = 100$, then $n = \dots$

(A) 5 or 15 (B) 5 or 20 (C) 5 or 10 (D) 10 or 20

$$\text{Sol. : } s^2 = \frac{\sum_{i=1}^n x_i^2}{n} - \left(\frac{\sum_{i=1}^n x_i}{n}\right)^2$$

$$\therefore 4 = \frac{100}{n} - \left(\frac{20}{n}\right)^2$$

$$\therefore 4 = \frac{100n - 400}{n^2}$$

$$\therefore n^2 - 25n + 100 = 0$$

$$\therefore n = 20 \text{ or } n = 5$$

Ans. : (B)

(18) Variance of first n natural numbers is

- (A) $\frac{n^2 - 1}{12}$ (B) $\frac{n^2 + 1}{12}$ (C) $\frac{(n + 1)(2n + 1)}{6}$ (D) $\frac{n(n + 1)^2}{4}$

Sol. : 1, 2, 3, ..., n are the first n natural numbers.

$$\bar{x} = \frac{\sum x_i}{n} = \frac{1 + 2 + 3 + \dots + n}{n} = \frac{\frac{n}{2}(n + 1)}{n} = \frac{n + 1}{2}$$

$$\sum x_i^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n + 1)(2n + 1)}{6}$$

$$s^2 = \frac{\sum x_i^2}{n} - \bar{x}^2 = \frac{n(n + 1)(2n + 1)}{6n} - \frac{(n + 1)^2}{4} = \frac{(n + 1)[4n + 2 - 3n - 3]}{12} = \frac{n^2 - 1}{12} \quad \text{Ans. : (A)}$$

(19) The mean of 10 observations is 50. The sum of the squares of the deviations from the mean is 250. Then the coefficient of variation is

- (A) 40 % (B) 50 % (C) 10 % (D) 20 %

Sol. : $n = 10$ $\bar{x} = 50$. $\sum (x_i - \bar{x})^2 = 250$

$$\therefore s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} = \sqrt{\frac{250}{10}} = 5$$

$$\therefore \text{The coefficient of variation } V = \frac{s}{\bar{x}} \times 100$$

$$= \frac{5}{50} \times 100 = 10 \%$$

Ans. : (C)

(20) Find the average deviation from the median :

x_i	2	5	6	8	10	12
f_i	2	8	10	7	8	5

(A) 2.3

(B) 2.7

(C) 1.8

(D) 2

Sol. :

x_i	f_i	cf	$ x_i - M $	$f_i x_i - M $
2	2	2	6	12
5	8	10	3	24
6	10	20	2	20
8	7	27	0	0
10	8	35	2	16
12	5	40	4	20
	$n = 40$			$\sum f_i x_i - M = 92$

The median $M = \left(\frac{n+1}{2}\right)$ th observation $= \frac{41}{2} = 20.5$ th observation

In the cumulative frequency table,

For $cf = 27$, $x_i = 8$. So, $M = 8$

$$\delta M = \frac{\sum f_i |x_i - M|}{n} = \frac{92}{40} = 2.3$$

Ans. : (A)

- (21) A population A has 100 observations 101, 102, ..., 200 and another population B has 100 observations 151, 152, ..., 250. If V_A and V_B are the variances of those two populations respectively, then $\frac{V_A}{V_B} = \dots$ [AIEEE : 2006]

(A) 1 (B) $\frac{9}{4}$ (C) $\frac{4}{9}$ (D) $\frac{2}{3}$

Sol. : Subtract 100 from the each observation of population A and subtract 150 from the each observation of population B,

$$V_A = V_B = \text{Variance of } 1, 2, 3, \dots, 100$$

$$\therefore \frac{V_A}{V_B} = 1 \quad \text{Ans. : (A)}$$

- (22) If 5 is subtracted from each observation, then the coefficient of variation is 10 % and if 5 is added to each observation, then the coefficient of variation is 6 %, then the coefficient of variation of given observations is

(A) 4 % (B) 8 % (C) 7.5 % (D) 4.5 %

Sol. : $y = x - 5$

$$\bar{y} = \bar{x} - 5$$

$$s_y = s_x$$

$$\text{C.V.} = \frac{s}{\bar{x}} \times 100$$

$$10 = \frac{s}{\bar{x} - 5} \times 100 \quad (1)$$

$$y = x + 5$$

$$\bar{y} = \bar{x} + 5$$

$$s_y = s_x$$

$$\text{C.V.} = \frac{s}{\bar{x}} \times 100$$

$$6 = \frac{s}{\bar{x} + 5} \times 100 \quad (2)$$

Divide equation (1) by (2),

$$\frac{5}{3} = \frac{\bar{x} + 5}{\bar{x} - 5}$$

$$5\bar{x} - 25 = 3\bar{x} + 15$$

$$\therefore \bar{x} = 20$$

$$\text{From (1), } 10 = \frac{s}{15} \times 100$$

$$\therefore s = 1.5$$

$$\text{The coefficient of variation } V = \frac{s}{\bar{x}} \times 100 = \frac{1.5}{20} \times 100 = 7.5 \% \quad \text{Ans. : (C)}$$

- (23) If the standard deviation of observations $x_1, x_2, \dots, x_n$ is 4.5, then the standard deviation of the observations $-2x_1 + 5, -2x_2 + 5, \dots, -2x_n + 5$ is

(A) -4 (B) 9 (C) 14 (D) -9

Sol. : Here, observations $y_i = -2x_i + 5$.

$$s_y = |-2| s_x = 2 \times 4.5 = 9 \quad \text{Ans. : (B)}$$

- (24) Find the standard deviation of following data :

x_i	6	7	8	9	10	11	12
f_i	3	6	9	13	8	5	4

(A) 1.4 (B) 1.6 (C) 2 (D) 1.2

Sol. :

x_i	f_i	$f_i x_i$	$(x_i - \bar{x})^2$	$f_i (x_i - \bar{x})^2$
6	3	18	9	27
7	6	42	4	24
8	9	72	1	9
9	13	117	0	0
10	8	80	1	8
11	5	55	4	20
12	4	48	9	36
	$n = 48$	432		124

$$\bar{x} = \frac{\sum f_i x_i}{n} = \frac{432}{48} = 9$$

$$s = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{n}} = \sqrt{\frac{124}{48}} = \sqrt{2.58} = 1.6$$

Ans. : (B)

(25) Find the standard deviation :

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	5	8	15	16	6

(A) 188.66

(B) 177.33

(C) 11.49

(D) 78.00

Sol. :

Class	Frequency f_i	Central Value, x_i	$d_i = \frac{x_i - A}{c}$	$f_i d_i$	$f_i d_i^2$
0 – 10	5	5	-2	-10	20
10 – 20	8	15	-1	-8	8
20 – 30	15	$\boxed{25} = A$	0	0	0
30 – 40	16	35	1	16	16
40 – 50	6	45	2	12	24
	$\sum f_i = 50$			$\sum f_i d_i = 10$	$\sum f_i d_i^2 = 68$

Class length $c = 10$. Assumed mean $A = 25$

$$\bar{x} = A + \frac{\sum f_i d_i}{n} \times c = 25 + \frac{10}{50} \times 10 = 27$$

$$s = \sqrt{\frac{\sum f_i d_i^2}{n} - \left(\frac{\sum f_i d_i}{n}\right)^2} \times c$$

$$= \sqrt{\frac{68}{50} - \left(\frac{10}{50}\right)^2} \times 10$$

$$= \sqrt{1.36 - 0.04} \times 10$$

$$= \sqrt{1.32} \times 10 = 1.149 \times 10 = 11.49$$

Ans. : (C)

- (26) The average of marks of boys in a class is 52 and that of girls is 42. The average of marks of boys and girls together is 50. The percentage of boys in the class is [AIEEE : 2007]
 (A) 60 (B) 80 (C) 40 (D) 20

Sol. : n_1 and n_2 are the number of boys and girls respectively and $\bar{x}_1$ and $\bar{x}_2$ are their mean respectively.

$$\begin{aligned}\bar{x}_1 &= 52 & \bar{x}_2 &= 42 & \bar{x} &= 50 \\ \therefore \bar{x} &= \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} \\ \therefore 50 &= \frac{52n_1 + 42n_2}{n_1 + n_2} \\ \therefore 50n_1 + 50n_2 &= 52n_1 + 42n_2 \\ \therefore 8n_2 &= 2n_1 \\ \therefore \frac{n_1}{n_2} &= \frac{4}{1} \\ \therefore \frac{n_1}{n_1 + n_2} &= \frac{4}{4+1} = \frac{4}{5}\end{aligned}$$

$$\text{Percentage of boys} = \frac{n_1}{n_1 + n_2} \times 100 = \frac{4}{5} \times 100 = 80$$

Ans. : (B)

Second method :

$$\text{Let } n_1 + n_2 = 100$$

$$\text{Now, } 50 = \frac{52n_1 + 42n_2}{n_1 + n_2}$$

$$\therefore 5000 = 52n_1 + 42(100 - n_1)$$

$$\therefore 800 = 10n_1$$

$$\therefore n_1 = 80$$

$$\therefore 80 \% \text{ boys} \quad \text{Ans. : (B)}$$

- (27) 6 is the mean of the observations $a, b, 8, 5$ and 10 and 6.8 is the variance. The possible values of a and b are [AIEEE : 2008]

$$(A) a = 3, b = 4 \quad (B) a = 0, b = 7 \quad (C) a = 5, b = 2 \quad (D) a = 1, b = 6$$

Sol. : $\bar{x} = 6$

$$\therefore \frac{a + b + 8 + 5 + 10}{5} = 6$$

$$a + b + 23 = 30. \text{ So, } a + b = 7$$

$$\therefore b = 7 - a \quad (1)$$

$$\begin{aligned}\sum x_i^2 &= a^2 + b^2 + 64 + 25 + 100 \\ &= 189 + a^2 + (7 - a)^2 \\ &= 189 + a^2 + 49 - 14a + a^2 \\ &= 2a^2 - 14a + 238\end{aligned} \quad (\text{By (1)})$$

$$\text{Now, } s^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$$

$$6.8 + 36 = \frac{2a^2 - 14a + 238}{5}$$

$$\therefore 214 = 2a^2 - 14a + 238$$

$$\therefore 2a^2 - 14a + 24 = 0$$

$$\therefore a^2 - 7a + 12 = 0$$

$$\therefore a = 4 \text{ or } a = 3 \Rightarrow b = 3 \text{ or } b = 4$$

$$\therefore a = 3, b = 4$$

Ans. : (A)

- (28) 50 and 5 are the mean and the standard deviation of 100 observations respectively. The sum of the squares of all observations is

$$(A) 50,000 \quad (B) 2,50,000 \quad (C) 2,52,500 \quad (D) 2,55,000$$

Sol. : $\sum x_i^2 = n(s^2 + \bar{x}^2) = 100 [25 + 2500] = 100 [2525] = 2,52,500$ **Ans. : (C)**

- (29) The mean and the variance of the 8 observations are 9 and 9.25 respectively. If six observations are 6, 7, 10, 12, 12 and 13, then the remaining observations are and
 (A) 4, 6 (B) 3, 8 (C) 4, 7 (D) 4, 8

Sol. : Suppose a and b are required observations.

$$\sum x_i = n\bar{x}$$

$$\therefore a + b + 6 + 7 + 10 + 12 + 12 + 13 = 8 \times 9$$

$$a + b = 12 \quad (1)$$

$$\sum x_i^2 = n(s^2 + \bar{x}^2)$$

$$\therefore a^2 + b^2 + 36 + 49 + 100 + 144 + 144 + 169 = 8(81 + 9.25) = 8(90.25)$$

$$a^2 + b^2 + 642 = 722$$

$$a^2 + b^2 = 80 \quad (2)$$

$$a^2 + (12 - a)^2 = 80 \quad (\text{From (1)})$$

$$2a^2 - 24a + 64 = 0$$

$$a^2 - 12a + 32 = 0$$

$$(a - 8)(a - 4) = 0$$

$$a = 8 \text{ or } a = 4. \text{ So, } b = 4 \text{ or } b = 8$$

$\therefore$ Required remaining observations are 4 and 8.

Ans. : (D)

- (30) If the mean deviation of numbers 1, $1 + d$, $1 + 2d$, ..., $1 + 100d$ from their mean is 255, then d is equal to [AIEEE : 2009]

- (A) 20.0 (B) 10.1 (C) 20.2 (D) 10.0

Sol. : $T_n = a + (n - 1)d \Rightarrow 1 + 100d = 1 + (n - 1)d$

$$\therefore n - 1 = 100 \Rightarrow n = 101$$

$$\bar{x} = \frac{1 + (1 + d) + \dots + (1 + 100d)}{101}$$

$$= \frac{(1 + 1 + \dots + 1 \text{ 101 times}) + d[1 + 2 + \dots + 100]}{101} = 1 + \frac{(100)(101)}{2(101)}d$$

$$\bar{x} = 1 + 50d$$

$$\text{The mean deviation} = \frac{\sum |x_i - \bar{x}|}{n}$$

$$= \frac{1}{101} \left[\sum_{i=1}^{101} |x_i - 1 - 50d| \right]$$

$$= \frac{1}{101} [50d + 49d + \dots + d + 0 + d + 2d + \dots + 50d]$$

$$= \frac{2d}{101} [1 + 2 + 3 + 4 + \dots + 50] = \frac{2d}{101} \left[\frac{50}{2} \times 51 \right]$$

But given mean deviation = 255

$$255 = \frac{50 \times 51d}{101}$$

$$\therefore d = \frac{255 \times 101}{50 \times 51} = 10.1$$

Ans. : (B)

- (31) $x_1, x_2, \dots, x_n$ are n observations such that $\sum x_i^2 = 200$ and $\sum x_i = 60$. The possible value of n is

(A) 16 (B) 19 (C) 12 (D) 10

Sol. : $\frac{\sum x_i^2}{n} \geq \left(\frac{\sum x_i}{n}\right)^2$

$$\therefore \frac{200}{n} \geq \left(\frac{60}{n}\right)^2$$

$$\therefore 200n \geq 3600$$

$$\therefore n \geq 18 \quad \text{So, } n = 19$$

Ans. : (B)

- (32) The mean of 10 observations is 30. The sum of squares of deviations from the mean is 90. Then the C.V. of the data is

(A) 20 % (B) 10 % (C) 11 % (D) 12 %

Sol. : $s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} = \sqrt{\frac{90}{10}} = 3$

$$\text{C.V.} = \frac{s}{\bar{x}} \times 100 = \frac{3}{30} \times 100 = 10 \%$$

Ans. : (B)

- (33) If x and y are in the relation $4x - 3y = 10$, average deviation of x is 10, then the average deviation of y is

(A) 13 (B) 12.3 (C) 13.3 (D) 13.5

Sol. : $y = \frac{4}{3}x - \frac{10}{3}$

$$\delta \bar{y} = |k| \delta \bar{x}$$

$$\delta \bar{y} = \frac{4}{3}(10) = \frac{40}{3} = 13.3$$

Ans. : (C)

- (34) If the mean of the following frequency distribution is 2.6, then $y = \dots$

x_i	1	2	3	4	5
f_i	4	5	y	1	2

(A) 24 (B) 13 (C) 8 (D) 3

Sol. :

x_i	f_i	$f_i x_i$
1	4	4
2	5	10
3	y	$3y$
4	1	4
5	2	10
	$\sum f_i = 12 + y$	$\sum f_i x_i = 28 + 3y$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\therefore 2.6 = \frac{28 + 3y}{12 + y}$$

$$\therefore \frac{26}{10}(12 + y) = 28 + 3y$$

$$\therefore 156 + 13y = 140 + 15y$$

$$\therefore 16 = 2y$$

$$\therefore y = 8$$

Ans. : (C)

- (35) Out of $2n$ observations, half of the observations have value b and half of the observations have value $-b$. If the variance is 3, then $|b| = \dots$

(A) 3 (B) $\sqrt{3}$ (C) $\frac{\sqrt{3}}{n}$ (D) $\frac{1}{n}$

Sol. : $\bar{x} = \frac{\sum x_i}{n} = \frac{0}{n} = 0$

$$\sum x_i^2 = b^2 + \dots + b^2(n \text{ times}) + (-b)^2 + (-b)^2 + \dots + (-b)^2(n \text{ times}) = 2nb^2$$

$$s^2 = \frac{\sum x_i^2}{n} - \bar{x}^2 = \frac{2nb^2}{2n} - 0 = b^2 = 3$$

$$\therefore |b| = \sqrt{3}$$

Ans. : (B)

- (36) If first n natural number have the frequency ${}_nC_1, {}nC_2, {}nC_3, \dots, {}nC_n$ respectively, then the mean $\bar{x} = \dots$

(A) $\frac{n \cdot 2^{n-1}}{2^n - 1}$ (B) $\frac{3n(n+1)}{2(2n+1)}$ (C) $\frac{n \cdot 2^n}{n+1}$ (D) $\frac{(n+1)(2n+1)}{6}$

Sol. : $\bar{x} = \frac{n \cdot {}nC_1 + 2 \cdot {}nC_2 + 3 \cdot {}nC_3 + \dots + n \cdot {}nC_n}{{}_nC_1 + {}nC_2 + {}nC_3 + \dots + {}nC_n} \quad \left(\bar{x} = \frac{\sum f_i x_i}{n} \right)$

$$= \frac{n + n(n-1){}_nC_1 + n(n-1){}_nC_2 + \dots + n(n-1){}_nC_{n-1}}{{}_nC_0 + {}nC_1 + {}nC_2 + \dots + {}nC_n - {}nC_0}$$

$$= \frac{n[{}_nC_0 + {}nC_1 + {}nC_2 + \dots + {}nC_{n-1}]}{{}_nC_0 + {}nC_1 + \dots + {}nC_{n-1}}$$

$$\bar{x} = \frac{n \cdot 2^{n-1}}{2^n - 1}$$

Ans. : (A)

$$\left[2 \cdot {}nC_2 = \frac{2 \cdot n!}{(n-2)! 2!} = \frac{n(n-1)!}{(n-2)!} = n({}_{n-1}C_1) \text{ . Similarly, } 3 \cdot {}nC_3 = n({}_{n-1}C_2), 4({}_nC_4) = n({}_{n-1}C_3) \right]$$

and ${}_nC_0 + {}nC_1 + {}nC_2 + {}nC_3 + \dots + {}nC_n = 2^n$

- (37) If for the observations $x_1, x_2, \dots, x_n$, $\sum_{i=1}^n (x_i + 4) = 100$ and $\sum_{i=1}^n (x_i + 6) = 140$, then $n = \dots$ and

$$\bar{x} = \dots$$

(A) 20, 2 (B) 20, 1 (C) 22, 1 (D) 23, 1

Sol. : $\sum_{i=1}^n (x_i + 6) - \sum_{i=1}^n (x_i + 4) = 140 - 100$

$$\therefore 6n - 4n = 40$$

$$\therefore 2n = 40. \text{ So, } n = 20$$

Now, $\sum_{i=1}^{20} (x_i + 4) = 100$

$$\sum_{i=1}^{20} x_i + 80 = 100 \quad (4n = 80)$$

$$\sum_{i=1}^{20} x_i = 20$$

$$\therefore \bar{x} = \frac{\sum_{i=1}^{20} x_i}{20} = \frac{20}{20} = 1 \quad \text{Ans. : (B)}$$

(38) For the observations $x_1, x_2, \dots, x_n$, $\sum_{i=1}^n (x_i + 1)^2 = 9n$ and $\sum_{i=1}^n (x_i - 1)^2 = 5n$. The standard deviation of given data is

- (A) $\sqrt{10}$ (B) $\sqrt{5}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$

$$\text{Sol. : } \sum_{i=1}^n (x_i + 1)^2 = 9n \Rightarrow \sum_{i=1}^n x_i^2 + 2\sum_{i=1}^n x_i = 8n \quad (1)$$

$$\sum_{i=1}^n (x_i - 1)^2 = 5n \Rightarrow \sum_{i=1}^n x_i^2 - 2\sum_{i=1}^n x_i = 4n \quad (2)$$

$$\text{Add (1) and (2), } \sum_{i=1}^n x_i^2 = 6n$$

$$\text{Subtract (2) from (1), } \sum_{i=1}^n x_i = n$$

$$s = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2} = \sqrt{\frac{6n}{n} - \left(\frac{n}{n}\right)^2} = \sqrt{6-1} = \sqrt{5} \quad \text{Ans. : (B)}$$

(39) The standard deviation of observations 31, 32, 33, ..., 47 is

- (A) $4\sqrt{3}$ (B) $2\sqrt{6}$ (C) $\sqrt{\frac{47^2 - 1}{12}}$ (D) $\sqrt{\frac{17}{12}}$

$$\text{Sol. : Standard deviation of first } n \text{ natural numbers } 1, 2, 3, \dots, n = \sqrt{\frac{n^2 - 1}{12}}$$

If $y_i = x_i + 30$, we get observations 31, 32, 33, ..., 47.

The standard deviation of 31, 32, 33, ..., 47 = The standard deviation of 1, 2, 3, 4, ..., 17

$$s = \sqrt{\frac{17^2 - 1}{12}} = \sqrt{\frac{289 - 1}{12}} = \sqrt{24} = 2\sqrt{6} \quad \text{Ans. : (B)}$$

(40) Following data are obtained by Vaishali and Madhavi from results obtained in annual examination by four students :

Name of the students	Mean $\bar{x}$	Standard Deviation
Vimal	75	11.25
Hiren	65	5.98
Gaurang	48	8.88
Vijay	44	5.28

Who is the most and the least consistent ?

- (A) Vimal, Hiren (B) Hiren, Gaurang (C) Gaurang, Hiren (D) Hiren, Vijay

$$\text{Sol. : } C.V_{\text{Vimal}} = \frac{s}{\bar{x}} \times 100 = \frac{11.25}{75} \times 100 = 15$$

$$C.V_{\text{Hiren}} = \frac{s}{\bar{x}} \times 100 = \frac{5.98}{65} \times 100 = 9.2$$

$$C.V_{\text{Gaurang}} = \frac{s}{\bar{x}} \times 100 = \frac{8.88}{48} \times 100 = 18.5$$

$$C.V_{\text{Vijay}} = \frac{s}{\bar{x}} \times 100 = \frac{5.28}{44} \times 100 = 12$$

Gaurang and Hiren have the largest and the smallest C.V. respectively.

∴ So, Hiren is the most and Gaurang is the least consistent.

Ans. : (B)

- (41) If the mean of 31 observations is 60, the mean of first 16 observations is 58 and the mean of last 16 observations is 62, then the 16th observation is

(A) 50 (B) 60 (C) 70 (D) 80

Sol. : Let $x_1, x_2, \dots, x_{31}$ be the observations,

$$x_1 + x_2 + \dots + x_{15} + x_{16} = 16 \times 58 = 928 \quad (1)$$

$$x_{16} + x_{17} + \dots + x_{31} = 16 \times 62 = 992 \quad (2)$$

Add (1) and (2),

$$x_1 + x_2 + \dots + x_{31} + x_{16} = 1920$$

$$31 \times 60 + x_{16} = 1920$$

$$1860 + x_{16} = 1920$$

$$\therefore x_{16} = 60$$

Ans. : (B)

- (42) The mean of 23 observations is 34. The mean of first 12 observations is 32 and the mean of last 12 observations is 38. The 12th observation is

(A) 58 (B) 59 (C) 57 (D) 60

Sol. : $n = 23, \bar{x} = 34, n_1 = 12, \bar{x}_1 = 32, n_2 = 12, \bar{x}_2 = 38$

$$\therefore \text{12th observation} = n_1 \bar{x}_1 + n_2 \bar{x}_2 - n \bar{x}$$

$$= 12 \times 32 + 12 \times 38 - 23 \times 34 = 384 + 456 - 782 = 58$$

Ans. : (A)

- (43) If the sum of the deviations of 20 observations taken from 30 is 20, then the mean is

(A) 30 (B) 30.1 (C) 29 (D) 31

$$\text{Sol. : } \sum_{i=1}^{20} (x_i - 30) = 20$$

$$\therefore \sum_{i=1}^{20} x_i - \sum_{i=1}^{20} 30 = 20$$

$$\therefore 20\bar{x} - 20 \times 30 = 20$$

$$\therefore 20\bar{x} = 620$$

$$\therefore \bar{x} = 31$$

Ans. : (D)

- (44) In the frequency distribution of data of 40 students, the class length is 10. The mean and the variance are 40 and 49 respectively. At the time of proof reading observation 2 of class 21-30 was placed incorrectly in the class 31-40. The new mean and the variance after correction are and respectively .

(A) 39.5, 49.0 (B) 39.5, 58.25 (C) 40, 50 (D) 39, 49

Sol. : $n = 40, c = 10, \bar{x} = 40, s^2 = 49$

Observation 2 of the class 21 – 30 is placed incorrectly in class 31 – 40.

$$\begin{aligned}\text{Corrected } \sum f_i x_i &= n\bar{x} - \text{Incorrect } f_i x_i + \text{Correct } f_i x_i \\ &= 40 \times 40 - 2 \times 35.5 + 2 \times 25.5 \\ &= 1600 - 71 + 51 = 1580\end{aligned}$$

$$\therefore \text{Correct } \bar{x} = \frac{\text{Correct } \sum f_i x_i}{n} = \frac{1580}{40} = 39.5$$

$$\begin{aligned}\text{Correct } \sum f_i x_i^2 &= n(s^2 + \bar{x}^2) - \text{Incorrect } f_i x_i^2 + \text{Correct } f_i x_i^2 \\ &= 40(49 + 1600) - 2(35.5)^2 + 2(25.5)^2 \\ &= 65960 - 2520.5 + 1300.5 = 64740\end{aligned}$$

$$\begin{aligned}s^2 &= \frac{\text{Correct } \sum f_i x_i^2}{n} - (\text{Correct } \bar{x})^2 \\ &= \frac{64740}{40} - (39.5)^2 = 1618.5 - 1560.25 = 58.25\end{aligned}$$

$$\text{Correct } \bar{x} = 39.5, \text{ Correct variance } s^2 = 58.25$$

Ans. : (B)

- (45) If $\sum_{i=1}^{18} (x_i - 8) = 9$ and $\sum_{i=1}^{18} (x_i - 8)^2 = 45$, then standard deviation of $x_1, x_2, x_3, \dots, x_{18}$ is

(A) $\frac{4}{9}$ (B) $\frac{9}{4}$ (C) $\frac{3}{2}$ (D) $\frac{2}{3}$

Sol. : Suppose $d_i = x_i - 8$

$$\sum_{i=1}^{18} (x_i - 8) = 9 \quad \text{and} \quad \sum_{i=1}^{18} (x_i - 8)^2 = 45$$

$$\therefore \sum_{i=1}^{18} d_i = 9 \quad \text{and} \quad \sum_{i=1}^{18} d_i^2 = 45, \quad n = 18$$

By subtracting constant, standard deviation remains same.

$$\therefore s_{d_i} = s_{x_i} = s$$

$$s = \sqrt{\frac{\sum_{i=1}^{18} d_i^2}{n} - \left(\frac{\sum_{i=1}^{18} d_i}{n} \right)^2} = \sqrt{\frac{45}{18} - \left(\frac{9}{18} \right)^2} = \sqrt{\frac{5}{2} - \frac{1}{4}} = \sqrt{\frac{10-1}{4}} = \frac{3}{2} \quad \text{Ans. : (C)}$$

- (46) The mean and the variance of n observations $x_1, x_2, \dots, x_n$ are 5 and 0 respectively. If $\sum x_i^2 = 400$, then $n = \dots$

(A) 80 (B) 25 (C) 20 (D) 16

Sol. : Here, $\frac{\sum x_i}{n} = 5$, $s^2 = 0$

$$\text{Now, } s^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

$$\therefore 0 = \frac{400}{n} - 25$$

$$\therefore 25 = \frac{400}{n}. \text{ So, } n = \frac{400}{25} = 16$$

Ans. : (D)

(47) If $\sum_{i=1}^9 x_i = 36$ and $\sum_{i=1}^9 x_i^2 = 180$, then the coefficient of variation is

(A) 4 %

(B) 50 %

(C) 100 %

(D) 10 %

$$\text{Sol. : } \bar{x} = \frac{\sum_{i=1}^9 x_i}{9} = \frac{36}{9} = 4$$

$$s = \sqrt{\frac{\sum x_i^2}{n} - \bar{x}^2} = \sqrt{\frac{180}{9} - 16} = 2$$

$$\text{The coefficient of variation} = \frac{s}{\bar{x}} \times 100 = \frac{2}{4} \times 100 = 50 \%$$

Ans. : (B)

(48) If $\sum_{i=1}^n (x_i + 5) = 120$ and $\sum_{i=1}^n (x_i + 7) = 160$ for n observations $x_1, x_2, x_3, \dots, x_n$, then n and $\bar{x}$ are and respectively.

(A) 20, 1

(B) 20, 3

(C) 3, 17

(D) 1, 20

$$\text{Sol. : } \sum_{i=1}^n (x_i + 5) = 120$$

$$\therefore \sum_{i=1}^n x_i + 5n = 120 \quad (1)$$

$$\therefore \sum_{i=1}^n (x_i + 7) = \sum_{i=1}^n x_i + 7n = 160 \quad (2)$$

Subtract (1) from (2),

$$2n = 40. \quad \text{So, } n = 20$$

$$\text{From equation (1) and } n = 20, \sum_{i=1}^{20} x_i + 100 = 120$$

$$\therefore \sum_{i=1}^{20} x_i = 20. \quad \text{So, } \bar{x} = \frac{\sum x_i}{n} = \frac{20}{20} = 1$$

Ans. : (A)

- (49) If the sum of the deviations of n observations taken from 40 is 30 and the sum of deviations taken from 45 is -20 , then $n = \dots$ and mean = $\dots$
- (A) 10, 33 (B) 10, 43 (C) 9, 33 (D) 10, 42

Sol. : $\sum_{i=1}^n (x_i - 40) = 30$

$$\therefore \sum_{i=1}^n x_i - 40n = 30 \quad (1)$$

$$\therefore \sum_{i=1}^n (x_i - 45) = -20$$

$$\therefore \sum_{i=1}^n x_i - 45n = -20 \quad (2)$$

Subtract (2) from (1),

$$5n = 50. \text{ So, } n = 10.$$

From result (1) and $n = 10$, $\sum_{i=1}^{10} x_i - 400 = 30$

$$\therefore \sum_{i=1}^{10} x_i = 430$$

$$\therefore \bar{x} = \frac{\sum_{i=1}^{10} x_i}{10} = \frac{430}{10} = 43$$

Ans. : (B)

- (50) For two sets of data, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4 respectively. The variance of the combined data set is $\dots$. [AIEEE : 2010]

(A) $\frac{11}{2}$ (B) 6 (C) $\frac{13}{2}$ (D) $\frac{5}{2}$

Sol. : $\sum x_i^2 = n_1(s_1^2 + \bar{x}_1^2) \quad \sum y_i^2 = n_2(s_2^2 + \bar{x}_2^2)$

Here, $s_1^2 = 4$, $\bar{x}_1 = 2$, $s_2^2 = 5$, $\bar{x}_2 = 4$, $n = 5$

$$\begin{array}{l|l} \sum x_i^2 = 5(4 + 4), & \sum y_i^2 = 5(5 + 16) \\ \therefore \sum x_i^2 = 40, & \sum y_i^2 = 105 \end{array}$$

$$\therefore \sum (x_i^2 + y_i^2) = 40 + 105 = 145 \quad (1)$$

$$\sum x_i = n_1\bar{x}_1 = 10, \quad \sum y_i = n_2\bar{x}_2 = 20, \quad \sum (x_i + y_i) = 30$$

$$s^2 = \frac{\sum (x_i^2 + y_i^2)}{n} - \left[\frac{\sum (x_i + y_i)}{n} \right]^2 \quad n = n_1 + n_2 = 10$$

$$= \frac{145}{10} - \left(\frac{30}{10} \right)^2 = 14.5 - 9 = 5.5 = \frac{11}{2}$$

Ans. : (A)

(51) Find the average deviation from the median for the following data :

37, 70, 48, 50, 32, 56, 63, 46, 54, 44

(A) 7.6

(B) 49

(C) 8.6

(D) 9.6

Sol. : Here the total number of observations is 10. Arrange them in the ascending order as in the table :

x_i (Increasing order)	$ x_i - M $
32	17
37	12
44	5
46	3
48	1
50	1
54	5
56	7
63	14
70	21
	$\sum x_i - M = 86$

Median,

$$M = \frac{\text{5th observation} + \text{6th observation}}{2}.$$

$$\text{So, } n = 10, M = \frac{48 + 50}{2} = 49$$

$$\delta M = \frac{\sum |x_i - M|}{n} = \frac{86}{10} = 8.6 \quad \text{Ans. : (C)}$$

(52) If the mean of $\log x, \log 3, \log 27, \log 3x, \log 3, \log x$ is $\log 9$, then the value of x is

(A) 3

(B) 27

(C) 9

(D) 81

Sol. : Mean $\bar{x} = \log 9 = \frac{\log x + \log 3 + \log 27 + \log 3x + \log 3 + \log x}{6}$

$$\therefore 6\log 9 = \log[x \times 3 \times 27 \times 3x \times 3 \times x]$$

$$\therefore \log 9^6 = \log(3^6 x^3)$$

$$\therefore 9^6 = 3^6 x^3$$

$$\therefore 3^{12} = 3^6 x^3$$

$$\therefore 3^6 = x^3$$

$$\therefore x = 3^2 = 9$$

Ans. : (C)

(53) The mean of $\cos \frac{\pi}{8}, \cos \frac{3\pi}{8}, \cos \frac{5\pi}{8}, \cos \frac{7\pi}{8}, \cos \frac{9\pi}{8}, \cos \frac{15\pi}{8}$ is

(A) -1

(B) 0

(C) $\sqrt{2}$

(D) $\frac{1}{\sqrt{2}}$

Sol. : The mean $\bar{x} = \frac{\cos \frac{\pi}{8} + \cos \frac{3\pi}{8} + \cos \frac{5\pi}{8} + \cos \frac{7\pi}{8} + \cos \frac{9\pi}{8} + \cos \frac{15\pi}{8}}{6}$

$$= \frac{\cos \frac{\pi}{8} + \cos\left(\pi - \frac{\pi}{8}\right) + \cos \frac{3\pi}{8} + \cos\left(\pi - \frac{3\pi}{8}\right) + \cos\left(\pi + \frac{\pi}{8}\right) + \cos\left(2\pi - \frac{\pi}{8}\right)}{6}$$

$$= \frac{\cos \frac{\pi}{8} - \cos \frac{\pi}{8} + \cos \frac{3\pi}{8} - \cos \frac{3\pi}{8} - \cos \frac{\pi}{8} + \cos \frac{\pi}{8}}{6} = \frac{0}{6} = 0 \quad \text{Ans. : (B)}$$

(54) If the mean of x and $\frac{1}{x}$ is 2, then mean of x^3 and $\frac{1}{x^3}$ is

(A) 35

(B) 8

(C) 70

(D) 26

Sol. : $\frac{x + \frac{1}{x}}{2} = 2$

$\therefore x + \frac{1}{x} = 4$ (1)

Now, $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) = 4^3 - 3(4) = 64 - 12 = 52$ (By (1))

The mean of x^3 and $\frac{1}{x^3} = \frac{x^3 + \frac{1}{x^3}}{2} = \frac{52}{2} = 26$ **Ans. : (D)**

(55) If for n observations $x_1, x_2, x_3, \dots, x_n$, $\sum_{i=1}^n (x_i + 3) = 120$ and $\sum_{i=1}^n (x_i + 5) = 160$, then $n = \dots$ and $\bar{x} = \dots$

(A) 20, 5 (B) 20, 3 (C) 3, 17 (D) 3, 20

Sol. : $\sum_{i=1}^n (x_i + 3) = 120$

$\therefore n\bar{x} + 3n = 120$ (1)

$n\bar{x} + 5n = 160$ (2)

Subtract (1) from (2), $2n = 40$. So, $n = 20$

Substitute $n = 20$ in equation (2),

$20\bar{x} + 100 = 160$

$\therefore \bar{x} = 3$ **Ans. : (B)**

(56) If the mean deviation about the median of the numbers $a, 2a, 3a, \dots, 50a$ is 50, then $a = \dots$ [AIEEE : 2011]

(A) 2 (B) 5 (C) 4 (D) 3

Sol. : Here the total number of observations is 50.

$\therefore$ The median $= \frac{25\text{th observation} + 26\text{th observation}}{2} = \frac{25a + 26a}{2} = (25.5)a$

The average deviation about median, $\delta M = \frac{\sum_{i=1}^{50} |x_i - M|}{50}$

$50 = \frac{1}{50} [(24.5)a + (23.5)a + (0.5)a + (0.5)a + \dots + (23.5)a + (24.5)a]$

$= \frac{1}{50} [2(0.5)a + 2(1.5)a + 2(2.5)a + 2(24.5)a]$

$= \frac{a}{25} [0.5 + 1.5 + \dots + 24.5]$

$= \frac{a}{25} \times \frac{25}{2} [0.5 + 24.5]$

$\therefore a = 4$ **Ans. : (C)**

- (57) The mean and the variance of n observations $x_1, x_2, \dots, x_n$ are 10 and 0 respectively.
If $\sum x_i^2 = 1600$, then $n = \dots$
- (A) 16 (B) 20 (C) 25 (D) 80

Sol. : Here $\frac{\sum x_i}{n} = \bar{x} = 10$ and $s^2 = 0$

$$s^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n} \right)^2$$

$$0 = \frac{1600}{n} - 100$$

$$\therefore n = 16$$

Ans. : (A)

- (58) If a discrete variable takes the value $x + 4, x - \frac{7}{2}, x - \frac{5}{2}, x - 3, x - 2, x + \frac{1}{2}, x - \frac{1}{2}, x + 5$ ($x > 0$), then the median is
- (A) $x + \frac{5}{4}$ (B) $x - 2$ (C) $x - \frac{1}{2}$ (D) $x - \frac{5}{4}$

Sol. : Arrange observations in ascending order.

$x - \frac{7}{2}, x - 3, x - \frac{5}{2}, x - 2, x - \frac{1}{2}, x + \frac{1}{2}, x + 4, x + 5$ is the order.

Here, $n = 8$.

$$\therefore \text{The median} = \frac{4\text{th observation} + 5\text{th observation}}{2}$$

$$= \frac{x - 2 + x - \frac{1}{2}}{2}$$

$$\therefore \text{Median} = x - \frac{5}{4}$$

Ans. : (D)

- (59) The mean of $a, a + d, a + 2d, \dots, a + 2nd$ is
- (A) $a + (2n - 1)d$ (B) $a + (n + 1)d$ (C) $a + nd$ (D) $a + (n - 1)d$

Sol. : Here number of terms $= 2n + 1$

$$\sum x_i = a + (a + d) + \dots + a + 2nd$$

$$= \frac{2n+1}{2} [2a + (2n + 1 - 1)d]$$

$$= (2n + 1) [a + nd]$$

$$\therefore \bar{x} = \frac{\sum x_i}{2n+1} = \frac{(2n+1)(a+nd)}{(2n+1)}$$

$$\therefore \bar{x} = a + nd$$

Ans. : (C)

- (60) If the mean of the numbers $27 + x, 31 + x, 89 + x, 107 + x$ and $156 + x$ is 82, then the mean of $130 + x, 126 + x, 68 + x, 50 + x$ and $1 + x$ is
- (A) 75 (B) 80 (C) 157 (D) 79

Sol. : $\bar{x} = \frac{27 + x + 31 + x + 89 + x + 107 + x + 156 + x}{5} = 82$

$$\therefore 410 = 410 + 5x$$

$$\therefore x = 0$$

$$\text{Now, } \bar{x} = \frac{130 + 126 + 68 + 50 + 1}{5} = \frac{375}{5} = 75$$

Ans. : (A)

- (61) 38 is the mean of 50 numbers. If two numbers 55 and 45 are incorrectly introduced, then the new mean after removing these numbers is

(A) 38.5 (B) 37.5 (C) 36.5 (D) 38

Sol. : $\sum x_i = n\bar{x} = 50 \times 38 = 1900$

Correct $\sum x_i = 1900 - (55 + 45) = 1900 - 100 = 1800$

Correct $\bar{x} = \frac{\text{Correct } \sum x_i}{48} = \frac{1800}{48} = 37.5$ **Ans. : (B)**

- (62) The mean of the weight of 35 students is 40 kg. If the mean of weight of students increases by $\frac{1}{2}$ kg after adding teacher's weight. The weight of teacher is kg.

(A) 40.5 (B) 50 (C) 41 (D) 58

Sol. : Now, $\sum x_i = n\bar{x} = 35 \times 40 = 1400$

Suppose the weight of the teacher is w kg

$\therefore$ The new mean $= \bar{x} + \frac{1}{2} = \frac{\sum x_i + w}{36}$

$\therefore 40 + \frac{1}{2} = \frac{1400 + w}{36}$

$18(80 + 1) = 1400 + w$

$1458 - 1400 = w$

$\therefore$ The weight of the teacher $w = 58$ kg **Ans. : (D)**

- (63) The population A has 100 observations 151, 152, ..., 250 and another population B has 100 observations 101, 102, ..., 200. If V_A and V_B are variances of these two populations respectively, then $3V_A : 4V_B = \dots$

(A) 1 : 1 (B) 4 : 3 (C) 3 : 4 (D) 1 : 4

Sol. : If we subtract 50 from the observations of population A, we get the observations of population B.

$\therefore V_A = V_B = m$

$\frac{3V_A}{4V_B} = \frac{3m}{4m} = \frac{3}{4}$ **Ans. : (C)**

- (64) If the mean of marks of 10 students of a school, is $\bar{x} = 70$ and $\sum (x_i - \bar{x})^2 = 78$, then the standard deviation =

(A) 2.97 (B) 3.97 (C) 2.91 (D) 2.79

Sol. : Here, $n = 10$ and $\sum (x_i - \bar{x})^2 = 78$

Standard deviation $= \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} = \sqrt{\frac{78}{10}} = \sqrt{7.8} = 2.79$ **Ans. : (D)**

- (65) The variance of the observations 3, 4, 5 and 8 is

(A) 4.5 (B) 6.5 (C) 3.5 (D) 3.00

Sol. : $\bar{x} = \frac{\sum x_i}{n} = \frac{3+4+5+8}{4} = 5$

$\sum x_i^2 = 3^2 + 4^2 + 5^2 + 8^2 = 9 + 16 + 25 + 64 = 114$

$\therefore s^2 = \frac{\sum x_i^2}{n} - \bar{x}^2 = \frac{114}{4} - 25 = \frac{14}{4} = 3.5$ **Ans. : (C)**

(66) If the standard deviation of two observations is 3.5 and one of the observations is 3, then the other observation is

- (A) 9 (B) 10 (C) 7 (D) 3

Sol. : Suppose remaining observation is a ,

$$\bar{x} = \frac{a+3}{2}$$

$$s^2 = \frac{\sum x_i^2}{2} - \bar{x}^2$$

$$\therefore (3.5)^2 = \frac{a^2+9}{2} - \frac{(a+3)^2}{4}$$

$$\therefore 49 = 2a^2 + 18 - a^2 - 6a - 9$$

$$\therefore 49 = a^2 - 6a + 9 = (a-3)^2$$

$a-3 = 7$ or $a-3 = -7$. But $a = -4$ is not given in options.

$$\therefore a = 10$$

Ans. : (B)

Another method :

$$s^2 = \frac{(x_1 - x_2)^2}{4}$$

$$\therefore 4 \times (3.5)^2 = (a-3)^2$$

$$\therefore (a-3)^2 = 49$$

$$\therefore a-3 = \pm 7$$

$$\therefore a-3 = 7 \Rightarrow a = 10$$

Ans. : (B)

(67) The variance of the observations 2, 4, 6, ..., 100 is

[JEE : 2014]

- (A) $\frac{833}{4}$ (B) 833 (C) 437 (D) $\frac{437}{4}$

Sol. : 2, 4, 6, ..., 100 are given observations. $n = 50$

$$\bar{x} = \frac{2+4+6+\dots+100}{50} = 51$$

$$s^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$$

$$= \frac{2^2+4^2+6^2+\dots+100^2}{50} - (51)^2$$

$$= \frac{\sum_{i=1}^{50} 4i^2}{50} - 2601$$

$$= \frac{4 \times 50 \times 51 \times 101}{6 \times 50} - 2601 = 3434 - 2601 = 833$$

Ans. : (B)

Another method :

$$s^2 = \left(\frac{n^2-1}{12} \right) d^2 = \left[\frac{2500-1}{12} \right] \times 4 = \frac{2499}{3} = 833$$

Ans. : (B)

- (68) Find the mean and the variance of the following data :

x_i	4	8	12	16	20	24
f_i	6	14	4	8	11	7

(A) 16, 44.96

(B) 16, 48

(C) 14, 44.96

(D) 14, 45

Sol. :

x_i	f_i	$d_i = \frac{x_i - A}{c}$	$f_i d_i$	$f_i d_i^2$
4	6	-3	-18	54
8	14	-2	-28	56
12	04	-1	-4	4
A = 16	08	0	00	0
20	11	1	11	11
24	07	2	14	28
	$\sum f_i = 50$		$\sum f_i d_i = -25$	$\sum f_i d_i^2 = 153$

Let, $A = 16$, $d_i = \frac{x_i - A}{c}$, $c = 4$

$$\therefore \bar{x} = A + \frac{\sum f_i d_i}{n} \times c = 16 + \frac{-25}{50} \times 4 = 14$$

$$s^2 = \left[\frac{\sum f_i d_i^2}{n} - \left(\frac{\sum f_i d_i}{n} \right)^2 \right] \times c^2$$

$$= \left[\frac{153}{50} - \left(\frac{-25}{50} \right)^2 \right] \times 16 = [3.06 - 0.25] \times 16$$

$$\therefore s^2 = 2.81 \times 16 = 44.96$$

Mean $\bar{x} = 14$ and variance = 44.96

Ans. : (C)

- (69) If the variance of $6x_i + 3$ is 30, then the standard deviation of x_i is ($i = 1, 2, 3, \dots, n$)

(A) $\sqrt{30}$

(B) 30

(C) $\sqrt{\frac{5}{6}}$

(D) $\frac{5}{\sqrt{6}}$

Sol. : Standard deviation of $6x_i + 3 = \sqrt{30}$

$$\therefore \text{Standard deviation of } 6x_i = \sqrt{30}$$

$$\therefore \text{Standard deviation of } x_i = \frac{1}{6} \sqrt{30} = \sqrt{\frac{30}{36}} = \sqrt{\frac{5}{6}}$$

Ans. : (C)

- (70) If the median of $\frac{x}{8}, \frac{x}{4}, \frac{x}{2}, x, \frac{x}{16}$, where $x > 0$ is 8, then, the mean =

(A) 12.2

(B) 12.3

(C) 12.4

(D) 12.1

Sol. : Arrange the observations in ascending order, $\frac{x}{16}, \frac{x}{8}, \frac{x}{4}, \frac{x}{2}, x$

$$\text{Median} = \left(\frac{5+1}{2} \right) \text{th observation} = 3 \text{rd observation} = \frac{x}{4}$$

$$\therefore \frac{x}{4} = 8. \quad \text{So, } x = 32$$

So, 2, 4, 8, 16, 32 are given observations.

$$\therefore \text{Mean} = \frac{2+4+8+16+32}{5} = \frac{62}{5} = 12.4 \quad \text{Ans. : (C)}$$

(71) The variance of the observations 60, 45, 25, 40 and 70 is

- (A) 246 (B) 15.81 (C) 250 (D) 15.68

$$\text{Sol. : } \bar{x} = \frac{60+45+25+40+70}{5} = \frac{240}{5} = 48$$

$$\sum (x_i - \bar{x})^2 = 144 + 9 + 529 + 64 + 484 = 1230$$

$$\text{The variance} = \frac{\sum (x_i - \bar{x})^2}{5} = \frac{1230}{5} = 246 \quad \text{Ans. : (A)}$$

(72) If $n = 48$, $\sum f_i x_i = 432$ and $\sum f_i x_i^2 = 4012$, then the mean and the standard deviation are and respectively.

- (A) 9, 1.60 (B) 9.5, 1.06 (C) 9.5, 1.6 (D) 9.7, 2.58

$$\text{Sol. : } \bar{x} = \frac{\sum f_i x_i}{n} = \frac{432}{48} = 9 \quad (\text{Ans. (A) decided (9 is only in (A))})$$

$$s = \sqrt{\frac{\sum f_i x_i^2}{n} - \bar{x}^2} = \sqrt{\frac{4012}{48} - 81} = \sqrt{83.58 - 81} = \sqrt{2.58} \approx 1.6 \quad \text{Ans. : (A)}$$

(73) The average deviation of an A.P., $a, a + d, a + 2d, \dots, a + 2nd$ is ($d > 0$)

- (A) $\frac{n(n-1)d}{2n+1}$ (B) $\frac{n(n+1)d}{2n}$ (C) $\frac{n(n+1)d}{2n+1}$ (D) $n(n+1)d$

$$\text{Sol. : } \bar{x} = \frac{1}{2n+1} \left[\frac{2n+1}{2} (a + a + 2nd) \right] = a + nd$$

$$\text{Average deviation} = \frac{1}{2n+1} \sum_{i=0}^{2n} |a + id - (a + nd)|$$

$$= \frac{d}{2n+1} \sum_{i=0}^{2n} |i - n|$$

$$= \frac{d}{2n+1} [n + n - 1 + n - 2 + \dots + 1 + 0 + 1 + 2 + \dots + n]$$

$$= \frac{d}{2n+1} [2(1 + 2 + 3 + \dots + n)]$$

$$= \frac{d}{2n+1} \times \frac{2n(n+1)}{2}$$

$$\therefore \text{Average deviation} = \frac{n(n+1)d}{2n+1} \quad \text{Ans. : (C)}$$

- (74) The median of 21 observations is 15. After adding the observations 12 and 20, the new median of data is
 (A) 12 (B) 15 (C) 20 (D) 16

Sol. : Median of 21 observation is 15.

10 observations	15	10 observations
-----------------	----	-----------------

12 is smaller than 15 and 20 is greater than 15.

So, now we have 23 observations.

So, 11 observations	15	11 observations
---------------------	----	-----------------

The total number of observations is 23.

The median = $\left(\frac{n+1}{2}\right)$ th observation = 12th observation

The median is the 12th observation 15.

Ans. : (B)

- (75) If $L = 42.5$, $n = 50$, $F = 15$, $f = 5$ and $c = 20$, then the median of the data is
 (A) 52.5 (B) 62.5 (C) 72.5 (D) 82.5

Sol. : $M = L + \frac{\frac{n}{2} - F}{f} \times c$
 $= 42.5 + \frac{\frac{50}{2} - 15}{5} \times 20$
 $= 42.5 + 40 = 82.5$

$\therefore$ The median = 82.5

Ans. : (D)

- (76) Runs scored in last five innings by a batsman are 46, 34, 42, 38, 44. Then the mean deviation about the median is
 (A) 3.6 (B) 4.2 (C) 8.4 (D) 1.8

Sol. : Arrange the observations in ascending order,

x_i	$ x_i - M $
34	8
38	4
42	0
44	2
46	4
$\sum x_i - M = 18$	

Here, number of observations = 5

The median = $\left(\frac{5+1}{2}\right)$ th observation = 3rd observation = 42

$\therefore$ The mean deviation = $\frac{\sum |x_i - M|}{5} = \frac{18}{5} = 3.6$

Ans. : (A)

- (77) The standard deviation of data :

x_i	18	19	20	21	22
f_i	3	7	11	14	15

is

- (A) 1.21 (B) 1.25 (C) 1.3 (D) 0.95

Sol. :	x_i	f_i	$d_i = x_i - A$	$f_i d_i$	$f_i d_i^2$
	18	3	-2	-6	12
	19	7	-1	-7	7
	A = 20	11	0	0	0
	21	14	1	14	14
	22	15	2	30	60
		$\sum f_i = 50$		$\sum f_i d_i = 31$	$\sum f_i d_i^2 = 93$

Let, $A = 20$

$$s = \sqrt{\frac{\sum f_i d_i^2}{n} - \left(\frac{\sum f_i d_i}{n}\right)^2}$$

$$= \sqrt{\frac{93}{50} - \left(\frac{31}{50}\right)^2} = \sqrt{1.86 - 0.3844} = \sqrt{1.4756} \approx 1.21$$

Ans. : (A)

- (78) The sum of the squares of deviations from the mean of the observations of a group is 521. Also the variance of the given observations is 52.1. Then the number of observations is
 (A) 10 (B) 100 (C) 101 (D) 11

Sol. : $s^2 = \frac{\sum (x_i - \bar{x})^2}{n}$

$$\therefore 52.1 = \frac{521}{n}$$

$$\therefore n = \frac{521}{52.1}$$

$$\therefore n = 10$$

Ans. : (A)

- (79) The mean of 9 observations is 15. If after adding one more observation the new mean is 16, then the new observation added is
 (A) 25 (B) 37 (C) 40 (D) 15

Sol. : $\sum_{i=1}^9 x_i = n\bar{x} = 9 \times 15 = 135$

Let the observation added be a .

Then, $\sum_{i=1}^{10} x_i = 135 + a$

Corrected $\bar{x} = \frac{\text{Corrected } \sum_{i=1}^{10} x_i}{10}$

$$\therefore 16 = \frac{135 + a}{10}$$

$$\therefore 160 - 135 = a$$

$$\therefore a = 25$$

Ans. : (A)

- (80) The mean of the following frequency distribution is 8.3. Then $k = \dots$

x_i	4	6	7	k	12	14
f_i	5	6	8	10	2	9

- (A) 10 (B) 12 (C) 7 (D) 13

Sol. :

x_i	f_i	$f_i x_i$
4	5	20
6	6	36
7	8	56
k	10	$10k$
12	2	24
14	9	126
	40	$\sum f_i x_i = 262 + 10k$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\therefore 8.3 = \frac{262 + 10k}{40}$$

$$\therefore 332 = 262 + 10k$$

$$\therefore 70 = 10k$$

$$\therefore k = 7$$

Ans. : (C)

- (81) If 80, 86, 69, 74 and 91 are the seconds taken by 5 students in a 400 meter race, then the coefficient of variation is $\dots$

- (A) 0.125 (B) 0.099 (C) 0.091 (D) 1.02

Sol. :

x_i	$(x_i - \bar{x})^2$
80	0
86	36
69	121
74	36
91	121
$\sum x_i = 400$	$\sum (x_i - \bar{x})^2 = 314$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{400}{5} = 80$$

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} = \sqrt{\frac{314}{5}} = \sqrt{62.8} = 7.925$$

$$\text{So, } V = \frac{s}{\bar{x}} = \frac{7.925}{80} \approx 0.099$$

Ans. : (B)

- (82) If the mean and the median of a given frequency distribution are 20 and 21, then approximate value of mode is $\dots$

- (A) 24 (B) 24.5 (C) 23 (D) 23.5

Sol. : Mode = $3(\text{Median}) - 2(\text{Mean}) = 3(21) - 2(20) = 63 - 40 = 23$

Ans. : (C)

- (83) Find the mean deviation of given frequency distribution from median :

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	6	7	8	5	4

- (A) 20 (B) 22.5 (C) 24 (D) 20.5

Sol. :

Class	f	cf
00 – 10	6	6
10 – 20	7	13 = F
20 – 30	8 = f	21
30 – 40	5	26
40 – 50	4	30

$\frac{n}{2} = 15$ th observation is in the class 20 – 30.

So, the median class is 20 – 30. $L = 20$, $f = 8$, $F = 13$

$$\therefore M = L + \frac{\frac{n}{2} - F}{f} \times c = 20 + \frac{15 - 13}{8} \times 10$$

$$= 20 + 2.5 = 22.5 \quad \text{Ans. : (B)}$$

- (84) The mean and variance of 10 observations are 45 and 16 respectively. Later on it was found that one observation was taken incorrectly to be 52 instead of 25. The correct mean and variance are and
 (A) 42, 43 (B) 41, 45 (C) 42.3, 43.81 (D) 43.3, 42.81

Sol. : $\bar{x} = 45, s^2 = 16$

$$\begin{aligned}\text{Correct } \sum x_i &= n\bar{x} - (\text{Incorrect observation}) + \text{Correct observation} \\ &= 10 \times 45 - 52 + 25 = 450 - 27 = 423\end{aligned}$$

$$\text{Correct } \bar{x} = \frac{\text{Correct } \sum x_i}{n} = \frac{423}{10} = 42.3$$

Note : It is clear that option (C) is the correct option.

$$\begin{aligned}\text{Correct } \sum x_i^2 &= n(s^2 + \bar{x}^2) - (\text{Incorrect observation})^2 + (\text{Correct observation})^2 \\ &= 10(16 + 2025) - (52)^2 + (25)^2 = 20410 - 2704 + 625 = 18331\end{aligned}$$

$$\begin{aligned}\text{Correct } s^2 &= \frac{1}{n} \text{Correct } \sum x_i^2 - (\text{Correct } \bar{x})^2 \\ &= \frac{18331}{10} - (42.3)^2 = 1833.1 - 1789.29 = 43.81\end{aligned} \quad \text{Ans. : (C)}$$

- (85) Variance of the natural numbers 1, 2, 3, ..., $2n + 1$

$$(A) \frac{n(n-1)}{2} \quad (B) \frac{n(n+1)}{3} \quad (C) \frac{n(n-1)(2n+1)}{3} \quad (D) \frac{n(n+1)}{2}$$

Sol. : For sequence 1, 2, 3, ..., $2n + 1$,

$$s^2 = \frac{(2n+1)^2 - 1}{12} = \frac{4n^2 + 4n}{12} = \frac{n(n+1)}{3} \quad \text{Ans. : (B)}$$

: Assertion and Reason Type Questions :

Explanation : Each of these questions contains statement 1 (Assertion) and statement 2 (Reason). Each question has four choices, option (A), (B), (C) and (D), which are as follows. You have to select the correct choice.

- (A) Statement 1 is true, statement 2 is true. Statement 2 is the correct explanation for statement 1.
 (B) Statement 1 is true, statement 2 is true. Statement 2 is not a correct explanation for statement 1.
 (C) Statement 1 is true, statement 2 is false.
 (D) Statement 1 is false, statement 2 is true.

- (86) Suppose mean, mode and variance of n observations $x_1, x_2, \dots, x_n$ are $\bar{x}$, Z and s^2 respectively and $d_i = -x_i + a$ where $i = 1, 2, 3, \dots, n$ and $a \in \mathbb{R}$.

Statement 1 : The variance of $d_1, d_2, \dots, d_n$ is s^2 .

Statement 2 : The mean and the mode of $d_1, d_2, \dots, d_n$ are $-\bar{x} + a$ and $-Z + a$ respectively.

Sol. : Multiply by -1 and add a to each of the observations $x_1, x_2, \dots, x_n$. We get new set of observations $d_1, d_2, \dots, d_n$.

$$\text{The new mean} = (-1)\bar{x} + a = -\bar{x} + a$$

$$\text{The new mode} = (-1)Z + a = -Z + a$$

$\therefore$ Statement 2 is true.

Similarly variance = $(-1)^2 s^2 = s^2$

Statement 1 and 2 are true, but statement 2 is not sufficient to explain statement 1.

Ans. : (B)

- (87) **Statement 1** : The arithmetic mean of the sequence 1, 3, 9, 27, ..., 3^n is $\frac{3^{n+1} - 1}{2(n+1)}$

Statement 2 : Sum of n terms of increasing G.P. is $a \left(\frac{r^n - 1}{r - 1} \right)$. ($r \neq 1$).

Sol. : In statement 1, for sequence 1, 3, 9, 27, ..., 3^n ,

$$\text{Arithmetic mean} = \frac{1 + 3 + 3^2 + \dots + 3^n}{n + 1} = \frac{3^{n+1} - 1}{2(n + 1)}$$

So, statement 1 is true.

$a, ar, ar^2, \dots, ar^{n-1}$ is an increasing G.P. $r > 1$

$$s_n = a + ar + ar^2 + \dots + ar^{n-1}$$

$$s_n = \frac{a(r^n - 1)}{r - 1} \quad (r \neq 1)$$

So, statement 2 is true.

To explain statement 1, statement 2 is sufficient.

Ans. : (A)

- (88) Suppose the mean and the variance of n observations $x_1, x_2, \dots, x_n$ are $\bar{x}$ and s^2 .

[AIEEE : 2012]

Statement 1 : The variance of $2x_1, 2x_2, \dots, 2x_n$ is $4s^2$.

Statement 2 : The mean of $2x_1, 2x_2, \dots, 2x_n$ is $4\bar{x}$.

Sol. : Variance of $2x_1, 2x_2, \dots, 2x_n = 2^2 \times s^2 = 4s^2$

$\therefore$ Statement 1 is true.

Mean of $2x_1, 2x_2, \dots, 2x_n$ observations

$$= 2[\text{mean of observations } x_1, x_2, \dots, x_n]$$

$$= 2\bar{x}$$

Statement 2 is false.

Ans. : (C)

- (89) **Statement 1** : In a class of 100 students average of marks of 80 boys is 50 and average of marks of 20 girls is 40. Also average of marks of all the students is 48.

Statement 2 : The number of observations in given data are n_1 and n_2 respectively. Also $\bar{x}_1$ and $\bar{x}_2$ are means of both the data respectively and the mixed mean is $\bar{x}$.

$$\text{where, } \bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

Sol. : $n_1 = 80, \bar{x}_1 = 50, n_2 = 20, \bar{x}_2 = 40$

$$\begin{aligned} \bar{x} &= \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} \\ &= \frac{80 \times 50 + 20 \times 40}{100} = \frac{4000 + 800}{100} = 48 \end{aligned}$$

Statement 1 is true.

In the first data, the number of observations n_1 and mean $\bar{x}_1$

In the second data, the number of observations n_2 and mean $\bar{x}_2$

$$\therefore \text{ The mixed mean } \bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

Statement 2 is true.

Statement 2 is sufficient to explain statement 1.

Ans. : (A)

(90) **Statement 1 :** The variance of first n odd natural numbers is $\frac{1}{3}(n^2 - 1)$.

Statement 2 : The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first

$$n \text{ natural numbers is } \frac{n}{6}(n+1)(2n+1).$$

Sol. : Statement 1 : For 1, 3, 5, 7, ..., $2n-1$,

$$\bar{x} = \frac{1+3+5+\dots+2n-1}{n} = \frac{n^2}{n} = n$$

$$\begin{aligned} \frac{1}{n} \sum x_i^2 &= \frac{1}{n} [1^2 + 3^2 + 5^2 + \dots + (2n-1)^2] \\ &= \frac{1}{n} \sum (2n-1)^2 \\ &= \frac{1}{n} \sum (4n^2 - 4n + 1) \\ &= \frac{1}{n} \left[\frac{4}{6} n(n+1)(2n+1) - \frac{4}{2} n(n+1) + n \right] \\ &= \frac{1}{3} [2(2n^2 + 3n + 1) - 6(n+1) + 3] \\ &= \frac{1}{3} [4n^2 + 6n + 2 - 6n - 6 + 3] \\ &= \frac{1}{3} [4n^2 - 1] \end{aligned}$$

$$s^2 = \frac{\sum x_i^2}{n} - \bar{x}^2 = \frac{1}{3} (4n^2 - 1) - n^2 = \frac{1}{3} (n^2 - 1)$$

Statement 1 is true.

$\therefore$ Statement 1 is true and statement 2 is also true and statement 2 is sufficient to explain statement 1.

Ans. : (A)

(91) **Statement 1 :** The variance of first n even natural numbers is $\frac{n^2-1}{4}$.

Statement 2 : Sum of first n natural number is $\frac{n(n+1)}{2}$ and sum of the squares of first n

$$n \text{ natural numbers is } \frac{n}{6}(n+1)(2n+1).$$

[AIEEE : 2009]

Sol. : Given observations are 2, 4, 6, 8, ..., $2n$.

$$\bar{x} = \frac{2+4+6+\dots+2n}{n} = \frac{2\sum n}{n} = n+1$$

$$\begin{aligned}\sum x_i^2 &= 2^2 + 4^2 + 6^2 + \dots + 4n^2 \\ &= 4[1^2 + 2^2 + 3^2 + \dots + n^2] \\ &= \frac{4}{6}n(n+1)(2n+1)\end{aligned}$$

$$\begin{aligned}\text{The variance} &= \frac{\sum x_i^2}{n} - \bar{x}^2 \\ &= \frac{2n(n+1)(2n+1)}{3n} - (n+1)^2 \\ &= \frac{n+1}{3} [2(2n+1) - 3(n+1)] \\ &= \frac{n+1}{3} [4n+2 - 3n-3] \\ &= \frac{1}{3}(n^2 - 1)\end{aligned}$$

Statement 1 is false.

Using statement 2, $\sum n = \frac{n(n+1)}{2}$; $\sum n^2 = \frac{n}{6}(n+1)(2n+1)$. Statement 2 is true.

$\therefore$ Statement 1 is false and statement 2 is true.

Ans. : (D)

: 'Match the following' Type Questions :

(92) Match the data given in column I with the proper statements in column II :

	Column I	Column II
(1)	The mean of 1, 2, 3, ..., 10	(P) 2
(2)	The median of 11, 12, 13, ..., 20	(Q) $\frac{33}{4}$
(3)	The variance of 1, 2, 3, 4, ..., 10	(R) 15.5
(4)	The mode of 2, 3, 5, 2, 6, 5, 3, 2, 8, 6, 2, 5	(S) 5.5

(A) 1 - S, 2 - R, 3 - Q, 4 - P

(B) 1 - S, 2 - Q, 3 - R, 4 - P

(C) 1 - S, 2 - P, 3 - Q, 4 - R

(D) 1 - R, 2 - S, 3 - Q, 4 - P

Sol. : (1) $\bar{x} = \frac{1+2+3+\dots+10}{10} = \frac{\frac{10}{2}(11)}{10} = \frac{11}{2} = 5.5$ (1) - S

(2) For the median of observations; arrange them in ascending order. Here, $n = 10$.

$$\text{The median} = \frac{5\text{th observation} + 6\text{th observation}}{2}$$

$$= \frac{15+16}{2} = 15.5 \quad (2) - R$$

So, (A) is the correct answer.

$$(3) s^2 = \frac{n^2 - 1}{12}$$

$$\text{Here, } n = 10, \text{ so } s^2 = \frac{(10)^2 - 1}{12} = \frac{99}{12} = \frac{33}{4} \quad (3) - Q$$

(4) In the given ungrouped data 2 occurs maximum number of times, So mode is 2.

$$\therefore \text{Mode} = 2$$

(4) – P

Ans. : (A)

Short method :

All the options of (2) are different.

$\therefore$ Only by taking the calculation of (2); we get (2) $\rightarrow$ R. Hence option (A) is the correct answer.

Ans. : (A)

(93) Match the information of column A with column B :

Column A		Column B	
(1)	The mean of 2, 6, 10, 14, 18	(a)	$\sqrt{10}$
(2)	The median of 2, 6, 10, 14, 18, 22, 26	(b)	5
(3)	The mode of 1, 2, 3, 4, 5, 6, 7, 8, 9	(c)	14
(4)	The standard deviation of 1, 2, 3, ..., 10, 11	(d)	10

(A) (1) $\rightarrow$ (d) (2) $\rightarrow$ (b) (3) $\rightarrow$ (c) (4) $\rightarrow$ (a)

(B) (1) $\rightarrow$ (d) (2) $\rightarrow$ (c) (3) $\rightarrow$ (b) (4) $\rightarrow$ (a)

(C) (1) $\rightarrow$ (d) (2) $\rightarrow$ (a) (3) $\rightarrow$ (c) (4) $\rightarrow$ (b)

(D) (1) $\rightarrow$ (c) (2) $\rightarrow$ (b) (3) $\rightarrow$ (c) (4) $\rightarrow$ (d)

Sol. : (1) The mean of 2, 6, 10, 14, 18

$$\text{The mean} = \frac{\frac{5}{2}[2+18]}{5} = \frac{5(20)}{2 \times 5} = 10 \quad (1) \rightarrow (d)$$

(2) Arrange the given 7 observations in ascending order. 2, 6, 10, 14, 18, 22, 26.

$$\text{The median} = \frac{7+1}{2} \text{th observation} = 4\text{th observation} = 14 \quad (2) \rightarrow (c)$$

Note : (1) $\rightarrow$ (d), (2) $\rightarrow$ (c). So, option (B) is the correct answer.

(3) Mean and median of 1, 2, 3, ..., 9 are 5.

$$Z = 3M - 2\bar{x} = 5 \quad (3) \rightarrow (b)$$

(4) For 1, 2, 3, ..., 10, 11

$$\text{Standard deviation of observations} = \sqrt{\frac{n^2-1}{12}}$$

Here, $n = 11$.

$$\text{Standard deviation} = \sqrt{\frac{(11)^2-1}{12}} = \sqrt{\frac{120}{12}} = \sqrt{10} \quad (4) \rightarrow (a)$$

Ans. : (1) $\rightarrow$ (d); (2) $\rightarrow$ (c); (3) $\rightarrow$ (b); (4) $\rightarrow$ (a)

Ans. : (B)

