

Total No. of Questions—15

Total No. of Printed Pages—3

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Part III

MATHEMATICS

(Bridge Course) (For B.P.C. Candidates)

Paper I

(English Version)

Time : 3 Hours

Max. Marks : 75

Note :— This question paper consists of TWO Sections A and B.

SECTION-A

10×3=30

I. Short answer type questions :

- (i) Answer ALL the questions.
(ii) Each question carries THREE marks.

1. If $A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$, then show that $A^2 = -I$.

2. Find the unit vector in the direction of the vector
 $\vec{a} = 2\vec{i} + 3\vec{j} + \vec{k}$.

3. If $\vec{a} = 2\vec{i} + 2\vec{j} - 3\vec{k}$, $\vec{b} = 3\vec{i} - \vec{j} + 2\vec{k}$, then find the angle
between $2\vec{a} + \vec{b}$ and $\vec{a} + 2\vec{b}$.

4. Find the maximum and minimum values of :

$$\cos\left(x + \frac{\pi}{3}\right) + 2\sqrt{2}\sin\left(x + \frac{\pi}{3}\right) - 3.$$

5. Evaluate :

$$\sin^2 82\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ.$$

6. Find the equation of the straight line which makes an angle 135° with the positive x -axis and passing through the point $(3, -2)$.
7. Find the point of intersection of the straight lines $4x + 8y - 1 = 0$ and $2x - y + 1 = 0$.
8. Find 'x' if the distance between the points $(5, -1, 7)$ and $(x, 5, 7)$ is '9' units.
9. Evaluate :

$$\lim_{x \rightarrow 0} \left[\frac{e^x - \sin x - 1}{x} \right].$$

10. If $f(x) = 2x^2 + 3x - 5$, then prove that $f'(0) + 3 \cdot f'(-1) = 0$.

SECTION-B

3×15=45

- II. Long answer type questions :

- (i) Attempt ANY THREE questions.
 (ii) Each question carries FIFTEEN marks.

11. (a) Solve :

$$\begin{aligned} x + y + z &= 1 \\ 2x + 2y + 3z &= 6 \\ x + 4y + 9z &= 3 \end{aligned}$$

by matrix-inversion method.

- (b) Show that :

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a).$$

12. (a) If the points whose position vectors are $3\bar{i} - 2\bar{j} - \bar{k}$, $2\bar{i} + 3\bar{j} - 4\bar{k}$, $-\bar{i} + \bar{j} + 2\bar{k}$ and $4\bar{i} + 5\bar{j} + \lambda\bar{k}$ are coplanar, then show that $\lambda = \frac{-146}{17}$.
- (b) Prove that for any three vectors $\bar{a}$, $\bar{b}$ and $\bar{c}$, $[\bar{b} + \bar{c} \quad \bar{c} + \bar{a} \quad \bar{a} + \bar{b}] = 2[\bar{a} \quad \bar{b} \quad \bar{c}]$.
13. (a) Prove that : $\sin^4\left(\frac{\pi}{8}\right) + \sin^4\left(\frac{3\pi}{8}\right) + \sin^4\left(\frac{5\pi}{8}\right) + \sin^4\left(\frac{7\pi}{8}\right) = \frac{3}{2}$.
- (b) If A, B, C are angles of a triangle, then prove that : $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$.
14. (a) Find the circumcenter of the triangle whose sides are given by $3x - y - 5 = 0$, $x + 2y - 4 = 0$ and $5x + 3y + 1 = 0$.
- (b) Find the value of 'k' if the angle between the straight lines $4x - y + 7 = 0$ and $kx - 5y - 9 = 0$ is 45° .
15. (a) If $x^y = e^{x-y}$, then show that : $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$.
- (b) Find the equations of the tangent and normal to the curve $y = 5x^4$ at the point (1, 5).