Paper Code - 5303

M.Sc. Semester-III (Mathmatics) New Course
July-Septermber-2021
MSM1302-Complex Analysis

કુલ સમય :– : ૬૦ મીનીટ **કુલ પ્રશ્નો : ૫૦** **વિદ્યાર્થીએ ટીક કરવાના પ્રશ્નો : ૩૫**

નોંધ : વિદ્યાર્થીએ માત્ર ૩૫ પ્રશ્નો જ ટીક કરવાના રહેશે. જો વધારે ટીક કરેલ હશે તો પ્રથમ ૩૫ પ્રશ્નો જ ગણતરીમાં લેવામાં આવશે.

1. If $z = re^{i\theta}$, then $\left|e^{iz}\right| =$_____ .
- **(a)** $e^r \sin\theta$
- **(b)** $e^{-r} \sin\theta$
- **(c)** $e^r \cos\theta$
- **(d)** $e^{-r} \cos\theta$

2. The value of $\left|\frac{z-1}{\bar{z}-1}\right|$ is _____.
- **(a)** i
- **(b)** 1
- **(c)** -1
- **(d)** None of these

3. If $i^2 = -1$ then $\sum_{n=1}^{66} i^n$=_____ .
- **(a)** i
- **(b)** -1
- **(c)** $-1 + i$
- **(d)** $1 - i$

4. A function is analytic at a point z_0 of domain D, than it is _____ .
- **(a)** Differentiable in D.
- **(b)** Not differentiable in D.
- **(c)** Differentiable at z_0 .
- **(d)** Not differentiable at z_0 .

5. If a function f is continuous throughout a region R which is closed and bounded then
- **(a)** f is analytic.
- **(b)** $|f(z)|$ is bounded in R.
- **(c)** f is entire function.
- **(d)** f is differentiable.

6. Which of the following is Cauchy-Riemann equation in polar form?
- **(a)** $u_x = v_y$ and $u_y = v_x$.
- **(b)** $u_x = v_y$ and $u_y = -v_x$.
- **(c)** $ru_r = v_\theta$ and $u_\theta = -rv_r$.
- **(d)** $ru_r = -v_\theta$ and $u_\theta = rv_r$.

7. If $f'(z) = 0$ everywhere in a domain D, then
- **(a)** $f(z)$ is an identity function.
- **(b)** $f(z)$ is an hyperbolic function.
- **(c)** $f(z)$ is an trigonometric function.
- **(d)** $f(z)$ is an constant function.

8. What is the harmonic conjugate of $u(x, y) = x^2 - y^2$?
- **(a)** $v(x, y) = xy$
- **(b)** $v(x, y) = 2xy$
- **(c)** $v(x, y) = x^2 + y^2$
- **(d)** $v(x, y) = (x - y)^2$

9. For a function $f(z) = u(x, y) + iv(x, y)$, which of the following condition is not sufficient for analyticity of f in domain D?
- **(a)** f satisfies CR-equations.
- **(b)** First order partial derivatives of u and v exists and continuous and satisfy CR-equations.

(c) v is harmonic conjugate of u.
(d) $f'(z)$ exists at neighborhood of each point of D.

10. $\int_{C:|z|=1} \frac{dz}{z-2} =$
(a) $2\pi i$
(b) 0
(c) $4\pi i$
(d) $-3\pi i$

11. What is the value of i^i?
(a) $e^{-\pi}/_3$
(b) $e^{-3\pi}/_2$
(c) $e^{-\pi}/_2$
(d) $e^{-\pi}/_6$

12. What is the value of $\log -1 - \sqrt{3}i$?
(a) $\ln 2 - \frac{2\pi i}{3}$
(b) $\ln 2 + \frac{2\pi i}{3}$
(c) $\ln 2 - \frac{\pi i}{3}$
(d) $\ln 4 - \frac{2\pi i}{3}$

13. $\text{Log } i^3 =$
(a) $3\text{Log } i$
(b) $\frac{3\pi i}{2}$
(c) $i\text{Log } 3$
(d) None of these

14. Evaluate $\int_0^{1+i} z^2\, dz$ along the line $y = \frac{x}{2}$.
(a) $\frac{1}{3}\left(\frac{1}{4} + \frac{11}{8}i\right)$
(b) $\frac{2i+2}{3}$
(c) $\frac{2i-3}{3}$
(d) $\frac{3i-2}{2}$

15. If C is any simple closed contour, in either direction then $\int_C e^{z^2}\, dz =$ _____ .
(a) 1
(b) 4
(c) 2
(d) 0.

16. Evaluate $\int_C x\, dz$ along the parabola $y = 2x^2$ from $(1,2)$ to $(2,8)$.
(a) $\frac{5}{2} + \frac{28i}{3}$
(b) $\frac{3}{2} + \frac{28i}{3}$
(c) $\frac{3}{2} - \frac{28i}{3}$

17. If $f(z)$ is continuous on the curve C of length L and $|f(z)| \leq M$ then $\left|\int_C f(z)dz\right|$ is _____ .
(a) $\leq M$.
(b) $\leq ML$.
(c) $\geq ML$.
(d) $= 0$.

18. $\oint_{C:|Z|=4} \frac{zdz}{\sin z} =$
(a) $2\pi i$
(b) 0

(c) $-2\pi i$
(d) πi

19. The value of the integral $\int_C \frac{dz}{z(z-2)}, C:|z|=1$ is

(a) 0
(b) πi
(c) $-\pi i$
(d) $2\pi i$

20. The value of the integral $\int_C \frac{3z+4}{z(2z+1)} dz, C:|z|=1$ is

(a) $3\pi i$
(b) 4
(c) -4
(d) $2\pi i$

21. The value of the integral $\oint_C \frac{dz}{z^2-2z}, C: |z-2| = 1$ is

(a) $-\pi i$
(b) $2\pi i$
(c) πi
(d) None of these

22. Evaluate $\oint_C \frac{\cos \pi z}{z-1}\, dz$ if $C: |z| = 2$.

(a) $-2\pi i$
(b) $2\pi i$
(c) πi
(d) 0

23. By Cauchy integral formula if a function $f(z)$ is analytic in the region R then its derivative at any point $z = a$ of R is _____ .

(a) $f'(a) = \frac{1}{\pi i}\oint_C \frac{f(z)}{(z+a)^2} dz$
(b) $f'(a) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{(z-a)^2} dz$
(c) $f'(a) = \frac{1}{\pi i}\oint_C \frac{f(z)}{(z-a)^2} dz$
(d) $f'(a) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{(z+a)^2} dz$

24. Which of the following is known as Liouville theorem?

(a) If $f(z)$ is analytic and bounded in the entire $z-plane$ for all z then $f(z)$ is not constant.
(b) If $f(z)$ is continuous and bounded in the entire $z-plane$ for all z then $f(z)$ is constant.
(c) If $f(z)$ is analytic and bounded in the entire $z-plane$ for all z then $f(z)$ is constant.
(d) If $f(z)$ is analytic in the entire $z-plane$ for all z then $f(z)$ is constant.

25. Find $\oint_C \frac{dz}{z}$, where C is the right half of the circle $|z| = 2$.

(a) 0
(b) $2\pi i$
(c) πi
(d) 3

26. For $f(z) = (z - \sin z)/z^3$, the point $z = 0$ is

(a) A pole
(b) A removable singularity
(c) An essential singularity

(d) A zero.

27. When any point is said to be a singular point?

(a) if the function is analytic at that point.
(b) if the function is differentiable at that point.
(c) if the function is not continuous at that point.
(d) if the function is not analytic at that point.

28. If $f(z)$ has a pole of order m at $z = a$ then $Res_{z=a}\, f(z) =$

(a) $\frac{1}{m!}\lim_{z\to a}\left[\frac{d^{m-1}}{dz^{m-1}}(z-a)^m f(z)\right]$
(b) $\frac{1}{(m-1)!}\lim_{z\to a}\left[\frac{d^{m-1}}{dz^{m-1}}(z-a)^m f(z)\right]$
(c) $\frac{1}{(m+1)!}\lim_{z\to a}\left[\frac{d^{m-1}}{dz^{m-1}}(z-a)^m f(z)\right]$
(d) $\frac{1}{(m-1)!}\lim_{z\to a}\left[\frac{d^{m}}{dz^{m}}(z-a)^m f(z)\right]$

29. The value of the integral $\oint_C \frac{dz}{\left(z^2+4\right)^2}, C: |z-i| = 2$ is

(a) $\frac{\pi}{4}$
(b) $\frac{\pi}{8}$
(c) 2π
(d) π

30. The function $f(z) = \ln z$ has _____ isolated singularities.

(a) 1
(b) 2
(c) 3
(d) None of these

31. For the function $f(z) = (1 - e^{2z})/z^4$ has pole of order ____ at $z = 0$.

(a) 4
(b) 3
(c) 2
(d) 1

32. What is the formula to find residue at simple pole a?

(a) $Res = \lim_{z\to -a}(z+a)f(z)$
(b) $Res = \lim_{z\to -a}(z-a)f(z)$
(c) $Res = \lim_{z\to a}(z+a)f(z)$
(d) $Res = \lim_{z\to a}(z-a)f(z)$

33. Which of the following term is used to find residue at $z = a$ in Laurent's series $f(z) = \sum_{n=0}^{\infty} a_n(z-a)^n + \sum_{n=1}^{\infty}\frac{b_n}{(z-a)^n}$?

(a) b_1
(b) b_2
(c) a_0
(d) a_1

34. Let $f(z) = \frac{z}{(z-3)^4(z-5)(z-2)^2}$ then which of the following is simple pole?

(a) 3
(b) 2
(c) 5
(d) 0

35. Find the residue of $f(z) = \frac{z}{z^2+1}$ about $z = i$.

(a) $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. 0

****