Paper Code – 5403

M.Sc. Semester-III (Mathematics) (OLD)
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MTHP-6 (Functional Analysis)

કુલ સમય : ૩૦ મીનીટ **કુલ પ્રશ્નો : ૩૫** **વિદ્યાર્થીએ ટીક કરવાના પ્રશ્નો : ૧૮**

નોંધ : વિદ્યાર્થીએ માત્ર ૧૮ પ્રશ્નો જ ટીક કરવાના રહેશે. જો વધારે ટીક કરેલ હશે તો પ્રથમ ૧૮ પ્રશ્નો જ ગણતરીમાં લેવામાં આવશે.

Q. 1 Which of the following conditions must hold in Normed Linear Space N over field F?
(a) $\|x+y\| \leq \|x\| + \|y\|\ ;\ \forall x, y \in N$ (b) $\|\alpha.x\| < |\alpha|.\|x\|\ ;\ \alpha \epsilon F$
(c) $\|\alpha.x\| > |\alpha|.\|x\|\ ;\ \alpha \epsilon F, x, y \in N$ (d) $\|x+y\| \geq \|x\| + \|y\|\ ;\ \forall x, y \in N$

Q. 2 Which of the following is True Statement?
(a) Every Linear Space is Metric Space. (b) Every metric space is Linear Space.
(c) Every Normed Linear Space is metric Space. (d) Both (a) and (b) are True Statements.

Q. 3 If N is Normed Linear Space,then_____
(a) $|\|x\| - \|y\|| = \|x-y\|\ ;\ \forall x, y \in N$ (b) $|\|x\| - \|y\|| \leq \|x-y\|\ ;\ \forall x, y \in N$
(c) $|\|x\| - \|y\|| \geq \|x-y\|\ ;\ \forall x, y \in N$ (d) $|\|x\| - \|y\|| \neq \|x-y\|\ ;\ \forall x, y \in N$

Q. 4 If M is subspace of linear space L, then the coset of an element $x \in L$ is defined as_____.
(a) $x + M = \{x + M / x \in L\}$ (b) $x + M = \{x + M / x \notin L\}$
(c) $x + M = \{x + M / m \in M\}$ (d) $x + M = \{x + M / m \notin M\}$

Q. 5 Which of the following is True Statement?
(a) Every Normed Linear Space is Banach Space. (b) Every Complete Normed Linear Space is Banach Space.
(c) Every Linear Space is Banach Space. (d) Every Incomplete Normed Linear Space is Banach Space.

Q. 6 Choose Correct option from the following.
(a) R^2 is Normed Linear Space under Norm $\|x\| = |x|\ ;\ \forall x \in R$.
(b) R^2 is Complete Normed Linear Space under Norm $\|x\| = |x|\ ;\ \forall x \in R$.
(c) R^2 is Banach Space under the Norm $\|x\| = |x|\ ;\ \forall x \in R$.
(d) All (a), (b), (c) are True Statements.

Q. 7 Choose Correct option from the following.
(a) $\mathbb{C}^2$ is Banach Space under Norm $\|x\| = |x|\ ;\ \forall x \in \mathbb{C}$. (b) $\mathbb{C}^2$ is Linear Space over $\mathbb{C}$.
(c) $\mathbb{C}^2$ is complete Linear Space over R. (d) All (a), (b), (c) are True Statements.

Q. 8 If $\alpha, \beta \epsilon R^+ \cup \{0\}$, and if $p > 1$, such that $\frac{1}{p} + \frac{1}{q} = 1$, then_____
(a) $\alpha \cdot \beta \leq \frac{\alpha^p}{p} + \frac{\beta^p}{p}$ (b) $\alpha \cdot \beta \geq \frac{\alpha^p}{p} + \frac{\beta^q}{q}$ (c) $\alpha \cdot \beta \leq \frac{\alpha^p}{p} + \frac{\beta^q}{q}$ (d) $\alpha \cdot \beta \geq \frac{\alpha^q}{q} + \frac{\beta^q}{q}$

Q. 9 If $x \in l_p^n, y \in l_q^n\ ;\ \frac{1}{p} + \frac{1}{q} = 1$ and $p > 1$, then_____
(a) $\sum_{i=1}^n |x_i \cdot y_i| \leq \|x\|_p \cdot \|y\|_q$ (b) $\sum_{i=1}^n |x_i \cdot y_i| \geq \|x\|_p \cdot \|y\|_q$
(c) $\sum_{i=1}^n |x_i \cdot y_i| \leq \|x\|_q \cdot \|y\|_p$ (d) $\sum_{i=1}^n |x_i \cdot y_i| \geq \|x\|_q \cdot \|y\|_p$

Q. 10 Consider the statement $\sum_{i=1}^n |x_i \cdot y_i| \leq (\sum_{i=1}^n |x_i|^2)^{\frac{1}{2}} \cdot (\sum_{i=1}^n |y_i|^2)^{\frac{1}{2}}$, $\forall (x_1, \ldots x_n) \in R^n, \forall\ (y_1, \ldots y_n) \in R^n$
(a) Given statement is known as " Cauchy's Inequality"
(b) Given statement is known as " Minkowski's Inequality"
(c) Given statement is known as " Holder's Inequality"
(d) Given statement is known as " Euclid's Lemma"

Q. 11 If $x, y \in l_p^n$, where $p \geq 1$, then_____
(a) $\|x+y\|_p = \|x\|_p + \|y\|_p$ (b) $\|x+y\|_p \neq \|x\|_p + \|y\|_p$
(c) $\|x+y\|_p \leq \|x\|_p + \|y\|_p$ (d) $\|x+y\|_p \geq \|x\|_p + \|y\|_p$

Q. 12 If $N$and N' are Normed Linear Space, and $T: N \to N'$ is Linear Transformation, then_____
(a) If T is continuous on N then T is continuous at origin. (b) T is continuous at origin $\Rightarrow$ T is continuous on N.
(c) T is continuous on $N \Leftrightarrow T$ is continuous at origin. (d) All (a), (b), (c) are True Statements.

Q. 13 If $N$and N' are Normed Linear Space, and $T: N \to N'$ is Linear Transformation, then_____
(a) If T is continuous at origin then T is not bounded. (b) If T is Bounded then T is continuous on N.
(c) T is continuous on N then T is not bounded. (d) If T is Bounded then T is not continuous at origin.

Q. 14 If $N$and N' are Normed Linear Space, and $T: N \to N'$ is Linear Transformation, then which of the following is false.
(a) $\|T\| = Sup\{\|T(x)\| : x \epsilon N and \|x\| < 1\}$ (b) $\|T\| = Sup\{\|T(x)\| : x \epsilon N and \|x\| = 1\}$ where $N = \{\bar{0}\}$.
(c) $\|T\| = Sup\{k : k \geq 0\ and \|T(x)\| \leq k\|x\|; \forall x \in N\}$. (d) $\|T\| = Inf\{k : k \geq 0\ and \|T(x)\| \leq k\|x\|; \forall x \in N\}$.

Q. 15 If $N$and N' are Normed Linear Space, and $T: N \to N'$ is continuous Linear Transformation, then_____
(a) $\|T(x)\| \leq \|T\| \cdot \|x\|; \forall x \in N$ (b) $\|T(x)\| = \|T\| \cdot \|x\|; \forall x \in N$
(c) $\|T(x)\| \geq \|T\| \cdot \|x\|; \forall x \in N$ (d) $\|T(x)\| \cdot \|T\| \leq \|x\|; \forall x \in N$

Q. 16 If $T \in \mathfrak{B}(N)$ and $T' \in \mathfrak{B}(N)$; then_____
(a) $\|T^{\circ}T'\| \geq \|T\| \cdot \|T'\|$. (b) $\|T^{\circ}T'\| \leq \|T\| \cdot \|T'\|$ (c) $\|T'^{\circ}T\| \neq \|T\| \cdot \|T'\|$ (d) $\|T'^{\circ}T\| = \|T'\| \cdot \|T\|$.

Q. 17 Which of the following is dual space of R^n?
(a) C^n (b) Q^n (c) R^n (d) $R^n - R$

Q. 18 Which of the following is dual space of l^1?
(a) l^1 (b) l_p (c) l_p^n (d) l^{∞}

Q. 19 Which of the following is dual space of l_p, where $p > 1$, and q is conjugate of p.
(a) l^{∞} (b) l_q (c) l_p (d) l^1

Q. 20 Consider the statement:" If M is Linear subspace of a Normed Linear Space N; and f is a functional defined on M, then f can be extended to a functional f_0 defined on N, such that $\|f_0\| = \|f\|$. This statement is related to_____
(a) Hahn-Banach Theorem (b) Closed graph Theorem
(c) Open-mapping Theorem (d) Uniform Bounded Theorem

Q. 21 If N is Normed Linear Space, and $0 \neq x_0 \in N$, then_____
(a) ∃ a functional f_0 in N^* such that $f_0(x_0) = \|x_0\|$ and $\|f_0\| = 0$.
(b)∀ functional f_0 in N^*, $f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$.
(c) ∃ a functional f_0 in N^* such that $f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$.
(d) ∀ functional f_0 in N^*, $f_0(x_0) \neq \|x_0\|$ and $\|f_0\| \neq 1$.

Q. 22 If M is a closed Linear Subspace of a Normed Linear Space N, and $x_0 \notin M$; then_____
(a) ∃ a functional f_0 in N^* such that $f_0(M) = 0$ and $f_0(x_0) = 0$.
(b) ∃ a functional f_0 in N^* such that $f_0(M) = 0$ and $f_0(x_0) \neq 0$.
(c) ∀functional f_0 in N^*, $f_0(M) = 0$ and $f_0(x_0) \neq 0$.
(d) ∀functional f_0 in N^*, $f_0(M) = 0$ and $f_0(x_0) = 0$.

Q. 23 For fixed $x \epsilon N$ (Normed Linear Space); if we defined $F_x: N^* \to R$ as $F_x(f) = f(x); \forall f \in N^*$, then which of the following is false option?
(a) F_x is Linear (b) F_x is not bounded (c) F_x is bounded (d) $F_x \in N^{**}$

Q. 24 Normed Linear Space N is said to be Reflexive if _____
(a) $N^{**} = N^*$ (b) $N \neq N^*$ (c) $N^* \neq N^{**}$ (d) $N = N^{**}$

Q. 25 Which of the following is correct option?
(a) $c_0^{**} = l_{\infty}$ (b) $l_1^* \neq l_{\infty}$ (c) $l_p^{**} \neq l_p$ (d) $c_0^{**} \neq l_{\infty}$

Q. 26 Which of the following is false option? $1 \leq p \leq \infty$
(a) $l_p^{**} = l_p$ (b) $\left(l_p^n\right)^{**} = l_p^n$ (c) $c_0^{**} \neq l_{\infty}$ (d) $l_1^* = l_{\infty}$

Q. 27 If (X, d_1) and (Y, d_2) are metric space, then T is open mapping if____
(a) ∀ open set O in $X, T(O)$ is open set in Y. (b) ∀ open set O in $Y, T^{-1}(O)$ is open set in X.
(c) ∃open set O in $X such that T(O)$ is open set in Y. (d)∃ open set O in Y such that $T^{-1}(O)$ is open set in X.

Q. 28 Consider the statement " If B and B' are Banach Spaces, and if $T: B \to B'$ is continuous onto Linear Transformation then T is an open mapping". This statement is related to_____
(a) Hahn-Banach Theorem (b) Closed Graph Theorem
(c) Open-mapping Theorem (d) Uniform Boundedness Theorem

Q. 29 If B and B' are Banach Spaces and if $T: B \to B'$ is continuous Bijective Linear Transformation, then_____
(a) T^{-1}does not exists. (b) T^{-1} is continuous. (c) T^{-1} is not continuous. (d) $T^{-1}$is not bounded.

Q. 30 If P is projection on a Banach Space B, and if M and N are its range and null-space of P, then_____
(a) M is open (b) M and N both are open (c) N is open (d) $M$and$N$ both are closed.

Q. 31 If B and B' are Banach Spaces, and if $T: B \to B'$ is Linear Transformation, then_____
(a) T is continuous ⇒ its graph is open. (b) Graph of T is open ⇒ T is continuous.
(c) T is not continuous ⇒ its graph is open. (d) Graph of T is closed ⇒ T is continuous.

Q. 32 Consider the statement: 'If B and B' are Banach Spaces, and if $T: B \to B'$ is Linear Transformation, then T is continuous ⇔ its graph is closed, This statement is related to_____
(a) Hahn Banach Theorem (b) Open-mapping Theorem
(c) Closed graph Theorem (d) Ascoli's Theorem

Q. 33 Consider the statement:" If B is Banach Space and N is a Normed Linear Space.If $\{T_i\}$ is non-empty set of continuous Linear Transformations of B into N with the property that $\{T_i(x)\}$ is a bounded subset of N, $\forall x \in B$; then $\{\|T_i\|\}$ is bounded set of numbers". This statement is related to_____
(a) Uniform Boundedness Theorem (b) Ascoli's Theorem
(c) Open mapping Theorem (d) Hahn-Banach Theorem

Q. 34 A non-empty subset X of a Normed Linear Space N is Bounded if and only if_____
(a) $f(X)$ is a bounded set of number for some $f \in N^*$. (b) $f(X)$ is a bounded set of number for all $f \in N^*$.
(c) $f(X)$ is unbounded set of numbers for some $f \in N^*$. (d) $f(X)$ is unbounded set of numbers for all $f \in N^*$.

Q. 35 If $T: N \to N$ is continuous Linear Transformation; where N is Normed Linear Space, and If we define $T^*: N^* \to N^*$ as $[T^*(f)](x) = f\big(T(x)\big); \forall f \in N^*, x \in N$, then_____
(a) $\|T^*\| \neq \|T\|$ (b) T^* is not continuous (c) $\|T\| = \|T^*\|$ (d) T^* is not bounded