Total Pages : 8

H-5323

M.A./M.Sc. (Semester-II) Examination, 2022

MATHEMATICS

(Topology-II)

Time Allowed : Three Hours

Maximum Marks : 70

Minimum Passing Marks : 25

Note : This question paper is divided into four sections. All sections are **compulsory**. Attempt questions as per instructions given in each section.

SECTION-A

(Objective Type Questions)

Note : Attempt **any ten** questions. Each question carries **1** mark. [1x10=10]

1. (A) Fill in the blanks :

(i) The closure of connected set is .....

(ii) Every components of a locally connected space is ..... .

(iii) A compact metric space is .....

(iv) The product space of two Hausdorff spaces is .....

(v) Every second countable normal space is .....

(B) Multiple Choice questions :

(vi) Every metrizable space is :

(a) First Countable

(b) Second Countable

(c) Metric Space

(d) Compact Space

(vii) If (X, J) is an indiscrete topological space then every net in X converge to :

(a) every point in X

(b) only one point in X

(c) a point to $x_0 \notin X$

(d) none of these

(viii) Any covering map $P : \overline{X} \to X$ is :

(a) Close

(b) Open

(c) Close-Open both are true

(d) None of these

(ix) The product space $X_1 \times X_2$ is connected iff both X_1 and X_2 are :

(a) Disconnected

(b) Connected

(c) Closed

(d) Opened

(x) $\mathbb{R}$ is :

(a) Locally compact

(b) Countably compact

(c) Sequentially compact

(d) Totally bounded

(xi) Let (X,Z) be a topological space, then :

(a) ext (X) = ϕ

(b) ext (X) = X

(c) ext (X) $\neq \phi$

(d) none of these

(xii) The product of any non-empty class of compact space is :

(a) Hausdroff

(b) Normal

(c) Compact

(d) None of these

SECTION-B

(Very Short Answer Type Questions)

Note : Attempt **any five** questions. Each question carries **02** marks. **(25-30 words)** [5x2=10]

2. (i) Define Homotopy of paths.

(ii) Define Filter with example.

(iii) The unit interval Q = [0, 1] is metrizable.

(iv) Define Tychonoff space with example.

(v) Define connected spaces.

(vi) Define covering space.

(vii) Every closed subspace of a compact space is compact.

SECTION-C

(Short Answer Type Questions)

Note : Attempt **any five** questions. Each question carries **04** marks. **(250 words)** : [5x4=20]

3. (i) A topological space X is locally connected iff the components of every open subspace of X are open in X.

(ii) Every filter F on set X is the intersection of all the ultrafilters finer than F.

(iii) Show that a space X is compact iff every filter on X has a cluster point.

(iv) State and prove that fundamental theorem of Algebra.

(v) Prove that a subset of R^n is closed and bounded iff it is compact.

(vi) Every Tychonoff space X can be embedded as a subspace of a cube.

(vii) A continuous image of a compact set is compact.

SECTION-D

(Essay Type Questions)

Note : Attempt **any three** questions. Each question carries **10** marks. **(more than 500 words)** [3x10=30]

4. (i) State and prove Urysohn's metrization theorem.

(ii) State and prove Smirnov metrization theorem.

(iii) State and prove Embedding Lemma.

(iv) The product space $X = \Pi(X_i : i \in I)$ is connected iff each co-ordinate space is connected.

----x----