

- **Sequence** : A function with the domain $\mathbb{N}$ or $\{1, 2, 3, \dots, n\}$ is called a sequence. i.e. $f: \mathbb{N} \rightarrow \mathbb{R}$ or $f: \{1, 2, 3, \dots, n\} \rightarrow \mathbb{R}$ is called a sequence.

$f: \mathbb{N} \rightarrow \mathbb{N}$ is a sequence of natural numbers, $f: \mathbb{N} \rightarrow \mathbb{Z}$ is a sequence of integers,

$f: \mathbb{N} \rightarrow \mathbb{R}$ is a sequence of real numbers, $f: \mathbb{N} \rightarrow \mathbb{Q}$ is a sequence of rational numbers.

$f(1), f(2), \dots, f(n) \dots$ are called the terms of the sequence.

$f(n)$ is denoted by a_n or t_n or u_n or T_n .

- **Series** : A series is the sum of first n terms of a sequence. i.e. if the terms of a sequence are $a_1, a_2, a_3, \dots, a_n$ then, $a_1 + a_2 + a_3 + \dots + a_n$ is the corresponding series. The sum of the first n terms is denoted by S_n .

The sequence corresponding to the given series is obtained by the formula $S_n - S_{n-1} = a_n$; $n \geq 2$ and $S_1 = a_1$.

- **Arithmetic Progression** : Sequence $f: \mathbb{N} \rightarrow \mathbb{R}$, $f(n) = an + b$; $a, b \in \mathbb{R}$; $a \neq 0$ is called an Arithmetic Progression (A.P.).

The difference between two consecutive terms in an arithmetic progression will be a non-zero constant. This common difference is denoted by d . If the first term of an arithmetic progression is a , its n th term will be $t_n = f(n) = a_n = a + (n-1)d$.

i.e., the terms of an arithmetic progression will be $a, a + d, a + 2d, a + 3d, \dots, a + (n-1)d, \dots$

Note : (1) If a, b, c are in A.P., then $2b = a + c$.

(2) In an A.P., if 3 or 5 or 7 or 9... terms are to be considered, then take the middle term as a , reduce by d to get the preceding term, add d to get the succeeding term etc. i.e. $\dots a - d, a, a + d$ or $\dots a - 2d, a - d, a, a + d, a + 2d \dots$ are in A.P.

(3) In an A.P., if 4 or 6 or 8... terms are to be considered, then there are two middle terms and keep the difference $2d$ between two consecutive terms. i.e. $\dots a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d, \dots$ etc. are in A.P.

- **Arithmetic Series** : The series corresponding to the arithmetic sequence is called arithmetic series. i.e. if the sum of first n terms of the A.P. $a, a + d, a + 2d, \dots, a + (n-1)d$ is taken, it is called the arithmetic series. It is denoted by S_n .

$$\therefore S_n = a + (a + d) + (a + 2d) + \dots + (a + (n-1)d)$$

$$\therefore S_n = \frac{n}{2} [2a + (n-1)d] \text{ is the sum of first } n \text{ terms of an A.P.}$$

If we take $l = a + (n-1)d$, then S_n can be written as $S_n = \frac{n}{2}(a+l)$.

- **Geometric Progression** : Sequence $f: \mathbb{N} \rightarrow \mathbb{R}$, $f(n) = ar^{n-1}$; $a, r \in \mathbb{R} - \{0\}$ is called a Geometric Progression (G.P.).

In a G.P., the ratio of two consecutive terms will be a non-zero constant and is denoted by r . If the first term of a G.P. is a and the common ratio is r , its n th term is $a_n = t_n = f(n) = a \cdot r^{n-1}$.

i.e. $a, ar, ar^2, \dots, ar^{n-1}$ are in G.P.

Note : (1) If a, b, c are in G.P., then $\frac{b}{a} = \frac{c}{b}$ i.e. $b^2 = ac$, $a, b, c \in \mathbb{R}^+$

(2) In a G.P., if 3 or 5 or 7,... terms are to be considered, then take the middle term as a , multiply by $\frac{1}{r}$ to get the preceding term and by r to get the succeeding term. i.e. ... $\frac{a}{r^2}$,

$\frac{a}{r}$, a , ar , ar^2 ... are in G.P. For two, four, six,...terms ... $\frac{a}{r^3}$, $\frac{a}{r}$, ar , ar^3 ... are in G.P.

- **Geometric Series** : The series corresponding to the geometric progression is called the geometric series. i.e. if the sum of first n terms of a G.P. a , ar , ar^2 ... is taken, it is called the geometric series. It is denoted by S_n .

Here, $S_n = \frac{a(r^n - 1)}{r - 1}$, $r > 1$ and $S_n = \frac{a(1 - r^n)}{1 - r}$, $r < 1$ and $S_n = n \cdot a$, $r = 1$

- **Harmonic Progression** : If $a_1, a_2, a_3, \dots, a_n$ are n non-zero terms of an A.P., then the terms $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \dots, \frac{1}{a_n}$ are in Harmonic Progression (H.P.). So, if the n th term of an A.P. is

$t_n = a + (n-1)d$, the n th term of the corresponding H.P. is $\frac{1}{a + (n-1)d}$, where $(a + (n-1)d \neq 0)$.

- **Arithmetic Mean** : If the numbers a , A and b are in A.P., then A is called the arithmetic mean of a and b . We have $A = \frac{a+b}{2}$

- **n Arithmetic Means** : If $a, A_1, A_2, A_3, \dots, A_n, b$ are in A.P., then $A_1, A_2, A_3, \dots, A_n$ are called n arithmetic means between a and b .

$A_1 = a + d$, $A_2 = a + 2d$, $A_3 = a + 3d$, ..., $A_n = a + nd$ and $d = \frac{b-a}{n+1}$

- **Geometric Mean** : If a, G, b are in G.P., then G is called the geometric mean of positive numbers a and b . $G = \sqrt{ab}$

- **n Geometric Means** : If $a, G_1, G_2, G_3, \dots, G_n, b$ are in G.P., then $G_1, G_2, G_3, \dots, G_n$ are called n geometric means between a and b . $a, b \in \mathbb{R}^+$

Here, $G_1 = ar$, $G_2 = ar^2$, $G_3 = ar^3$, ..., $G_n = ar^n$ and $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

- **Harmonic Mean** : If a, H, b are in H.P., then H is called the harmonic mean of a and b .

In this case, $\frac{1}{a}, \frac{1}{H}, \frac{1}{b}$ are in A.P.. Also $H = \frac{2ab}{a+b}$.

- **n Harmonic Means** : If $a, H_1, H_2, H_3, \dots, H_n, b$ are in H.P., then $H_1, H_2, H_3, \dots, H_n$ are called n harmonic means between a and b .

$\therefore \frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \dots, \frac{1}{H_n}, \frac{1}{b}$ will be in A.P.. Also, $d = \frac{a-b}{(n+1)ab}$

and $\frac{1}{H_1} = \frac{1}{a} + d$, $\frac{1}{H_2} = \frac{1}{a} + 2d$, $\frac{1}{H_3} = \frac{1}{a} + 3d$, ..., $\frac{1}{H_n} = \frac{1}{a} + nd$

Note : For two numbers a and b , if the A.M., G.M. and H.M. are A , G and H respectively, then $AH = G^2$.

$\therefore A, G, H$ are in geometric progression and $A > G > H$.

- **Arithmetico - Geometric Series :** The series $a + (a + d)r + (a + 2d)r^2 + (a + 3d)r^3 + \dots$ is called arithmetico-geometric series, where $t_n = [a + (n - 1)d]r^{n-1}$.

$$S_n = \frac{a}{1-r} + \frac{d \cdot r(1-r^{n-1})}{(1-r)^2} - \frac{[a + (n-1)d]r^n}{(1-r)}$$

When $n \rightarrow \infty$, then $r^n \rightarrow 0$, if $|r| < 1$. So, $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$.

- **Power Series :** Notation Σ is used to denote a sum like some special series, the sum of first n natural numbers, their squares, their cubes etc.

Important results of Σ :

$$(1) \quad \sum_{i=1}^n (a_i \pm b_i) = \sum_{i=1}^n a_i \pm \sum_{i=1}^n b_i$$

$$(2) \quad \sum_{i=1}^n ka_i = k \sum_{i=1}^n a_i$$

$$(3) \quad \sum_{i=1}^n 1 = (1 + 1 + 1 + \dots n \text{ times}) = n = \Sigma 1$$

$$(4) \quad \sum_{i=1}^n i = \Sigma n = \frac{n(n+1)}{2}$$

$$(5) \quad \sum_{i=1}^n i^2 = \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$(6) \quad \sum_{i=1}^n i^3 = \Sigma n^3 = \frac{n^2(n+1)^2}{4}$$

Note : (i) $\sum_{i=1}^n (a_i b_i) \neq \sum_{i=1}^n a_i \cdot \sum_{i=1}^n b_i ; \forall i \in \mathbb{N}$

$$(ii) \quad \sum_{i=1}^n \left(\frac{a_i}{b_i} \right) \neq \frac{\sum_{i=1}^n a_i}{\sum_{i=1}^n b_i} ; \forall i \in \mathbb{N}$$

: Multiple Choice Questions :

(1) If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P., then

(A) a^2, b^2, c^2 are in A.P.

(B) a, b, c are in A.P.

(C) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

(D) $\frac{1}{a^2}, \frac{1}{b^2}, \frac{1}{c^2}$ are in A.P.

Sol. : Here $\frac{2}{c+a} = \frac{1}{b+c} + \frac{1}{a+b} = \frac{a+b+b+c}{(a+b)(b+c)}$

$$\therefore \frac{2}{c+a} = \frac{a+c+2b}{ab+ac+b^2+bc}$$

$$\therefore 2ab + 2ac + 2b^2 + 2bc = ac + a^2 + c^2 + ac + 2bc + 2ab$$

$$\therefore 2b^2 = a^2 + c^2$$

$$\therefore a^2, b^2, c^2 \text{ are in A.P.}$$

Ans. : (A)

- (2) If the n th terms of two series $3 + 10 + 17 + \dots$ and $63 + 65 + 67 + \dots$ are equal, then $n = \dots\dots$
 (A) 9 (B) 11 (C) 13 (D) 19

Sol. : Here, 3, 10, 17,... and 63, 65, 67, ... are arithmetic progressions.

For the first A.P. 3, 10, 17,..., $a = 3$, $d = 7$

$$t_n = 3 + (n - 1)7 = 7n - 4$$

For the second A.P., 63, 65, 67,..., $a = 63$, $d = 2$

$$\therefore T_n = 63 + (n - 1)2 = 2n + 61$$

$$\text{Given, } t_n = T_n. \quad \text{So, } 7n - 4 = 2n + 61$$

$$\therefore 5n = 65$$

$$\therefore n = 13$$

Ans. : (C)

- (3) If $\log 2$, $\log (2^x - 1)$ and $\log (2^x + 3)$ are in A.P., then $x = \dots\dots$

- (A) $\frac{2}{5}$ (B) $\log_5 2$ (C) $\log_2 5$ (D) $\log_3 5$

Sol. : $2 \log (2^x - 1) = \log 2 + \log (2^x + 3)$

$$\therefore \log (2^x - 1)^2 = \log (2 \cdot (2^x + 3))$$

$$\therefore (2^x - 1)^2 = 2 \cdot (2^x + 3)$$

$$\therefore (2^x)^2 - 2 \cdot 2^x + 1 = 2 \cdot 2^x + 6$$

$$\therefore (2^x)^2 - 4 \cdot 2^x - 5 = 0$$

$$\therefore (2^x + 1) (2^x - 5) = 0$$

$$\therefore 2^x = -1 \text{ or } 2^x = 5$$

But $2^x = -1$ is not possible. So, $2^x = 5$

$$\therefore x = \log_2 5$$

Ans. : (C)

- (4) If the interior angles of a convex polygon are in A.P. with the smallest angle measuring 120 (in degrees) and the common difference between measures of angles being 5, then the polygon will have sides.

- (A) 9, 16 (B) 16 (C) 10 (D) 9

Sol. : Here, the smallest measure of an angle is 120. So, $a = 120$ and $d = 5$

Let the polygon have n sides.

Then, sum of its interior angles will be $(2n - 4) \times 90$.

$$\therefore \frac{n}{2} [2 (120) + (n - 1)5] = (2n - 4) 90. \quad \text{So, } \frac{n}{2} (240 + 5n - 5) = (n - 2) 180$$

$$\therefore n (235 + 5n) = (n - 2) 360. \quad \text{So, } 5n^2 - 125n + 720 = 0$$

$$\therefore n^2 - 25n + 144 = 0$$

$$\therefore (n - 16) (n - 9) = 0$$

$$\therefore n = 16 \text{ or } n = 9$$

If $n = 16$, $t_{16} = a + 15d = 120 + 75 = 195$, which is not possible. So, $n = 9$ **Ans. : (D)**

(5) If $5^{1+x} + 5^{1-x}; \frac{a}{2}; 25^x + 25^{-x}$ are in A.P., the minimum value of a is

- (A) 5 (B) 7 (C) 10 (D) 12

Sol. : Here $2\left(\frac{a}{2}\right) = 5^{1+x} + 5^{1-x} + 25^x + 25^{-x} = 5 \cdot 5^x + \frac{5}{5^x} + (5^2)^x + (5^2)^{-x}$

$$= 5 \cdot 5^x + \frac{5}{5^x} + (5^x)^2 + (5^x)^{-2}$$

Let, $5^x = m$

$$\therefore a = 5m + \frac{5}{m} + m^2 + \frac{1}{m^2} = 5\left(m + \frac{1}{m}\right) + \left(m^2 + \frac{1}{m^2}\right)$$

$$= 5\left(\sqrt{m} - \frac{1}{\sqrt{m}}\right)^2 + \left(m - \frac{1}{m}\right)^2 + 12 \geq 12$$

$\therefore$ The minimum value of a is 12. (It corresponds to $x = 0$)

Ans. : (D)

(6) If $a_1, a_2, a_3, \dots$ are in A.P., and $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 240$, then the sum of the first 24 terms will be

- (A) 950 (B) 860 (C) 900 (D) 960

Sol. : In an A.P., the sum of terms in equidistant positions from the beginning and the end will be the same as the sum of first and last terms.

Here $a_1, a_2, a_3, \dots, a_{24}$ are the first 24 terms of A.P.

And $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 240$ is given.

$$\therefore (a_1 + a_{24}) + (a_5 + a_{20}) + (a_{10} + a_{15}) = 240$$

$$\therefore (a_1 + a_{24}) + (a_1 + a_{24}) + (a_1 + a_{24}) = 240$$

$$\therefore 3(a_1 + a_{24}) = 240$$

$$\therefore a_1 + a_{24} = 80$$

$$\text{Now, } S_{24} = \frac{24}{2} [a_1 + a_{24}] = 12(80) = 960$$

Second method :

$$a + a + 4d + a + 9d + a + 14d + a$$

$$+ 19d + a + 23d = 240$$

$$\therefore 6a + 69d = 240$$

$$\therefore 2a + 23d = 80$$

$$\therefore S_{24} = \frac{24}{2}(2a + 23d)$$

$$= (12)(80) = 960$$

Ans. : (D)

(7) If $\langle a_n \rangle$ is an A.P., then $D = \begin{vmatrix} a_p & a_q & a_r \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = \dots$

- (A) 0 (B) -1 (C) 1 (D) pqr

Sol. : Here $\langle a_n \rangle$ is an A.P.

$$\therefore a_p = a + (p-1)d, a_q = a + (q-1)d \text{ and } a_r = a + (r-1)d$$

$$\therefore D = \begin{vmatrix} a+(p-1)d & a+(q-1)d & a+(r-1)d \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a & a & a \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} \quad R_{31}(d), R_{21}(-d)$$

$$= a \begin{vmatrix} 1 & 1 & 1 \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = 0 \quad \text{Ans. : (A)}$$

(8) If $\frac{a+b}{1-ab}$, b , $\frac{b+c}{1-bc}$ are an A.P., then $\frac{1}{a}$, b , $\frac{1}{c}$ will be in sequence.

(A) Arithmetic (B) Geometric (C) Harmonic (D) Arithmetico-Geometric

Sol. : Here $\frac{a+b}{1-ab}$, b , $\frac{b+c}{1-bc}$ are in A.P.

$$\therefore b - \left(\frac{a+b}{1-ab} \right) = \frac{b+c}{1-bc} - b$$

$$\therefore \frac{b-ab^2-a-b}{1-ab} = \frac{b+c-b+b^2c}{1-bc}$$

$$\therefore \frac{-ab^2-a}{1-ab} = \frac{c+b^2c}{1-bc}$$

$$\therefore \frac{-a(b^2+1)}{1-ab} = \frac{c(1+b^2)}{1-bc}$$

$$\therefore \frac{-a}{1-ab} = \frac{c}{1-bc}$$

$$\therefore -a + abc = c - abc$$

$$\therefore 2abc = a + c$$

$$\therefore 2b = \frac{a}{ac} + \frac{c}{ac} = \frac{1}{c} + \frac{1}{a}$$

$$\therefore \frac{1}{a}, b, \frac{1}{c} \text{ are in A.P.} \quad \text{Ans. : (A)}$$

(9) If $\log(a+c) + \log(a-2b+c) = 2 \log(a-c)$, then $\frac{1}{a}$, $\frac{1}{b}$, $\frac{1}{c}$ are in progression.

(A) Harmonic (B) Geometric (C) Arithmetic (D) Arithmetico-Geometric

Sol. : $\log(a+c) + \log(a-2b+c) = 2 \log(a-c)$

$$\therefore \log \{(a+c)(a-2b+c)\} = \log(a-c)^2$$

$$\therefore (a^2 - 2ab + ac + ac - 2bc + c^2) = (a^2 - 2ac + c^2)$$

$$\therefore (a^2 - 2ab + 2ac - 2bc + c^2) = (a^2 - 2ac + c^2)$$

$$\therefore 4ac = 2ab + 2bc.$$

$$\text{So, } 2ac = ab + bc$$

$$\therefore \frac{2}{b} = \frac{1}{c} + \frac{1}{a}$$

$$\therefore \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \text{ are in A.P.}$$

Ans. : (C)

(10) If $a_1, a_2, a_3, \dots, a_n$ are in A.P. and $a_i > 0, \forall i$, then

$$\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} = \dots\dots$$

$$(A) \frac{1}{\sqrt{a_1} + \sqrt{a_n}}$$

$$(B) \frac{n}{\sqrt{a_1} + \sqrt{a_n}}$$

$$(C) \frac{n+1}{\sqrt{a_1} + \sqrt{a_n}}$$

$$(D) \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}}$$

Sol. : Let the difference between two consecutive terms of the A.P. be d .

$$\text{Now, } \frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}}$$

$$= \frac{\sqrt{a_1} - \sqrt{a_2}}{a_1 - a_2} + \frac{\sqrt{a_2} - \sqrt{a_3}}{a_2 - a_3} + \dots + \frac{\sqrt{a_{n-1}} - \sqrt{a_n}}{a_{n-1} - a_n}$$

$$= \frac{\sqrt{a_1} - \sqrt{a_2}}{-d} + \frac{\sqrt{a_2} - \sqrt{a_3}}{-d} + \dots + \frac{\sqrt{a_{n-1}} - \sqrt{a_n}}{-d}$$

$$= -\frac{1}{d} [\sqrt{a_1} - \sqrt{a_2} + \sqrt{a_2} - \sqrt{a_3} + \dots + \sqrt{a_{n-1}} - \sqrt{a_n}]$$

$$= -\frac{1}{d} [\sqrt{a_1} - \sqrt{a_n}] = -\frac{1}{d} \left(\frac{a_1 - a_n}{\sqrt{a_1} + \sqrt{a_n}} \right)$$

$$\text{Since } (a_n - a_1) = (n-1)d, \text{ as } a_n = a_1 + (n-1)d, -\frac{1}{d} \left(\frac{-(n-1)d}{\sqrt{a_1} + \sqrt{a_n}} \right) = \frac{(n-1)}{\sqrt{a_1} + \sqrt{a_n}}$$

Ans. : (D)

(11) In an A.P., if the first term is m , the last term is p and the sum of the first n terms is S , then its common difference is

$$(A) \frac{p+m}{2S}$$

$$(B) \frac{m^2 - p^2}{2S - m + p}$$

$$(C) \frac{p^2 - m^2}{2S - (m+p)}$$

$$(D) \frac{p^2 + m^2}{2S + (m+p)}$$

Sol. : Here $a = m$ and $p = m + (n-1)d$ and $S_n = S = \frac{n}{2} (m + p)$.

$$\therefore \text{From } p = m + (n-1)d, \text{ we get } d = \frac{p-m}{n-1} \text{ and } n = \frac{2S}{m+p}.$$

$$\therefore d = \frac{p-m}{\frac{2S}{m+p} - 1} = \frac{(p-m)(m+p)}{2S - m - p} = \frac{p^2 - m^2}{2S - (m+p)}$$

Ans. : (C)

(12) If $a_1, a_2, a_3, \dots, a_n$ are n terms of an A.P., then

$$a_1^2 - a_2^2 + a_3^2 - a_4^2 + \dots + a_{2n-1}^2 - a_{2n}^2 = \dots$$

(A) $\frac{n}{(2n-1)} (a_1^2 - a_{2n}^2)$

(B) $\frac{1}{(2n-1)} (a_1^2 + a_{2n}^2)$

(C) $\frac{1}{(2n-1)} (a_1^2 - a_{2n}^2)$

(D) $\frac{2n}{2n+1} (a_1^2 - a_{2n}^2)$

Sol. : Let the common difference of the A.P. be d .

$$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \dots = a_n - a_{n-1} = d$$

$$\text{Now } (a_1^2 - a_2^2) + (a_3^2 - a_4^2) + \dots + (a_{2n-1}^2 - a_{2n}^2)$$

$$= (a_1 - a_2)(a_1 + a_2) + (a_3 - a_4)(a_3 + a_4) + \dots + (a_{2n-1} - a_{2n})(a_{2n-1} + a_{2n})$$

$$= -d[a_1 + a_2 + a_3 + a_4 + \dots + a_{2n-1} + a_{2n}]$$

$$= -nd[a_1 + a_{2n}] \quad (S_{2n} = \frac{1}{2}(2n)(a_1 + a_{2n}))$$

$$= -nd \frac{(a_1 + a_{2n})(a_1 - a_{2n})}{(a_1 - a_{2n})}$$

$$= -nd \frac{(a_1^2 - a_{2n}^2)}{(a_1 - a_{2n})} = \frac{nd(a_1^2 - a_{2n}^2)}{(a_{2n} - a_1)}$$

$$\begin{aligned} \therefore (a_1^2 - a_2^2) + (a_3^2 - a_4^2) + \dots + (a_{2n-1}^2 - a_{2n}^2) &= \frac{nd(a_1^2 - a_{2n}^2)}{(2n-1)d} \quad (a_{2n} = a_1 + (2n-1)d) \\ &= \frac{n(a_1^2 - a_{2n}^2)}{(2n-1)} \end{aligned}$$

Ans. : (A)

(13) If p, q, r, s, t form an A.P., then $p - 4q + 6r - 4s + t = \dots$

(A) 4

(B) 0

(C) 3

(D) 2

Sol. : Since p, q, r, s, t are in A.P., they are $p, p + d, p + 2d, p + 3d, p + 4d$ respectively.

$$\text{Here, } q - p = r - q = s - r = t - s = d$$

$$\therefore p - 4q + 6r - 4s + t = p - 4(p + d) + 6(p + 2d) - 4(p + 3d) + p + 4d$$

$$= p \cdot 0 + d \cdot 0 = 0$$

Ans. : (B)

(14) If the sum of the first n terms of an A.P. is n^2 , then the difference between two consecutive terms of that A.P. is

(A) 1

(B) 2

(C) 4

(D) 6

Sol. : Here $S_n = n^2$. So, $S_{n-1} = (n-1)^2$

$$\therefore a_n = S_n - S_{n-1} = n^2 - (n-1)^2 = 2n - 1. \quad n \geq 2 \text{ and } S_1 = a_1 = 1$$

$$\therefore \text{The A.P. is } 1, 3, 5, 7, \dots$$

$$\therefore \text{Common difference } d = 2$$

Ans. : (B)

- (15) If the equation $(x + 1) + (x + 4) + (x + 7) + \dots + (x + 34) = 216$ represents the sum of the first n terms of an A.P., then $x = \dots$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2

Sol. : Here $(x + 1) + (x + 4) + (x + 7) + \dots + (x + 34) = 216$ and $(x + 34)$ is the last term.

$$\text{From } l = a + (n - 1)d, x + 34 = (x + 1) + (n - 1)3.$$

$$\text{So, } 33 = (n - 1)3$$

$$\therefore n - 1 = 11 \quad \text{So, } n = 12.$$

Sum of the first 12 terms of the A.P. is 216.

$$\therefore S_{12} = \frac{12}{2} [(x + 1) + (x + 34)] = 216$$

$$\therefore 6(2x + 35) = 216$$

$$\therefore 2x + 35 = 36. \quad \text{So, } 2x = 1. \quad \text{So, } x = \frac{1}{2} \quad \text{Ans. : (B)}$$

- (16) If S_n is the sum of the first n terms of an A.P. and $S_{3n} = 4S_n$, then $\frac{S_{4n}}{S_n} = \dots$

(A) 2 (B) 4 (C) 6 (D) 8

Sol. : Here $S_{3n} = 4S_n$.

$$\therefore \frac{3n}{2} [2a + (3n - 1)d] = 4 \cdot \frac{n}{2} [2a + (n - 1)d]$$

$$\therefore 3 [2a + (3n - 1)d] = 4 \cdot [2a + (n - 1)d]$$

$$\therefore 6a + (9n - 3)d = 8a + (4n - 4)d$$

$$\therefore 2a = (5n + 1)d \quad (1)$$

$$\text{Now, } \frac{S_{4n}}{S_n} = \frac{\frac{4n}{2} [2a + (4n - 1)d]}{\frac{n}{2} [2a + (n - 1)d]} = \frac{4[2a + (4n - 1)d]}{[2a + (n - 1)d]}$$

$$= \frac{4[5nd + d + 4nd - d]}{5nd + d + nd - d} \quad (\text{From (1)})$$

$$= 4 \left(\frac{9nd}{6nd} \right) = 6 \quad \text{Ans. : (C)}$$

- (17) The difference between two consecutive roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$ is

(A) 1 (B) 2 (C) 3 (D) 4

Sol. : For the equation $x^3 - 6x^2 + 11x - 6 = 0$,

$$\therefore (x - 1)(x^2 - 5x + 6) = 0$$

$$\therefore (x - 1)(x - 2)(x - 3) = 0$$

$$\therefore x = 1, 2 \text{ or } 3.$$

So, the difference between the consecutive roots is 1.

Ans. : (A)

(18) If $\log_{\sqrt{2}} x + \log_{\sqrt[4]{2}} x + \log_{\sqrt[6]{2}} x + \dots + \log_{\sqrt[32]{2}} x = 136$, then $x = \dots$.

- (A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) $\frac{1}{2}$ (D) 2

Sol. : $\log_{\sqrt{2}} x + \log_{\sqrt[4]{2}} x + \log_{\sqrt[6]{2}} x + \dots + \log_{\sqrt[32]{2}} x = 136$

$$\therefore \frac{\log x}{\log \sqrt{2}} + \frac{\log x}{\log \sqrt[4]{2}} + \frac{\log x}{\log \sqrt[6]{2}} + \dots + \frac{\log x}{\log \sqrt[32]{2}} = 136$$

$$\therefore \frac{\log x}{\frac{1}{2} \log 2} + \frac{\log x}{\frac{1}{4} \log 2} + \frac{\log x}{\frac{1}{6} \log 2} + \dots + \frac{\log x}{\frac{1}{32} \log 2} = 136$$

$$\therefore \frac{2 \log x}{\log 2} + \frac{4 \log x}{\log 2} + \frac{6 \log x}{\log 2} + \dots + \frac{32 \log x}{\log 2} = 136$$

$$\therefore (2 + 4 + 6 + 8 + \dots + 32) \log_2 x = 136$$

$$\therefore \frac{16}{2} (2+32) \log_2 x = 136$$

$$\therefore 8(34) \log_2 x = 136$$

$$\therefore \log_2 x = \frac{136}{34 \times 8} = \frac{1}{2}$$

$$\therefore x = 2^{\frac{1}{2}} = \sqrt{2}$$

Ans. : (B)

(19) If the sixth term of an A.P. is 2 and the product of its first, fourth and fifth terms is minimum, then the difference between its consecutive terms is

- (A) $\frac{8}{5}$ (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{5}{8}$

Sol. : Suppose the common difference is d and the first term is a .

$$\text{Let, } t_6 = 2. \quad \text{So, } a + 5d = 2 \quad (1)$$

$$t_4 = a + 3d \quad \text{and} \quad t_5 = a + 4d,$$

$$\text{Let } t_1 \cdot t_4 \cdot t_5 = a(a + 3d)(a + 4d) = F(d),$$

$$F(d) = (2 - 5d)(2 - 5d + 3d)(2 - 5d + 4d) \quad (\text{From (1)})$$

$$= (2 - 5d)(2 - 2d)(2 - d)$$

$$= (2 - 5d)(4 - 6d + 2d^2)$$

$$= -2(5d^3 - 17d^2 + 16d - 4)$$

For minimum value, $F'(d) = 0$

$$\therefore -2(15d^2 - 34d + 16) = 0$$

$$\therefore -2(5d - 8)(3d - 2) = 0$$

$$\therefore d = \frac{8}{5} \text{ or } d = \frac{2}{3}$$

$$\text{Now, } F''(d) = -2(30d - 34)$$

$$\text{Taking } d = \frac{8}{5}, F''(d) = -2(30d - 34) = -28 < 0$$

$\therefore F(d)$ is local maximum.

$$\therefore \text{Taking } d = \frac{2}{3}, F''(d) = 28 > 0. \quad \text{So, } F(d) \text{ is local minimum for } d = \frac{2}{3}. \quad \text{Ans. : (B)}$$

(20) If $\frac{a-x}{px} = \frac{a-y}{qy} = \frac{a-z}{rz}$ and p, q, r are in A.P., then x, y, z will be in sequence.

- (A) Geometric (B) Arithmetico-Geometric (C) Arithmetic (D) Harmonic

Sol. : Here, p, q, r are in A.P.

$$\therefore 2q = p + r \quad (1)$$

$$\text{Let, } \frac{a-x}{px} = \frac{a-y}{qy} = \frac{a-z}{rz} = k,$$

$$\frac{a-x}{kx} = p, \frac{a-y}{ky} = q, \frac{a-z}{kz} = r.$$

$$\therefore \text{ By (1), } 2\left(\frac{a-y}{ky}\right) = \left(\frac{a-x}{kx}\right) + \left(\frac{a-z}{kz}\right). \quad \text{So, } 2\left(\frac{a-y}{y}\right) = \left(\frac{a-x}{x}\right) + \left(\frac{a-z}{z}\right)$$

$$\therefore \frac{2a-2y}{y} = \frac{az - zx + ax - zx}{zx} \quad \text{So, } \frac{2a}{y} = \frac{a}{x} + \frac{a}{z}$$

$$\therefore \frac{2}{y} = \frac{1}{x} + \frac{1}{z}.$$

So, x, y, z are in H.P.

Ans. : (D)

(21) In an increasing A.P., the sum of three consecutive terms is 51 and the product of first and third terms is 273. Its third term is

- (A) 9 (B) 13 (C) 17 (D) 21

Sol. : Let the three consecutive terms of the A.P. be $a-d, a, a+d$. Their sum is 51.

$$\therefore 3a = 51. \quad \text{So, } a = 17$$

$$\text{Also } (a-d)(a+d) = 273$$

$$\therefore a^2 - d^2 = 273$$

$$\therefore 289 - 273 = d^2. \quad \text{So, } d^2 = 16. \quad \text{So, } d = \pm 4.$$

$d = 4$ for the A.P. to be increasing. $a-d = 13, a = 17$. So, required term $a+d = 21$. **Ans. : (D)**

(22) If $x^2(y+z), y^2(z+x), z^2(x+y)$ are in A.P., then x, y, z are in progression. ($xy + yz + zx \neq 0$)

- (A) Harmonic (B) Arithmetic (C) Geometric (D) Arithmetico-Geometric

Sol. : $2y^2(z+x) = z^2(x+y) + x^2(y+z)$

$$\therefore z^2x + z^2y + x^2y + x^2z - 2y^2z - 2y^2x = 0$$

$$\therefore xz(x+z-2y) + y(z^2+x^2+2xz-4y^2) - 2y^2(z+x-2y) = 0$$

$$\therefore xz(x+z-2y) + y(z+x-2y)(z+x+2y) - 2y^2(z+x-2y) = 0$$

$$\therefore (z+x-2y)(zx+yz+xy+2y^2-2y^2) = 0$$

$$\therefore z+x = 2y \text{ or } xy + yz + zx = 0$$

$$\text{But } xy + yz + zx \neq 0. \quad \text{So, } z+x = 2y$$

$\therefore x, y, z$ are in A.P.

Ans. : (B)

(23) For an A.P. $\{a_n\}$, $a_1^2 - a_2^2 + a_3^2 - a_4^2 + \dots + a_{99}^2 - a_{100}^2 = \dots$

- (A) $\frac{50}{99} (a_1^2 - a_{100}^2)$ (B) $\frac{100}{99} (a_{100}^2 - a_1^2)$ (C) $\frac{50}{51} (a_1^2 - a_{100}^2)$ (D) $\frac{51}{50} (a_1^2 - a_{50}^2)$

Sol. : For the A.P., $a_1, a_2, a_3, \dots, a_{100}$,

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \dots = a_{100} - a_{99} = d \quad (1)$$

$$\begin{aligned} \therefore (a_1^2 - a_2^2) + (a_3^2 - a_4^2) + \dots + (a_{99}^2 - a_{100}^2) \\ = (a_1 - a_2)(a_1 + a_2) + (a_3 - a_4)(a_3 + a_4) + \dots + (a_{99} - a_{100})(a_{99} + a_{100}) \\ = (-d)(a_1 + a_2) + (-d)(a_3 + a_4) + \dots + (-d)(a_{99} + a_{100}) \\ = -d(a_1 + a_2 + a_3 + \dots + a_{100}) \\ = -d \cdot \frac{100}{2} [a_1 + a_{100}] \\ = -50d \frac{(a_1 + a_{100})(a_1 - a_{100})}{(a_1 - a_{100})} \end{aligned}$$

$$= \frac{-50d(a_1^2 - a_{100}^2)}{a_1 - (a_1 + 99d)} = \frac{-50d(a_1^2 - a_{100}^2)}{-99d} = \frac{50}{99} (a_1^2 - a_{100}^2) \quad \text{Ans. : (A)}$$

(24) If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of reciprocals of their squares, then bc^2, ca^2, ab^2 are in sequence.

(A) Arithmetic (B) Geometric (C) Harmonic (D) Arithmetico-Geometric

Sol. : Suppose the roots of $ax^2 + bx + c = 0$ are α and β .

$$\text{Here } \alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}.$$

$$\therefore \alpha + \beta = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} \quad (1)$$

$$\text{Since } \alpha + \beta = \frac{-b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

$$\text{From (1), } \frac{-b}{a} = \frac{\frac{b^2}{a^2} - 2\left(\frac{c}{a}\right)}{\frac{c^2}{a^2}} = \frac{b^2 - 2ac}{c^2}$$

$$\therefore -bc^2 = ab^2 - 2a^2c$$

$$\therefore 2a^2c = ab^2 + bc^2$$

$$\therefore bc^2, ca^2, ab^2 \text{ are in A.P.}$$

Ans. : (A)

(25) If $a_1, a_2, \dots, a_{10}$ are in A.P. and $\frac{1}{h_1}, \frac{1}{h_2}, \dots, \frac{1}{h_{10}}$ are in A.P. and $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$,

then $a_4 h_7 = \dots$

(A) $\frac{1}{6}$

(B) 6

(C) 3

(D) 2

Sol. : $a_1, a_2, a_3, \dots, a_{10}$ are in A.P. $a_1 = 2, a_{10} = 3$.

$$\therefore a_{10} = a_1 + 9d.$$

$$\text{So, } 3 = 2 + 9d.$$

$$\text{Hence, } d = \frac{1}{9}$$

$$\therefore a_4 = a_1 + 3d = 2 + \frac{3}{9} = 2 + \frac{1}{3} = \frac{7}{3}$$

Now, $\frac{1}{h_1}, \frac{1}{h_2}, \dots, \frac{1}{h_{10}}$ are in A.P., where $h_1 = 2$ and $h_{10} = 3$.

$$\therefore \frac{1}{h_{10}} = \frac{1}{h_1} + 9D. \quad \text{So, } 9D = \frac{1}{3} - \frac{1}{2} = \frac{-1}{6}. \quad \text{So, } D = \frac{-1}{54}$$

$$\therefore \frac{1}{h_7} = \frac{1}{h_1} + 6D = \frac{1}{2} - \frac{6}{54} = \frac{1}{2} - \frac{1}{9} = \frac{7}{18}. \quad \text{So, } h_7 = \frac{18}{7}$$

$$\therefore a_4 \cdot h_7 = \frac{7}{3} \cdot \frac{18}{7} = 6$$

Ans. : (B)

$$(26) \quad 3 \cdot \overline{537} = \dots\dots$$

$$(A) \quad \frac{333}{1178}$$

$$(B) \quad \frac{1187}{333}$$

$$(C) \quad \frac{1178}{333}$$

$$(D) \quad \frac{1871}{333}$$

$$\text{Sol. : } 3 \cdot \overline{537} = 3.537537537\dots$$

$$= 3 + (0.537 + 0.000537 + 0.000000537 + \dots)$$

$$= 3 + \left[\frac{0.537}{1-0.001} \right] \quad (a = 0.537, r = 0.001)$$

$$= 3 + \left(\frac{537}{999} \right) = 3 + \left(\frac{179}{333} \right) = \frac{999+179}{333} = \frac{1178}{333}$$

$$\text{Note : } 3 + \left(\frac{537}{999} \right) = \text{non-recurring part} + \frac{\text{recurring part}}{\text{as many nines as number of digits in recurring part}}$$

Ans. : (C)

(27) In a G.P., the sum of three consecutive terms is 38. If 1 is added to the first and second term and 1 is subtracted from the third term, the resulting numbers are in A.P., then the smallest of the three numbers is

$$(A) \quad 6$$

$$(B) \quad 8$$

$$(C) \quad 12$$

$$(D) \quad 18$$

Sol. : Suppose the three consecutive terms of the G.P. are a, ar, ar^2 .

$$\therefore a + ar + ar^2 = 38 \quad (1)$$

Now if 1 is added in first and second term and 1 is subtracted from its third term, then the numbers $a + 1, ar + 1$ and $ar^2 - 1$ are in A.P.

$$\therefore 2(ar + 1) = a + 1 + ar^2 - 1$$

$$\therefore 2ar + 2 = 38 - ar \quad (\text{From (1)})$$

$$\therefore 3ar = 36. \quad \text{So, } ar = 12 \quad (2)$$

$$\text{From (1), } a + 12 + 12r = 38. \quad \text{So, } a + 12r = 26$$

$$\therefore a + 12\left(\frac{12}{a}\right) = 26 \quad \left(r = \frac{12}{a}\right)$$

$$\therefore a^2 - 26a + 144 = 0. \quad \text{Hence, } (a - 18)(a - 8) = 0$$

$$\therefore a = 8 \text{ or } a = 18. \quad \text{So, the smallest number is 8. (numbers are 8, 12, 16)} \quad \text{Ans. : (B)}$$

- (28) If a, b, c are in G.P. and one root of $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ is common, then $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in sequence. ($a, b, c \in R^+$)

(A) Arithmetic (B) Geometric (C) Harmonic (D) Arithmetico-Geometric

Sol. : Since a, b, c are in G.P. $b^2 = ac$

Now, $ax^2 + 2bx + c = 0$

$$\therefore ax^2 + 2\sqrt{ac}x + c = 0 \quad (b^2 = ac)$$

$$\therefore (\sqrt{a}x + \sqrt{c})^2 = 0$$

$$\therefore \text{Only one root} = -\sqrt{\frac{c}{a}}$$

So, $x = -\sqrt{\frac{c}{a}}$ is a root of $dx^2 + 2ex + f = 0$.

$$\therefore d\left(\frac{c}{a}\right) - 2e\sqrt{\frac{c}{a}} + f = 0$$

$$\therefore \frac{dc}{a} + f = 2e\sqrt{\frac{c}{a}}$$

$$\therefore \left(\frac{d}{a} + \frac{f}{c}\right) = 2e\sqrt{\frac{1}{ac}} = 2e\left(\sqrt{\frac{1}{b^2}}\right) = \frac{2e}{b}$$

$$\therefore \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in A.P.}$$

Ans. : (A)

- (29) The sum of the first 20 terms of $\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \left(x^3 + \frac{1}{x^3}\right)^2 + \dots$ is

(A) $\frac{(x^{40}-1)}{x^2-1} + 40$ (B) $\left(\frac{x^{42}+1}{x^{40}}\right) - 40$ (C) $\frac{(x^{40}-1)}{x^2-1} \left(\frac{x^{42}+1}{x^{40}}\right) + 40$ (D) $\left(\frac{x^{40}-1}{x^2-1}\right) \left(\frac{x^{42}+1}{x^{40}}\right)$

Sol. : $\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \left(x^3 + \frac{1}{x^3}\right)^2 + \dots + \left(x^{20} + \frac{1}{x^{20}}\right)^2$

$$= (x^2 + x^4 + x^6 + \dots + x^{40}) + \left(\frac{1}{x^2} + \frac{1}{x^4} + \frac{1}{x^6} + \dots + \frac{1}{x^{40}}\right) + 40$$

$$= \frac{x^2[(x^2)^{20}-1]}{x^2-1} + \frac{\frac{1}{x^2}\left(1-\frac{1}{(x^2)^{20}}\right)}{\left(1-\frac{1}{x^2}\right)} + 40$$

$$= \frac{x^2(x^{40}-1)}{x^2-1} + \frac{(x^{40}-1)}{(x^2-1)x^{40}} + 40$$

$$= \left(\frac{x^{40}-1}{x^2-1}\right) \left(x^2 + \frac{1}{x^{40}}\right) + 40 = \left(\frac{x^{40}-1}{x^2-1}\right) \left(\frac{x^{42}+1}{x^{40}}\right) + 40$$

Ans. : (C)

- (30) If the sides a, b, c of a ΔABC are in G.P. and $\log a - \log 2b, \log 2b - \log 3c, \log 3c - \log a$ are in A.P., then ΔABC is triangle.

(A) a right angled (B) an acute angled (C) an obtuse angled (D) an equilateral

Sol. : a, b, c are in G.P.

$$\therefore b^2 = ac \quad (1)$$

Now, $\log a - \log 2b, \log 2b - \log 3c, \log 3c - \log a$ are in A.P.

$$\therefore \log \left(\frac{a}{2b} \right), \log \left(\frac{2b}{3c} \right), \log \left(\frac{3c}{a} \right) \text{ are in A.P.}$$

$$\therefore 2\log \left(\frac{2b}{3c} \right) = \log \left(\frac{a}{2b} \right) + \log \left(\frac{3c}{a} \right)$$

$$\therefore \log \left(\frac{2b}{3c} \right)^2 = \log \left(\frac{a}{2b} \times \frac{3c}{a} \right)$$

$$\therefore \frac{4b^2}{9c^2} = \frac{3c}{2b}$$

$$\therefore 8b^3 = 27c^3$$

$$\therefore \left(\frac{b}{c} \right)^3 = \frac{27}{8} = \left(\frac{3}{2} \right)^3$$

$$\therefore \frac{b}{c} = \frac{3}{2} \text{ and } \frac{b}{c} = \frac{a}{b} \text{ So, } \frac{a}{b} = \frac{3}{2}$$

$$\therefore \text{Let } a = 9k, b = 6k, c = 4k$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{k^2 [36 + 16 - 81]}{k^2 (2)(6)(4)} = \frac{-29}{48} < 0$$

$$\therefore A \in \left(\frac{\pi}{2}, \pi \right)$$

$\therefore \Delta ABC$ is an obtuse angled triangle.

Ans. : (C)

- (31) If $a_1, a_2, a_3, \dots, a_9$ are in G.P., then $\begin{vmatrix} \log a_1 & \log a_2 & \log a_3 \\ \log a_4 & \log a_5 & \log a_6 \\ \log a_7 & \log a_8 & \log a_9 \end{vmatrix} = \dots$

(A) 0 (B) -1 (C) $\log a_1 a_9 - 2 \log a_5$ (D) $\log a_1 a_9 - 2 \log a_1 + 4$

Sol. : Suppose $D = \begin{vmatrix} \log a_1 & \log a_2 & \log a_3 \\ \log a_4 & \log a_5 & \log a_6 \\ \log a_7 & \log a_8 & \log a_9 \end{vmatrix}$

Now, $a_2 = ar, a_3 = ar^2, \dots, a_9 = ar^8$.

$$D = \begin{vmatrix} \log a & \log a + \log r & \log a + 2 \log r \\ \log a + 3 \log r & \log a + 4 \log r & \log a + 5 \log r \\ \log a + 6 \log r & \log a + 7 \log r & \log a + 8 \log r \end{vmatrix}$$

$$= \begin{vmatrix} \log a & \log r & \log r \\ \log a + 3 \log r & \log r & \log r \\ \log a + 6 \log r & \log r & \log r \end{vmatrix}$$

$$C_{12}(-1), C_{23}(-1) = 0$$

Ans. : (A)

- (32) $(444 \dots n \text{ times } 4) + (6666 \dots n \text{ times } 6)^2 = \dots$

(A) $\frac{4}{9} (10^n - 1)$ (B) $\frac{4}{9} (10^n - 1)^2$ (C) $\frac{9}{4} \cdot 10^n (10^n - 1)^2$ (D) $\frac{4}{9} \cdot 10^n (10^n - 1)$

$$\text{Sol. : } (444 \dots n \text{ times } 4) = (4 + 40 + 400 + 4000 + \dots + 400 \dots 0 (n-1) \text{ times}) = \frac{4[10^n - 1]}{9}$$

$$(666 \dots n \text{ times } 6) = (6 + 60 + 600 + 6000 + \dots + 600 \dots 0 (n-1) \text{ times}) = \frac{6[10^n - 1]}{9}$$

$$= \frac{2}{3}(10^n - 1)$$

$$\therefore (666 \dots n \text{ times } 6)^2 = \frac{4}{9} (10^n - 1)^2$$

$$\begin{aligned} \therefore (444 \dots n \text{ times } 4) + (666 \dots n \text{ times } 6)^2 &= \frac{4}{9} (10^n - 1) + \frac{4}{9} (10^n - 1)^2 \\ &= \frac{4}{9} (10^n - 1) [1 + 10^n - 1] \\ &= \frac{4}{9} \cdot 10^n (10^n - 1) \end{aligned}$$

Ans. : (D)

(33) If the roots of the quadratic equation $x^2 - 3x + p = 0$ are a and b and roots of $x^2 - 12x + q = 0$ are c and d and if a, b, c, d are in G.P., then $\frac{q+p}{q-p} = \dots$

(A) $\frac{7}{8}$

(B) $\frac{10}{11}$

(C) $\frac{15}{17}$

(D) $\frac{17}{15}$

Sol. : Since a and b are roots of $x^2 - 3x + p = 0$, $a + b = 3$ and $ab = p$. (1)

Since c and d are roots of $x^2 - 12x + q = 0$, $c + d = 12$ and $cd = q$. (2)

a, b, c, d are in G.P.

$$\therefore b = ar, c = ar^2, d = ar^3$$

$$\therefore a(r+1) = 3, ar^2(r+1) = 12, \quad (3)$$

From (1), $a + ar = 3$ and from (2) $ar^2 + ar^3 = 12$

$$\therefore r^2 = 4$$

$$\therefore ab = a^2r = p, \quad cd = a^2r^5 = q. \quad \text{So, } \frac{q}{p} = r^4 = \frac{16}{1} \quad (\text{by (3)})$$

$$\therefore \frac{q+p}{q-p} = \frac{17}{15} \quad \text{Ans. : (D)}$$

(34) If the roots of the cubic equation $px^3 + qx^2 + rx + s = 0$ are in G.P., then which one of the following is true ?

(A) $pr^3 = q^3s$

(B) $p^3r = qs^3$

(C) $p^3q = r^3s$

(D) $pq^3 = rs^3$

Sol. : Suppose $\frac{a}{r}, a, ar$ are the roots of $px^3 + qx^2 + rx + s = 0$.

$$\therefore \frac{a}{r} \cdot a \cdot ar = \frac{-s}{p}. \quad \text{So, } a^3 = \frac{-s}{p}$$

Since a is a root, it must satisfy the cubic equation.

$$\therefore p \left\{ \left(-\frac{s}{p} \right)^{\frac{1}{3}} \right\}^3 + q \left\{ \left(-\frac{s}{p} \right)^{\frac{1}{3}} \right\}^2 + r \left(-\frac{s}{p} \right)^{\frac{1}{3}} + s = 0$$

$$\therefore p \left(-\frac{s}{p} \right) + q \left(-\frac{s}{p} \right)^{\frac{2}{3}} + r \left(-\frac{s}{p} \right)^{\frac{1}{3}} + s = 0$$

$$\therefore q \left(-\frac{s}{p} \right)^{\frac{2}{3}} + r \left(-\frac{s}{p} \right)^{\frac{1}{3}} = 0$$

$$\therefore \frac{q^3 s^2}{p^2} = r^3 \left(\frac{s}{p} \right)$$

$$\therefore pr^3 = q^3 s$$

Ans. : (A)

$$(35) \quad x^{\frac{1}{2}} \cdot x^{\frac{1}{4}} \cdot x^{\frac{1}{8}} \cdot x^{\frac{1}{16}} \dots = \dots$$

(A) 0

(B) 1

(C) x

(D) $\frac{1}{x}$

$$\text{Sol. : } x^{\frac{1}{2}} \cdot x^{\frac{1}{4}} \cdot x^{\frac{1}{8}} \cdot x^{\frac{1}{16}} \dots = x^{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots} = x^{\left[\frac{\frac{1}{2}}{1 - \frac{1}{2}} \right]} = x^{\frac{\frac{1}{2}}{\frac{1}{2}}} = x$$

Ans. : (C)

(36) In a G.P., if the sum of squares of three consecutive terms is s^2 and the sum of those terms is ps , then $p^2 \in \dots$, where $r \neq \pm 1$

(A) (1, 3)

(B) $\left(\frac{1}{3}, 3 \right)$

(C) $\left(\frac{1}{3}, 1 \right)$

(D) (0, 3)

Sol. : Suppose $\frac{a}{r}$, a , ar are three consecutive terms of a G.P.

$$\frac{a^2}{r^2} + a^2 + a^2 r^2 = s^2 \quad (1)$$

$$\text{and } \frac{a}{r} + a + ar = ps \quad (2)$$

Let $r + \frac{1}{r} = x$; since $r \neq \pm 1$, $x \neq \pm 2$

$$\text{From (1), } a^2 \left(r^2 + 1 + \frac{1}{r^2} \right) = s^2$$

$$\therefore a^2 \left[\left(r + \frac{1}{r} \right)^2 - 1 \right] = s^2. \quad \text{So, } a^2 (x^2 - 1) = s^2 \quad (3)$$

$$\text{From (2), } a \left(r + 1 + \frac{1}{r} \right) = ps. \quad \text{So, } a (x + 1) = ps \quad (4)$$

$$\therefore a^2 (x + 1)^2 = p^2 s^2$$

$$\therefore p^2 (x^2 - 1) = (x + 1)^2 \quad (p^2 s^2 = p^2 a^2 (x^2 - 1) = a^2 (x + 1)^2)$$

$$\therefore p^2 = \frac{(x+1)^2}{x^2-1} = \frac{x+1}{x-1}$$

$$\therefore x = \frac{p^2+1}{p^2-1} \quad \text{So, } r + \frac{1}{r} = \frac{p^2+1}{p^2-1}$$

$$\therefore (p^2-1)r^2 - (p^2+1)r + (p^2-1) = 0. \quad \text{So, } b^2 - 4ac = (p^2+1)^2 - 4(p^2-1)^2 \geq 0$$

$$\therefore (p^2+1-2(p^2-1))(p^2+1+2(p^2-1)) \geq 0 \text{ i.e. } (3-p^2)(3p^2-1) \geq 0$$

$$\therefore p^2 \leq 3 \text{ and } p^2 \geq \frac{1}{3}$$

$$\text{But } x \neq \pm 2. \quad \text{So, } p^2 \neq \frac{1}{3}; 3$$

$$\therefore \frac{1}{3} < p^2 < 3 \quad \text{So, } p^2 \in \left(\frac{1}{3}, 3\right)$$

Ans. : (B)

(37) If a, b, c and d are in G.P., then $(a-c)^2 + (b-c)^2 + (b-d)^2 - (a-d)^2 = \dots$

- (A) 0 (B) $a(1-r)$ (C) 1 (D) $a^2(1-r^2)$

Sol. : a, b, c, d are in G.P.,

$$\therefore b = ar, c = ar^2; d = ar^3$$

$$\begin{aligned} &\therefore (a-c)^2 + (b-c)^2 + (b-d)^2 - (a-d)^2 \\ &= (a-ar^2)^2 + (ar-ar^2)^2 + (ar-ar^3)^2 - (a-ar^3)^2 \\ &= a^2 [(1-r^2)^2 + r^2(1-r)^2 + r^2(1-r^2)^2 - (1-r^3)^2] \\ &= a^2 (1-r)^2 [(1+r)^2 + r^2 + r^2(1+r)^2 - (1+r+r^2)^2] \\ &= a^2 (1-r)^2 [1+2r+r^2+r^2+2r^3+r^4-1-r^2-r^4-2r-2r^3-2r^2] \\ &= a^2 (1-r)^2 (0) = 0 \end{aligned}$$

Ans. : (A)

Second method : a, b, c, d are in G.P.

$$\therefore b^2 = ac, c^2 = bd; ad = bc \quad (1)$$

$$\begin{aligned} &\therefore (a-c)^2 + (b-c)^2 + (b-d)^2 - (a-d)^2 \\ &= a^2 - 2ac + c^2 + b^2 - 2bc + c^2 + b^2 - 2bd + d^2 - a^2 + 2ad - d^2 \\ &= a^2 - 2b^2 + c^2 + b^2 - 2bc + c^2 + b^2 - 2c^2 + d^2 - a^2 + 2bc - d^2 \\ &= 0 \end{aligned} \quad (\text{From (1)})$$

(38) If $1 + a + a^2 + a^3 + \dots + a^{x-1} = (1+a)(1+a^2)(1+a^4)$, then $x = \dots$

- (A) 6 (B) 7 (C) 8 (D) 9

Sol. : $1 + a + a^2 + a^3 + \dots + a^{x-1} = (1+a)(1+a^2)(1+a^4)$

$$\therefore \frac{1 \cdot [a^x - 1]}{a - 1} = (1+a)(1+a^2)(1+a^4)$$

$$\therefore 1 - a^x = (1-a)(1+a)(1+a^2)(1+a^4) = 1 - a^8$$

$$\therefore x = 8$$

Ans. : (C)

(39) If p th, q th and r th terms of a G.P. are a, b, c ; then $\left(\frac{c}{b}\right)^p \left(\frac{b}{a}\right)^r \left(\frac{a}{c}\right)^q = \dots$

- (A) 0 (B) 1 (C) abc (D) $\frac{1}{abc}$

Sol. : Here, $t_p = A \cdot R^{p-1} = a, t_q = A \cdot R^{q-1} = b$ and $t_r = A \cdot R^{r-1} = c$

$$\therefore \frac{c}{b} = R^{r-1-q+1} = R^{r-q} \text{ and } \frac{b}{a} = R^{q-1-p+1} = R^{q-p} \text{ and } \frac{a}{c} = R^{p-1-r+1} = R^{p-r}$$

$$\therefore \left(\frac{c}{b}\right)^p \left(\frac{b}{a}\right)^r \left(\frac{a}{c}\right)^q = (R^{r-q})^p (R^{q-p})^r (R^{p-r})^q = R^0 = 1 \quad \text{Ans. : (B)}$$

(40) In an infinite G.P., if the sum of all the terms is 7 and the sum of cubes of all the terms is 49, then the common ratio between its consecutive terms is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{3}{2}$ (D) $\frac{5}{2}$

Sol. : Suppose $a, ar, ar^2, \dots$ is an infinite G.P.

$$\text{Now, } S_{\infty} = \frac{a}{1-r} = 7 \quad (1)$$

Sum of cubes of the terms is 49.

$$a^3 + a^3r^3 + a^3r^6 + \dots = 49$$

$$\therefore S'_{\infty} = \frac{a^3}{1-r^3} = 49$$

$$\therefore \frac{a}{1-r} \cdot \frac{a^2}{(1+r+r^2)} = 49$$

$$\therefore \frac{7 \cdot a^2}{1+r+r^2} = 49. \quad \text{Hence } \frac{a^2}{1+r+r^2} = 7 \quad (2)$$

$$\text{From, (1), } a^2 = 49 (1-r)^2 = 49 (1-2r+r^2)$$

$$\therefore 49 (1-2r+r^2) = 7 (1+r+r^2) \quad (\text{From (2)})$$

$$\therefore 7 - 14r + 7r^2 = 1 + r + r^2$$

$$\therefore 2r^2 - 5r + 2 = 0$$

$$\therefore r = 2 \text{ or } r = \frac{1}{2}. \text{ But } |r| < 1$$

$$\therefore r = \frac{1}{2} \quad \text{Ans. : (A)}$$

(41) If $0 < x, y, a, b < 1$, then $\sqrt{x} (\sqrt{a} + \sqrt{x}) + \sqrt{x} (\sqrt{ab} + \sqrt{xy}) + \sqrt{x} (b\sqrt{a} + y\sqrt{x}) + \dots = \dots$

(A) $\frac{\sqrt{x}}{1+\sqrt{b}} + \frac{\sqrt{x}}{1+\sqrt{y}}$ (B) $\frac{\sqrt{ax}}{1-\sqrt{b}} + \frac{x}{1-\sqrt{y}}$ (C) $\frac{ax}{1-\sqrt{b}} + \frac{\sqrt{x}}{1+\sqrt{y}}$ (D) $\frac{\sqrt{ax}}{1+\sqrt{b}} + \frac{x}{1+\sqrt{y}}$

Sol. : $\sqrt{x} (\sqrt{a} + \sqrt{x}) + \sqrt{x} (\sqrt{ab} + \sqrt{xy}) + \sqrt{x} (b\sqrt{a} + y\sqrt{x}) + \dots$

$$= (\sqrt{ax} + \sqrt{abx} + \sqrt{ab^2x} + \dots) + (x + x\sqrt{y} + xy + \dots)$$

$$= \left(\frac{\sqrt{ax}}{1-\sqrt{b}} \right) + \left(\frac{x}{1-\sqrt{y}} \right) \quad \text{Ans. : (B)}$$

(42) If $a = 0.2, b = \sqrt{5}$ and $x = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots \infty$, then $a^{\log_b x} = \dots$

(A) 0 (B) 2 (C) 4 (D) $\frac{1}{2}$

$$\begin{aligned}
\text{Sol. : } a^{\log_b x} &=_{(0.2)} \log_{\sqrt{5}} \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots \right) \\
&= \left(\frac{1}{5} \right)^{\log_{\sqrt{5}} \left(\frac{1}{4} + \frac{1}{8} + \dots \infty \right)} \\
&= \left(\sqrt{5} \right)^{-2 \log_{\sqrt{5}} \left(\frac{1}{1 - \frac{1}{2}} \right)} \\
&= \left(\sqrt{5} \right)^{-2 \log_{\sqrt{5}} \left(\frac{1}{\frac{1}{2}} \right)} \\
&= \left(\frac{1}{2} \right)^{-2} = 4
\end{aligned}$$

Ans. : (C)

(43) If x, y, z are in harmonic progression, then which of the following is true ?

(A) $x + z = 2y$ (B) $y^2 = zx$ (C) $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ (D) $\sqrt{zx} > y$

Sol. : x, y, z are in H.P.,

$$\therefore \frac{1}{x}, \frac{1}{y}, \frac{1}{z} \text{ are in A.P.}$$

$$\therefore \frac{1}{y} \text{ is the arithmetic mean of } \frac{1}{x} \text{ and } \frac{1}{z}.$$

$$\text{Since } A > G, \frac{1}{y} > \frac{1}{\sqrt{zx}}$$

$$\therefore y < \sqrt{zx}$$

Ans. : (D)

Note : $G > H$. So, $\sqrt{zx} > y$.

(44) If x, y, z are in H.P., then $\frac{x}{y+z}, \frac{y}{z+x}, \frac{z}{x+y}$ are in

(A) Arithmetic (B) Geometric (C) Harmonic (D) Arithmetico-Geometric

Sol. : x, y, z are in H.P.

$$\therefore \frac{1}{x}, \frac{1}{y}, \frac{1}{z} \text{ are in A.P.}$$

$$\therefore \frac{2}{y} = \frac{1}{x} + \frac{1}{z} = \frac{z+x}{zx}.$$

$$\text{So, } 2zx = xy + yz \quad (1)$$

$$\frac{y+z}{x} + \frac{x+y}{z} = \frac{yz+z^2+x^2+xy}{zx}$$

$$= \frac{z^2+x^2+2zx}{zx} \quad (\text{From (1)}) = \frac{(z+x)^2}{zx} = (z+x) \left(\frac{z+x}{zx} \right) = \frac{2(z+x)}{y}$$

$$\therefore \frac{y+z}{x}, \frac{z+x}{y}, \frac{x+y}{z} \text{ are in A.P.}$$

$$\therefore \frac{x}{y+z}, \frac{y}{z+x}, \frac{z}{x+y} \text{ are in H.P.}$$

Second method : $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

$$\therefore \frac{x+y+z}{x}, \frac{x+y+z}{y}, \frac{x+y+z}{z} \text{ are in A.P.}$$

$$\therefore 1 + \frac{y+z}{x}, 1 + \frac{z+x}{y}, 1 + \frac{x+y}{z} \text{ are in A.P.}$$

$$\therefore \frac{y+z}{x}, \frac{z+x}{y}, \frac{x+y}{z} \text{ are in A.P.}$$

$$\therefore \frac{x}{y+z}, \frac{y}{z+x}, \frac{z}{x+y} \text{ are in H.P.}$$

Ans. : (C)

(45) If a, b, c are in A.P.; b, c, d are in G.P. and c, d, e are in H.P., then a, c, e are in

(A) Arithmetic

(B) Geometric

(C) Harmonic

(D) Arithmetico-Geometric

Sol. : a, b, c are in A.P.

$$\therefore 2b = a + c \quad (1)$$

b, c, d are in G.P.

$$\therefore c^2 = bd \quad (2)$$

c, d, e are in H.P.

$$\therefore d = \frac{2ce}{c+e}$$

From (1) and (2),

$$c^2 = \left(\frac{a+c}{2} \right) \left(\frac{2ce}{c+e} \right) = \frac{ce(a+c)}{(c+e)}. \text{ So, } c = \frac{e(a+c)}{c+e} \quad (3)$$

$$\therefore c^2 + ce = ae + ec$$

$$\therefore c^2 = ae$$

$\therefore a, c, e$ are in G.P.

Ans. : (B)

Second method : From (3)

$$\begin{aligned} \frac{c}{e} &= \frac{a+c}{c+e} = \frac{a+c-e}{c+e-e} \\ &= \frac{a}{c} \\ \therefore c^2 &= ae \end{aligned}$$

(46) If x, y, z are in H.P., then $\frac{x+y}{y-x} + \frac{y+z}{y-z} = \dots\dots$

(A) 1

(B) 2

(C) 3

(D) 4

Sol. : x, y, z are in H.P.. So, $y = \frac{2zx}{z+x}$

$$\therefore \frac{y}{x} = \frac{2z}{z+x}$$

$$\therefore \frac{y+x}{y-x} = \frac{2z+z+x}{2z-z-x} = \frac{3z+x}{z-x} \quad (\text{Componendo-dividendo}) \quad (2)$$

$$\text{From (1), } \frac{y}{z} = \frac{2x}{z+x}$$

$$\therefore \frac{y+z}{y-z} = \frac{2x+z+x}{2x-z-x} = \frac{3x+z}{x-z} \quad (\text{Componendo-dividendo}) \quad (3)$$

$\therefore$ From (2) and (3),

$$\frac{y+x}{y-x} + \frac{y+z}{y-z} = \frac{3z+x}{z-x} + \frac{3x+z}{x-z}$$

$$= \frac{3z+x}{z-x} - \frac{3x+z}{z-x} = \frac{2z-2x}{z-x} = 2$$

Ans. : (B)

(47) If the p th, q th, r th terms of H.P. are x, y, z , then $\begin{vmatrix} yz & xz & xy \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = \dots$

- (A) 0 (B) 1 (C) $\frac{1}{xyz}$ (D) xyz

Sol. : Here t_p, t_q, t_r are in H.P.

$$\begin{vmatrix} yz & xz & xy \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = xyz \begin{vmatrix} \frac{1}{x} & \frac{1}{y} & \frac{1}{z} \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} \quad R_1 \left(\frac{1}{xyz} \right)$$

$$= xyz \begin{vmatrix} a+(p-1)d & a+(q-1)d & a+(r-1)d \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = 0 \quad \text{Ans. : (A)}$$

(48) If a, b, c are in A.P., p, q, r are in H.P. and pa, qb, cr are in G.P., then $\frac{p}{r} + \frac{r}{p} = \dots$

- (A) $\frac{a}{c} - \frac{c}{a}$ (B) $\frac{a}{b} - \frac{b}{c}$ (C) $\frac{a}{c} + \frac{c}{a}$ (D) $\frac{p}{q} + \frac{q}{r}$

Sol. : a, b, c are in A.P. So, $2b = a + c$ (1)

p, q, r are in H.P. So., $\frac{2}{q} = \frac{1}{p} + \frac{1}{r} = \frac{p+r}{pr}$

So, $p + r = \frac{2pr}{q}$ (2)

pa, qb, rc are in G.P.

$\therefore q^2 b^2 = pacr$ (3)

Now, $\frac{p}{r} + \frac{r}{p} = \frac{p^2 + r^2}{pr} = \frac{(p+r)^2 - 2pr}{pr}$

$$= \frac{\frac{4p^2 r^2}{q^2} - 2pr}{pr} \quad \text{(From (2))}$$

$$= \frac{4pr}{q^2} - 2$$

$$= \frac{4b^2}{ac} - 2 \quad \text{(From (3))}$$

$$= \frac{(a+c)^2}{ac} - 2 = \frac{a^2 + c^2}{ac} = \frac{a}{c} + \frac{c}{a} \quad \text{Ans. : (C)}$$

- (49) If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the arithmetic mean of a and b , then $n = \dots$ ($a \neq b$)
- (A) 0 (B) -1 (C) 1 (D) 2

Sol. : Arithmetic mean, $A = \frac{a+b}{2} = \frac{a^n + b^n}{a^{n-1} + b^{n-1}}$

$$\therefore a^n + ab^{n-1} + ba^{n-1} + b^n = 2a^n + 2b^n$$

$$\therefore ab^{n-1} + ba^{n-1} = a^n + b^n$$

$$\therefore ab^{n-1} - b^n = a^n - ba^{n-1}$$

$$\therefore b^{n-1}(a - b) = a^{n-1}(a - b)$$

$$\text{Since, } a \neq b, a^{n-1} = b^{n-1}$$

$$\text{Since } a \neq b, n - 1 = 0.$$

$$\text{So, } n = 1$$

Ans. : (C)

- (50) The sum of six arithmetic means between 2 and 23 is

- (A) 57 (B) 63 (C) 70 (D) 75

Sol. : $a = 2, b = 23; n = 6, d = \frac{b-a}{n+1} = \frac{21}{7} = 3$

If $a + d, a + 2d, a + 3d, a + 4d, a + 5d, a + 6d$ are the six A.M.'s between 2 and 23, their sum $= 6a + 21d = 6(2) + 21(3) = 12 + 63 = 75$

Ans. : (D)

- (51) If the sum of all terms of an A.P. obtained by placing n arithmetic means between 5 and 21 is 117, then $n = \dots$

- (A) 5 (B) 6 (C) 7 (D) 8

Sol. : Here, $a = 5, b = 21$ and $d = \frac{b-a}{n+1}$.

$$\text{Here, } S_{n+2} = \frac{n+2}{2} (a + b)$$

$$\therefore 117 = \frac{n+2}{2} (a + b) = \frac{n+2}{2} (26) = 13(n + 2)$$

$$\therefore 9 = n + 2. \quad \text{So, } n = 7$$

Ans. : (C)

- (52) If the A.M. and G.M. of two numbers a and b are 2 and 4 respectively, their H.M. is

- (A) 4 (B) 2 (C) 6 (D) 8

Sol. : Here, $A = \frac{a+b}{2} = 2$, we get $a + b = 4$ (1)

$$G = \sqrt{ab} = 4, \text{ we get } ab = 16 \quad (2)$$

$$\therefore H = \frac{2ab}{a+b} = \frac{2(16)}{4} = 8$$

$$\text{or from } AH = G^2, H = \frac{G^2}{A} = \frac{16}{2} = 8$$

Ans. : (D)

- (53) If arithmetic means between two numbers a and b are A_1 and A_2 , geometric means are G_1 and G_2 , harmonic means are H_1 and H_2 , then $\frac{A_1+A_2}{H_1+H_2} = \dots\dots$

(A) $\frac{A_1 A_2}{G_1 G_2}$ (B) $\frac{G_1 G_2}{A_1 A_2}$ (C) $\frac{G_1 G_2}{H_1 H_2}$ (D) $\frac{H_1 H_2}{G_1 G_2}$

Sol. : Here, a, A_1, A_2, b are in A.P.

$$\therefore A_1 = \frac{a+A_2}{2}, A_2 = \frac{A_1+b}{2}$$

$$\therefore A_1 + A_2 = \frac{a+A_1+A_2+b}{2}$$

$$\therefore A_1 + A_2 = a + b \quad (1)$$

Since, a, G_1, G_2, b are in G.P., $G_1 = \sqrt{aG_2}$ and $G_2 = \sqrt{bG_1}$

$$\therefore G_1 G_2 = \sqrt{abG_1 G_2} \quad \text{So, } G_1 G_2 = ab \quad (2)$$

Since a, H_1, H_2, b are in H.P., $\frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{b}$ are in A.P.

$$\therefore \frac{1}{H_1} + \frac{1}{H_2} = \frac{1}{a} + \frac{1}{b} \quad (3)$$

$$\text{From this, } \frac{H_1+H_2}{H_1 H_2} = \frac{a+b}{ab} = \frac{A_1+A_2}{G_1 G_2}$$

$$\therefore \frac{A_1+A_2}{H_1+H_2} = \frac{G_1 G_2}{H_1 H_2} \quad (\text{From (2)}) \quad \text{Ans. : (C)}$$

- (54) If the ratio of the harmonic mean and geometric mean of two positive real numbers a and b is 4:5, then the ratio of the numbers is

(A) 1 : 4 (B) 4:1 (C) 3:4 (D) 4:3

Sol. : Here, harmonic mean $H = \frac{2ab}{a+b}$ and geometric mean $G = \sqrt{ab}$.

$$\text{Also, } \frac{2ab}{\sqrt{ab}(a+b)} = \frac{4}{5}$$

$$\therefore \frac{2\sqrt{ab}}{a+b} = \frac{4}{5} \quad \text{So, } 2a - 5\sqrt{ab} + 2b = 0$$

$$\therefore (\sqrt{a} - 2\sqrt{b})(2\sqrt{a} - \sqrt{b}) = 0$$

$$\therefore \frac{a}{b} = 4 \text{ or } \frac{1}{4}$$

Second method :

$$\frac{A}{G} = \frac{5}{4} \quad \left(\frac{A}{G} = \frac{G}{H} \right)$$

$$\therefore \frac{a+b}{2\sqrt{ab}} = \frac{5}{4}$$

$$\therefore 4(a+b)^2 = 25ab$$

$$\therefore 4a^2 - 17ab + 4b^2 = 0$$

$$\therefore (4a-b)(a-4b) = 0$$

$$\therefore \frac{a}{b} = 4 \text{ or } \frac{1}{4} \quad \text{Ans. : (A), (B)}$$

- (55) If a, b, c are in G.P., x is the A.M. of a and b and y is the A.M. of b and c , then $\frac{1}{x} + \frac{1}{y} =$

..... .

(A) $\frac{1}{2}$ (B) 2 (C) $\frac{2}{a}$ (D) $\frac{2}{b}$

Sol. : a, b, c are in G.P. So, $b^2 = ac$ (1)

x is the A.M. of a and b . So, $x = \frac{a+b}{2}$ (2)

y is the A.M. of b and c . So, $y = \frac{b+c}{2}$ (3)

$$\begin{aligned} \text{Now, } \frac{1}{x} + \frac{1}{y} &= \frac{x+y}{xy} = \frac{\frac{a+b}{2} + \frac{b+c}{2}}{\left(\frac{a+b}{2}\right)\left(\frac{b+c}{2}\right)} \\ &= \frac{2(a+b+c)}{(ab+ac+bc+b^2)} = \frac{2(a+c+2b)}{(ab+b^2+bc+b^2)} \quad (\text{From (1)}) \\ &= \frac{2(a+c+2b)}{(ab+bc+2b^2)} = \frac{2(a+c+2b)}{b(a+c+2b)} \end{aligned}$$

$\therefore \frac{1}{x} + \frac{1}{y} = \frac{2}{b}$ **Ans. : (D)**

Second method :

$$\begin{aligned} \frac{1}{x} + \frac{1}{y} &= \frac{2}{a+b} + \frac{2}{b+c} = \frac{2}{a+\sqrt{a}\sqrt{c}} + \frac{2}{\sqrt{a}\sqrt{c}+c} \\ &= \frac{2}{\sqrt{a}(\sqrt{a}+\sqrt{c})} + \frac{2}{\sqrt{c}(\sqrt{a}+\sqrt{c})} \\ &= \frac{2(\sqrt{a}+\sqrt{c})}{\sqrt{a}\sqrt{c}(\sqrt{a}+\sqrt{c})} = \frac{2}{\sqrt{ac}} = \frac{2}{b} \quad \text{Ans. : (D)} \end{aligned}$$

(56) If $H_1, H_2, H_3, \dots, H_{10}$ are the harmonic means between 1 and $\frac{1}{a}$, then $\frac{H_1}{H_{10}} = \dots$

(A) $\frac{10+a}{10a+1}$ (B) $\frac{10+a}{a+1}$ (C) $\frac{10a+1}{10+a}$ (D) $\frac{10-a}{10a-1}$

Sol. : Here 1, $H_1, H_2, H_3, \dots, H_{10}, \frac{1}{a}$ are in H.P.

$\therefore 1, \frac{1}{H_1}, \frac{1}{H_2}, \dots, \frac{1}{H_{10}}, \frac{1}{a}$ are in A.P. So, $d = \frac{a-1}{11}$

Here $H_1 = \frac{1}{1+d}, H_2 = \frac{1}{1+2d}, \dots, H_{10} = \frac{1}{1+10d}$

$$\therefore \frac{H_1}{H_{10}} = \frac{1+10d}{1+d} = \frac{1+10\left(\frac{a-1}{11}\right)}{1+\left(\frac{a-1}{11}\right)} = \frac{11+10a-10}{11+a-1} = \frac{1+10a}{10+a} \quad \text{Ans. : (C)}$$

- (57) If A is the arithmetic mean of a and b and p and q are two geometric means between a and b , then $2A = \dots\dots\dots$

(A) $\frac{p^3 - q^3}{pq}$ (B) $\frac{p^3 + q^3}{pq}$ (C) $\frac{p^2 + q^2}{pq}$ (D) $\frac{p + q}{2}$

Sol. : a, p, q, b are in G.P.

$$\sqrt{aq} = p \text{ and } q = \sqrt{pb} \Rightarrow aq = p^2 \text{ and } q^2 = pb$$

$$\therefore a = \frac{p^2}{q} \text{ and } b = \frac{q^2}{p} \quad (1)$$

Since A is the A.M., $A = \frac{a+b}{2}$

$$\therefore 2A = a + b = \frac{p^2}{q} + \frac{q^2}{p} = \frac{p^3 + q^3}{pq} \quad \text{Ans. : (B)}$$

- (58) If p and q are the two arithmetic means and G is the geometric mean between two numbers a and b , then $G^2 = \dots\dots\dots$

(A) $(2p - q)(p - 2q)$ (B) $(2p - q)(2q + p)$ (C) $(2p + q)(2q + p)$ (D) $(2p - q)(2q - p)$

Sol. : Since p and q are the A.M.'s, a, p, q, b form an A.P.

$$\therefore p = \frac{a+q}{2}, q = \frac{b+p}{2}$$

$$\therefore a = 2p - q, b = 2q - p \quad (1)$$

Now, $G^2 = ab$ (G is the G.M.)

$$\therefore G^2 = (2p - q)(2q - p) \quad (\text{From (1)}) \quad \text{Ans. : (D)}$$

- (59) If $p = \sum_{n=0}^{\infty} x^n$, $q = \sum_{n=0}^{\infty} y^n$, $r = \sum_{n=0}^{\infty} (xy)^n$, (where $|x|, |y| < 1$) then which of the following is true ?

(A) $pq - r = pr - qr$ (B) $pq + r = pr + qr$ (C) $pr + q = pq + qr$ (D) $qr + p = pr + pq$

Sol. : Here $p = \sum_{n=0}^{\infty} x^n = (1 + x + x^2 + x^3 + \dots) = \frac{1}{1-x}$

$$q = \sum_{n=0}^{\infty} y^n = (1 + y + y^2 + y^3 + \dots) = \frac{1}{1-y}$$

$$r = \sum_{n=0}^{\infty} (xy)^n = (1 + xy + x^2y^2 + x^3y^3 + \dots) = \frac{1}{1-xy}$$

Now, $pq = \frac{1}{(1-x)(1-y)} = \frac{1}{1-x-y+xy}$

$$\begin{aligned} \therefore pq + r &= \frac{1}{1-x-y+xy} + \frac{1}{1-xy} = \frac{1-xy+1-x-y+xy}{(1-xy)(1-x-y+xy)} \\ &= \frac{2-x-y}{(1-x)(1-y)(1-xy)} \end{aligned} \quad (1)$$

$$pr = \frac{1}{(1-x)(1-yx)} \text{ and } qr = \frac{1}{(1-y)(1-xy)}$$

$$\therefore pr + qr = \frac{1}{(1-x)(1-xy)} + \frac{1}{(1-y)(1-xy)}$$

$$= \frac{1-y+1-x}{(1-x)(1-y)(1-xy)}$$

$$= \frac{2-x-y}{(1-x)(1-y)(1-xy)} \quad (2)$$

$\therefore$ From (1) and (2), $pq + r = pr + qr$

Ans. : (B)

(60) S_n for the series $(1^2 - 2^2) + (3^2 - 4^2) + (5^2 - 6^2) + \dots$ is

(A) $-n(n+1)$

(B) $-2n(n+1)$

(C) $-n(2n+1)$

(D) $2n+1$

Sol. : Here $(1^2 - 2^2) + (3^2 - 4^2) + (5^2 - 6^2) + \dots$ upto n terms

$$= (1^2 + 3^2 + 5^2 + \dots \text{ upto } n \text{ terms}) - (2^2 + 4^2 + 6^2 + \dots \text{ upto } n \text{ terms})$$

$$= \sum [1 + (n-1)2]^2 - 2^2 [1^2 + 2^2 + 3^2 + \dots \text{ upto } n \text{ terms}]$$

$$= \sum (2n-1)^2 - 4 \left(\sum n^2 \right)$$

$$= \sum (4n^2 - 4n + 1) - 4 \left(\sum n^2 \right)$$

$$= 4\sum n^2 - 4\sum n + \sum 1 - 4\sum n^2$$

$$= -4\sum n + \sum 1 = \frac{-4n(n+1)}{2} + n$$

$$= -2n(n+1) + n = n(-2n-2+1)$$

$$= -n(2n+1)$$

Second method :

$$\sum \left[(2n-1)^2 - (2n)^2 \right] = \sum (-4n+1)$$

$$= \frac{-4n(n+1)}{2} + n$$

$$= -2n^2 - n = -n(2n+1)$$

Ans. : (C)

Third method : $(1^2 - 2^2) + (3^2 - 4^2) + (5^2 - 6^2) + \dots + [(2n-1)^2 - (2n)^2]$

$$= (-1)[1 + 2 + 3 + 4 + \dots + 2n - 1 + 2n]$$

$$= \frac{(-1)2n(2n+1)}{2} = -n(2n+1)$$

(61) The sum of first 50 terms of $i, -2, -3i, 4, 5i \dots$ is

(A) $25 + 25i$

(B) $-25 + 25i$

(C) $-26 - 25i$

(D) $-26 + 25i$

Sol. : $i, -2, -3i, 4, 5i \dots = i, 2i^2, 3i^3, 4i^4, \dots$

$\therefore$ 50th term is $50i^{50}$.

$$\therefore S_{50} = i + 2i^2 + 3i^3 + 4i^4 + 5i^5 + \dots + 50i^{50} \quad (1)$$

$$\therefore iS_{50} = i^2 + 2i^3 + 3i^4 + 4i^5 + 5i^6 + \dots + 50i^{51} \quad (2)$$

$\therefore$ From (1) and (2),

$$S_{50} - iS_{50} = i + [i^2 + i^3 + i^4 + \dots + i^{50}] - 50i^{51}$$

$$\therefore (1-i) S_{50} = i + \frac{i^2(1-i^{49})}{(1-i)} - 50i^{51}$$

$$= i - \frac{(1-i^{48} \cdot i)}{(1-i)} - 50i^{51} = i - \frac{(1-i)}{(1-i)} - 50i^{48} \cdot i^2 \cdot i$$

$$\therefore (1-i) S_{50} = i - 1 + 50i = -1 + 51i$$

$$\therefore S_{50} = \frac{-1+51i}{1-i} = \frac{-1+51i}{(1-i)} \times \frac{(1+i)}{(1+i)}$$

$$= \frac{-1-i+51i+51i^2}{1-i^2} = \frac{-52+50i}{2} = -26 + 25i$$

Ans. : (D)

(62) The maximum sum of the terms of $20, 19\frac{1}{3}, 18\frac{2}{3}, \dots$ is

(A) 300

(B) 310

(C) 320

(D) 340

Sol. : $20, 19\frac{1}{3}, 18\frac{2}{3}, \dots$

$= \frac{60}{3}, \frac{58}{3}, \frac{56}{3}, \dots$. The maximum sum of terms is obtained when the n th term is positive and $(n+1)$ st term is negative

$$\begin{aligned} \therefore \text{Taking } a &= \frac{60}{3}, d = \frac{-2}{3}, t_n = \frac{60}{3} + (n-1) \left(\frac{-2}{3} \right) \\ &= \frac{60}{3} - \frac{2n}{3} + \frac{2}{3} \\ &= \frac{62}{3} - \frac{2n}{3} \end{aligned}$$

Taking $t_n \geq 0$, $62 - 2n \geq 0$. So, $62 \geq 2n$

$$\therefore n \leq 31$$

$\therefore$ For the maximum sum, 31 terms will be required.

$$\therefore S_{31} = \frac{31}{2} [2(20) + (31-1) \left(\frac{-2}{3} \right)] = \frac{31}{2} [40 - 20] = 310$$

Ans. : (B)

(63) The sum of first n terms of the series $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \dots$ is

(A) $\frac{n^2+1}{n^2+n+1}$

(B) $\frac{n(n+1)}{2(n^2+n+1)}$

(C) $\frac{n+1}{n^2+n+1}$

(D) $\frac{n-1}{2n^2+n+3}$

Sol. : Here $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \dots$ is given, the n th term of which is $\frac{n}{1+n^2+n^4}$.

The sum of all these terms, $S_n = \sum \frac{n}{1+n^2+n^4}$

$$= \sum \frac{n}{(n^2+1-n)(n^2+1+n)}$$

$$= \frac{1}{2} \sum \frac{2n}{(n^2+n+1)(n^2-n+1)}$$

$$\therefore S_n = \frac{1}{2} \sum \left[\frac{(n^2+n+1) - (n^2-n+1)}{(n^2+n+1)(n^2-n+1)} \right]$$

$$= \frac{1}{2} \sum \left[\frac{1}{n^2-n+1} - \frac{1}{n^2+n+1} \right]$$

$$= \frac{1}{2} \left[\left(1 - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{13}\right) + \dots + \left(\frac{1}{1+n(n-1)} - \frac{1}{1+n(n+1)}\right) \right]$$

$$= \frac{1}{2} \left[1 - \frac{1}{1+n(n+1)} \right] = \frac{1}{2} \left[\frac{1+n^2+n-1}{1+n(n+1)} \right] = \frac{n(n+1)}{2(1+n^2+n)}$$

Ans. : (B)

(64) The sum of first n terms of $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \frac{31}{32} + \dots$ is (IIT : 1988)

- (A) $2^{-n} + n - 1$ (B) 2^{-n} (C) $2^n \cdot n - 2^n + 1$ (D) $2^n - n + 1$

Sol. : $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \frac{31}{32} + \dots$ upto n terms.

$$= \left(1 - \frac{1}{2}\right) + \left(1 - \frac{1}{4}\right) + \left(1 - \frac{1}{8}\right) + \left(1 - \frac{1}{16}\right) + \left(1 - \frac{1}{32}\right) + \dots \text{ upto } n \text{ terms.}$$

$$= \left(1 - \frac{1}{2}\right) + \left(1 - \frac{1}{2^2}\right) + \left(1 - \frac{1}{2^3}\right) + \dots + \left(1 - \frac{1}{2^n}\right)$$

$$= \sum \left(1 - \frac{1}{2^n}\right) = \sum 1 - \sum 2^{-n}$$

$$= n - \left[\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \text{ upto } n \text{ terms} \right] = n - \left[\frac{\frac{1}{2} \cdot \left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}} \right] = n - (1 - 2^{-n}) = 2^{-n} + n - 1 \quad \text{Ans. : (A)}$$

(65) If $1^3 + 2^3 + 3^3 + \dots + n^3 = 2025$, then $n = \dots$

- (A) 8 (B) 9 (C) 10 (D) 11

Sol. : Here $1^3 + 2^3 + 3^3 + \dots + n^3 = 2025$

$$\therefore \sum n^3 = 2025$$

$$\therefore \frac{n^2(n+1)^2}{4} = 2025$$

$$\therefore \frac{n(n+1)}{2} = 45$$

$$\therefore n^2 + n - 90 = 0.$$

$$\text{So, } (n+10)(n-9) = 0.$$

$$\text{Hence, } n = 9$$

Ans. : (B)

$$(66) \quad \sum_{i=1}^n \sum_{j=1}^i \sum_{k=1}^j 1 = \dots$$

$$(A) \quad \frac{n(n+1)(2n+1)}{6}$$

$$(B) \quad \frac{n(n+1)}{2}$$

$$(C) \quad \frac{n(n+1)(n+2)}{6}$$

$$(D) \quad \frac{n^2(n+1)^2}{4}$$

$$\text{Sol. : } \sum_{i=1}^n \sum_{j=1}^i \sum_{k=1}^j 1$$

$$= \sum_{i=1}^n \sum_{j=1}^i j$$

$$(\sum 1 = n \text{ and } n = j)$$

$$= \sum_{i=1}^n \frac{i(i+1)}{2}$$

$$= \frac{1}{2} \sum_{i=1}^n (i^2 + i) = \frac{1}{2} [\sum n^2 + \sum n]$$

$$= \frac{1}{2} \left[\frac{n \cdot (n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \right]$$

$$= \frac{n(n+1)}{4} \left[\frac{2n+1}{3} + 1 \right] = \frac{n(n+1)}{4} \left(\frac{2n+4}{3} \right) = \frac{n(n+1)(n+2)}{6}$$

Ans. : (C)

$$(67) \quad \text{If } S_n = 1 + (1+2) + (1+2+3) + (1+2+3+4) + \dots \text{ upto } n \text{ brackets, then } S_n = \dots$$

$$(A) \quad \frac{n(n+1)(n+2)}{6}$$

$$(B) \quad \frac{n^2 - n + 1}{12}$$

$$(C) \quad \frac{n(2n+1)(2n-1)}{12}$$

$$(D) \quad \frac{n^2 + 4n + 1}{6}$$

$$\text{Sol. : Here } t_n = 1 + 2 + 3 + 4 + \dots + n = \sum n = \frac{n(n+1)}{2}.$$

$$S_n = \sum t_n = \sum \frac{n(n+1)}{2}$$

$$= \frac{1}{2} \sum n(n+1)$$

$$= \frac{1}{2} (\sum n^2 + \sum n) = \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{4} = \frac{n(n+1)(n+2)}{6}$$

Ans. : (A)

$$(68) \quad \text{The sum of } 1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + \dots + n \cdot (1) \quad \forall n \in \mathbb{N} \text{ is } \dots$$

$$(A) \quad \frac{n(n+1)(2n+1)}{6}$$

$$(B) \quad \frac{n(n+1)(2n-1)}{6}$$

$$(C) \quad \frac{n(n+1)(n+2)}{6}$$

$$(D) \quad \frac{n(n-1)(n-2)}{6}$$

$$\text{Sol. : Here } 1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + \dots + n \cdot (1) \text{ is to be evaluated.}$$

In each term, the first factor is 1, 2, 3... n and the second factor is $n, (n-1), (n-2) \dots 1$.

$$\therefore \text{ the } r\text{th term is } t_r = r [n - (r-1)].$$

$$\therefore \sum_{r=1}^n t_r = \sum_{r=1}^n r(n-r+1)$$

$$= (n+1) \sum_{r=1}^n r - \sum_{r=1}^n r^2$$

$$= (n+1) \cdot \frac{n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)}{6} [3(n+1) - (2n+1)]$$

$$= \frac{n(n+1)(n+2)}{6}$$

Ans. : (C)

$$(69) \quad \forall n \in \mathbb{N}, \frac{1 \cdot 2^2 + 2 \cdot 3^2 + 3 \cdot 4^2 + \dots \text{upto } n \text{ terms}}{1^2 \cdot 2 + 2^2 \cdot 3 + 3^2 \cdot 4 + \dots \text{upto } n \text{ terms}} = \dots$$

$$(A) \frac{5n+3}{5n-1}$$

$$(B) \frac{5n+1}{5n+3}$$

$$(C) \frac{3n+5}{3n+1}$$

$$(D) \frac{3n+1}{3n+5}$$

Sol. : Here $1 \cdot 2^2 + 2 \cdot 3^2 + 3 \cdot 4^2 + \dots$ upto n terms $= \sum n(n+1)^2$

and $1^2 \cdot 2 + 2^2 \cdot 3 + 3^2 \cdot 4 + \dots$ upto terms $= \sum n^2(n+1)$

$$\begin{aligned} \therefore \frac{1 \cdot 2^2 + 2 \cdot 3^2 + 3 \cdot 4^2 + \dots + \text{upto } n \text{ terms}}{1^2 \cdot 2 + 2^2 \cdot 3 + 3^2 \cdot 4 + \dots + \text{upto } n \text{ terms}} &= \frac{\sum n(n+1)^2}{\sum n^2(n+1)} \\ &= \frac{\sum n(n^2 + 2n + 1)}{\sum (n^3 + n^2)} \\ &= \frac{\sum (n^3 + 2n^2 + n)}{\sum (n^3 + n^2)} \\ &= \frac{\frac{n^2(n+1)^2}{4} + \frac{2 \cdot n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}}{\frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{6}} \\ &= \frac{\frac{n(n+1)}{2} \left[\frac{n(n+1)}{2} + \frac{2(2n+1)}{3} + 1 \right]}{\frac{n(n+1)}{2} \left[\frac{n(n+1)}{2} + \frac{2n+1}{3} \right]} \\ &= \frac{3n(n+1) + 4(2n+1) + 6}{3n(n+1) + 2(2n+1)} \end{aligned}$$

$$= \frac{3n^2 + 11n + 10}{3n^2 + 7n + 2}$$

$$= \frac{(3n+5)(n+2)}{(3n+1)(n+2)} = \frac{3n+5}{3n+1}$$

Ans. : (C)

(70) $1^3 + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \dots$ upto n terms is

(A) $\frac{2n^2+9n+13}{12}$ (B) $\frac{n(2n^2+9n+13)}{12}$ (C) $\frac{n(2n+9)(3n+2)}{24}$ (D) $\frac{n(2n^2+9n+13)}{24}$

Sol. : For $1^3 + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \dots$ upto n terms. Let us take,

$$t_n = \frac{1^3+2^3+3^3+\dots+n^3}{1+3+5+\dots+(2n-1)} = \frac{\Sigma n^3}{\Sigma (2n-1)} = \frac{n^2(n+1)^2}{4n^2} = \frac{n^2+2n+1}{4}$$

$$\begin{aligned} \therefore S_n &= \Sigma t_n = \frac{1}{4} \Sigma (n^2+2n+1) \\ &= \frac{1}{4} \left[\frac{n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2} + n \right] \\ &= \frac{n}{4} \left[\frac{2n^2+3n+1+6(n+1)+6}{6} \right] \\ &= \frac{n(2n^2+9n+13)}{24} \end{aligned}$$

Ans. : (D)

(71) The sum of first 20 terms of $2 + 12 + 36 + 80 + 150 + \dots$ is

(A) 64970 (B) 64790 (C) 49670 (D) 46970

Sol. : Here, to find the sum of $2 + 12 + 36 + 80 + 150 + \dots$ 20 terms; we will find the sum of $2 + 12 + 36 + 80 + 150 + \dots$ upto n terms and then substitute $n = 20$.

$$S_n = 1^2 \cdot 2 + 2^2 \cdot 3 + 3^2 \cdot 4 + 4^2 \cdot 5 + 5^2 \cdot 6 + \dots + n^2(n+1)$$

$$\therefore S_n = \Sigma n^2(n+1) = \Sigma (n^3+n^2)$$

$$\begin{aligned} \therefore S_{20} &= \sum_{i=1}^{20} i^3 + \sum_{i=1}^{20} i^2 = \left[\frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{6} \right]_{n=20} \\ &= \left[\frac{n(n+1)}{2} \right]_{n=20} \left[\frac{n(n+1)}{2} + \frac{2n+1}{3} \right]_{n=20} \\ &= \frac{20(21)}{2} \left[\frac{20(21)}{2} + \frac{41}{3} \right] \\ &= \frac{210}{6} [1260 + 82] = \frac{210(1342)}{6} = 70(671) = 46970 \end{aligned}$$

Ans. : (D)

(72) $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + n \cdot n! = \dots$

(A) $(n+1)! - n$ (B) $(n+1)! - 1$ (C) $n! - 1 + n$ (D) $n! + 1 - n$

$$\begin{aligned}
\text{Sol. : } 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + n \cdot n! &= \Sigma [(n+1) - 1] n! \\
&= \Sigma (n+1) \cdot n! - \Sigma n! \\
&= \Sigma (n+1)! - \Sigma (n!) \\
&= (2! - 1!) + (3! - 2!) + \dots + ((n+1)! - n!) \\
&= (n+1)! - 1
\end{aligned}$$

Ans. : (B)

(73) The sum of first n terms of $\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \dots$ is

(A) $\frac{6n^2}{n+1}$ (B) $\frac{6n}{n+1}$ (C) $\frac{6(2n-1)}{n+1}$ (D) $\frac{3(n^2+1)}{n+1}$

Sol. : Here 3, 5, 7, ... are in A.P.

$$\therefore t_n = 2n + 1$$

$$\begin{aligned}
\text{The } n\text{th term of the series } t_n &= \frac{2n+1}{1^2+2^2+\dots+n^2} = \frac{2n+1}{\Sigma n^2} \\
&= \frac{(2n+1) 6}{n(n+1)(2n+1)} = \frac{6}{n(n+1)}
\end{aligned}$$

The sum of n terms $= \Sigma t_n$

$$\begin{aligned}
&= 6 \Sigma \frac{1}{n(n+1)} \\
&= 6 \Sigma \left(\frac{n+1-n}{n(n+1)} \right) = 6 \Sigma \left(\frac{1}{n} - \frac{1}{n+1} \right) \\
&= 6 \left[\left(1 - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \dots + \left(\frac{1}{n} - \frac{1}{n+1} \right) \right] \\
&= 6 \left[1 - \frac{1}{n+1} \right] = \frac{6n}{n+1}
\end{aligned}$$

Ans. : (B)

(74) The sum of $\frac{1}{3} + \frac{2}{3^2} + \frac{1}{3^3} + \frac{2}{3^4} + \frac{1}{3^5} + \frac{2}{3^6} + \dots \infty$ is

(A) $\frac{1}{8}$ (B) $\frac{3}{8}$ (C) $\frac{5}{8}$ (D) $\frac{7}{8}$

$$\text{Sol. : } \frac{1}{3} + \frac{2}{3^2} + \frac{1}{3^3} + \frac{2}{3^4} + \frac{1}{3^5} + \frac{2}{3^6} + \dots \infty$$

$$\begin{aligned}
&= \left(\frac{1}{3} + \frac{1}{3^3} + \frac{1}{3^5} + \dots \infty \right) + 2 \left(\frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6} + \dots \infty \right) \\
&= \frac{\frac{1}{3}}{1 - \frac{1}{3^2}} + 2 \left(\frac{\frac{1}{3^2}}{1 - \frac{1}{3^2}} \right) = \frac{\frac{1}{3}}{1 - \frac{1}{9}} + 2 \left(\frac{\frac{1}{9}}{1 - \frac{1}{9}} \right) = \frac{3}{8} + \frac{2}{8} = \frac{5}{8}
\end{aligned}$$

Ans. : (C)

(75) The sum of n terms of $1 + 3 \cdot 2 + 5 \cdot 2^2 + 7 \cdot 2^3 + \dots$ is

(A) $n \cdot 2^{n+1} - 2^n + 1$ (B) $(n-1) 2^{n+1} - 2^n + 3$
(C) $(n+1) 2^n - 2^{n-1} + 1$ (D) $2^{n-1} n - n \cdot 2^n + 3$

Sol. : Here, first factors of the terms of the series are 1, 3, 5, 7, ... which are in A.P.

$$\therefore t_n = 1 + (n-1)2 = 2n-1$$

and second factors are $2^0, 2^1, 2^2, 2^3 \dots$ where $r = 2$ and so $t'_n = 2^{n-1}$.

For the given series, $T_n = (2n-1) \cdot 2^{n-1}$.

$$S_n = 1 + 3 \cdot 2 + 5 \cdot 2^2 + 7 \cdot 2^3 + \dots + (2n-1) \cdot 2^{n-1} \quad (1)$$

Multiplying (1) by 2,

$$2S_n = 1 \cdot 2 + 3 \cdot 2^2 + 5 \cdot 2^3 + 7 \cdot 2^4 + \dots + (2n-3) \cdot 2^{n-1} + (2n-1) \cdot 2^n \quad (2)$$

Subtracting (2) from (1),

$$-S_n = 1 - 1 \cdot 2 + 3 \cdot 2 - 5 \cdot 2^2 + 7 \cdot 2^3 - 5 \cdot 2^3 + \dots + (2n-1) \cdot 2^{n-1} - (2n-3) \cdot 2^{n-1} - (2n-1) \cdot 2^n$$

$$\therefore -S_n = 1 + 2 \cdot 2 + 2 \cdot 2^2 + 2 \cdot 2^3 + \dots + 2 \cdot 2^{n-1} - (2n-1) \cdot 2^n$$

$$\therefore -S_n = 1 + 2 [2 + 2^2 + 2^3 + \dots + 2^{n-1}] - (2n-1) \cdot 2^n$$

$$= 1 + 2 \left[\frac{2(2^{n-1}-1)}{2-1} \right] - (2n-1) \cdot 2^n$$

$$= 1 + 2^2 (2^{n-1} - 1) - (2n-1) \cdot 2^n$$

$$= 1 + 2^{n+1} - 4 - n \cdot 2^{n+1} + 2^n$$

$$= (1-n) \cdot 2^{n+1} + 2^n - 3$$

$$\therefore S_n = (n-1) 2^{n+1} - 2^n + 3$$

Ans. : (B)

Second method :

Given series is an arithmetico-geometric series.

$$\text{where } t_n = [a + (n-1)d]r^{n-1} \text{ and } S_n = \frac{a}{1-r} + \frac{d r (1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]r^n}{(1-r)}.$$

Here $a = 1$, $d = 2$ and $r = 2$.

$$\therefore S_n = \frac{1}{1-2} + \frac{2 \cdot 2(1-2^{n-1})}{(1-2)^2} - \frac{[1+(n-1)2] \cdot 2^n}{(1-2)}$$

$$\therefore S_n = -1 + 4(1-2^{n-1}) + (2n-1)2^n$$

$$= -1 + 4 - 2^{n+1} + n \cdot 2^{n+1} - 2^n$$

$$= 3 + (n-1) 2^{n+1} - 2^n$$

$$\therefore S_n = (n-1) 2^{n+1} - 2^n + 3$$

(76) The sum of first 20 terms of $3 \cdot 5 + 7 \cdot 5^2 + 11 \cdot 5^3 + \dots$ is

$$(A) \frac{2}{5} (1 + 20 \cdot 5^{39}) \quad (B) \frac{5}{2} (20 + 39 \cdot 5) \quad (C) \frac{5}{2} (1 + 39 \cdot 5^{20}) \quad (D) \frac{1}{2} (1 + 39 \cdot 5^{21})$$

Sol. : The given series $3 \cdot 5 + 7 \cdot 5^2 + 11 \cdot 5^3 + \dots$ is an arithmetico-geometric series. Its first factors 3, 7, 11... form an A.P. with $a = 3$, $d = 4$ and the second factors 5, 5^2 , 5^3 ... form a G.P. with $a = 5$, $r = 5$.

$\therefore$ n th term of A.P. is $3 + (n - 1)(4) = 4n - 1$ and n th term of G.P. is 5^n .

$\therefore$ n th term of the series is $t_n = (4n - 1)5^n$.

$$\text{So, } S_n = 3 \cdot 5 + 7 \cdot 5^2 + 11 \cdot 5^3 + \dots + (4n - 1) 5^n \quad (1)$$

$$5S_n = 3 \cdot 5^2 + 7 \cdot 5^3 + 11 \cdot 5^4 + \dots + (4n - 5) 5^n + (4n - 1) 5^{n+1} \quad (2)$$

Subtracting (2) from (1),

$$\therefore -4S_n = 3 \cdot 5 + 7 \cdot 5^2 - 3 \cdot 5^2 + 11 \cdot 5^3 - 7 \cdot 5^3 + \dots + (4n - 1) 5^n - (4n - 5) 5^n - (4n - 1) 5^{n+1}.$$

$$= 3 \cdot 5 + 4 \cdot 5^2 + 4 \cdot 5^3 + \dots + 4 \cdot 5^n - (4n - 1) 5^{n+1} \quad \left| \begin{array}{l} \text{Taking } n = 20, \\ S_{20} = \frac{-15}{4} - \frac{25}{4} (5^{19} - 1) + \frac{79}{4} 5^{21} \\ = \frac{-15}{4} - \frac{1}{4} 5^{21} + \frac{25}{4} + \frac{79}{4} 5^{21} \\ = \frac{10}{4} + \frac{78}{4} \cdot 5^{21} = \frac{5}{2} + \frac{39}{2} \cdot 5^{21} \\ = \frac{5}{2} (1 + 39 \cdot 5^{20}) \end{array} \right.$$

$$= 15 + 4 [5^2 + 5^3 + \dots + 5^n] - (4n - 1) 5^{n+1}$$

$$= 15 + 4 \left[\frac{25 (5^{n-1} - 1)}{4} \right] - (4n - 1) 5^{n+1}$$

$$= 15 + 25 (5^{n-1} - 1) - (4n - 1) 5^{n+1}$$

$$\therefore S_n = \frac{-15}{4} - \frac{25}{4} (5^{n-1} - 1) + \frac{(4n - 1)}{4} 5^{n+1}$$

Ans. : (C)

(77) The sum of the first 10 terms of $1 \cdot x + 4 \cdot x^2 + 7 \cdot x^3 + \dots$ is

(A) $28x^{16} + 31x^{10} - 2x + 1$

(B) $\frac{28x^{10} - x^9 + 1}{x - 1}$

(C) $\frac{82x^9 + 29x + 1}{(x - 1)^2}$

(D) $\frac{28x^{11} - 31x^{10} + 2x + 1}{(x - 1)^2}$

Sol. : $1 \cdot x + 4 \cdot x^2 + 7 \cdot x^3 + \dots$ is an arithmetico-geometric series.

where $a = 1$, $d = 3$, $r = x$ and $n = 10$.

$$\therefore S_{10} = \frac{1}{1 - x} + \frac{3 \cdot x(1 - x^{10-1})}{(1 - x)^2} - \frac{[1 + (10 - 1)3]x^{10}}{1 - x}$$

$$= \frac{1}{1 - x} + \frac{3x(1 - x^9)}{(1 - x)^2} - \frac{28x^{10}}{(1 - x)}$$

$$= \frac{(1 - x) + 3x(1 - x^9) - 28x^{10}(1 - x)}{(1 - x)^2}$$

$$= \frac{1 - x + 3x - 3x^{10} - 28x^{10} + 28x^{11}}{(1 - x)^2}$$

$$= \frac{28x^{11} - 31x^{10} + 2x + 1}{(x - 1)^2}$$

Ans. : (D)

- (78) If the 3rd term of a G.P. is 4, then the product of its first 5 terms is (IIT : 1982)
 (A) 64 (B) 1024 (C) 256 (D) 16

Sol. : Suppose $a, ar, ar^2 \dots$ are in G.P.
 where $ar^2 = 4$.

$$\begin{aligned} \text{Product of first 5 terms} &= (a) (ar) (ar^2) (ar^3) (ar^4) \\ &= a^5 (r^{10}) = (ar^2)^5 = 4^5 = 1024 \end{aligned} \quad \text{Ans. : (B)}$$

- (79) If a, b, c, d are in H.P., then which of the following is true ? (IIT : 1970)
 (A) $a + b > c + d$ (B) $a + c > b + d$ (C) $a + d > b + c$ (D) $b + d > a + c$

Sol. : If a, b, c, d are in H.P. b is the H.M. of a and c and c is the H.M. of b and d .
 Also the A.M. of a and c is greater than its H.M.

$$\begin{array}{l|l} \therefore \frac{a+c}{2} > b & \therefore a+c > 2b \text{ and } b+d > 2c \text{ ұқ.} \\ \text{Similarly, } \frac{b+d}{2} > c & \therefore a+b+c+d > 2b+2c \\ & \therefore a+d > b+c \end{array} \quad \text{Ans. : (C)}$$

- (80) If a, b, c, d are in H.P., then which of the following is true ? $a, b, c, d \in \mathbb{R}^+$ (IIT : 1970)
 (A) $ab > cd$ (B) $ac > bd$ (C) $ad > bc$ (D) $bc = ad$

Sol. : Here b is H.M. of a and c and c is the H.M. of b and d .
 G.M. of a and c is greater than its H.M.

$$\therefore \sqrt{ac} > b. \quad \text{So, } ac > b^2 \quad (1)$$

$$\text{Similarly, } \sqrt{bd} > c. \quad \text{So, } bd > c^2 \quad (2)$$

$$\text{From (1) and (2), } abcd > b^2c^2. \quad \text{So, } ad > bc \quad \text{Ans. : (C)}$$

- (81) If a, b, c, d are different real numbers such that
 $(a^2 + b^2 + c^2)p^2 - 2p(ab + bc + cd) + (b^2 + c^2 + d^2) \leq 0$, then a, b, c, d are in progression. (IIT : 1981)

(A) Arithmetic (B) Geometric (C) Harmonic (D) Arithmetico-Geometric

Sol. : Here $(a^2 + b^2 + c^2)p^2 - 2p(ab + bc + cd) + (b^2 + c^2 + d^2) \leq 0$

$$\therefore (a^2p^2 - 2pab + b^2) + (b^2p^2 - 2pbc + c^2) + (c^2p^2 - 2pcd + d^2) \leq 0$$

$$\therefore (ap - b)^2 + (bp - c)^2 + (cp - d)^2 \leq 0$$

$$(ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0 \text{ is the only possibility.}$$

$$\therefore ap = b; bp = c; cp = d. \quad \text{So, } \frac{b}{a} = \frac{c}{b} = \frac{d}{c} = p$$

$$\therefore a, b, c, d \text{ are in G.P.} \quad \text{Ans. : (B)}$$

- (82) If $x = \sum_{n=0}^{\infty} \cos^{2n} \theta$, $y = \sum_{n=0}^{\infty} \sin^{2n} \theta$, $z = \sum_{n=0}^{\infty} \cos^{2n} \theta \sin^{2n} \theta$, then which of the following is

true for $0 < \theta < \frac{\pi}{2}$? (IIT : 1993)

(A) $xyz = xz + y$ (B) $xyz = xy + z$ (C) $xyz = x + y + z$ (D) $xyz = yz + x$

Sol. : Here $x = \sum_{n=0}^{\infty} \cos^{2n} \theta$

$$= 1 + \cos^2 \theta + \cos^4 \theta + \cos^6 \theta + \dots = \frac{1}{1 - \cos^2 \theta} = \operatorname{cosec}^2 \theta \quad (1)$$

Similarly, $y = \sum_{n=0}^{\infty} \sin^{2n} \theta = 1 + \sin^2 \theta + \sin^4 \theta + \sin^6 \theta + \dots = \frac{1}{1 - \sin^2 \theta} = \sec^2 \theta \quad (2)$

$$z = \sum_{n=0}^{\infty} \cos^{2n} \theta \sin^{2n} \theta = 1 + \sin^2 \theta \cos^2 \theta + \sin^4 \theta \cos^4 \theta + \dots$$

$$\therefore z = \frac{1}{1 - \sin^2 \theta \cos^2 \theta} = \frac{1}{1 - \frac{1}{x} \cdot \frac{1}{y}} = \frac{xy}{xy - 1}$$

$$\therefore xyz - z = xy, \quad \text{So, } xyz = xy + z$$

$$\text{Now, } x + y = \frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta} = \frac{1}{\sin^2 \theta \cos^2 \theta}$$

$$\therefore x + y = xy$$

$$\text{By (1) and (2)}$$

$$xyz = x + y + z$$

Ans. : (B),(C)

(83) If a function $f(x)$ satisfies $f(x+y) = f(x) \cdot f(y)$; $\forall x, y \in \mathbb{N}$ and $f(1) = 3$ and $\sum_{x=1}^n f(x) = 120$,

then $n = \dots$

(IIT : 1992)

(A) 4

(B) 5

(C) 6

(D) 8

Sol. : $f(x+y) = f(x) \cdot f(y)$; $\forall x, y \in \mathbb{N}$ (1)

Taking, $x = y = 1$,

$$f(1+1) = f(2) = f(1) \cdot f(1) = f(1)^2$$

$$\text{and } f(1+2) = f(3) = f(1) \cdot f(2) = f(1) \cdot f(1)^2 = f(1)^3.$$

Proceeding in this way, $f(x) = [f(1)]^x$.

$$\therefore f(x) = 3^x \quad (f(1) = 3)$$

Now, $\sum_{x=1}^n f(x) = 120$

$$\therefore 3 + 3^2 + 3^3 + \dots + 3^n = 120$$

$$\therefore \frac{3[3^n - 1]}{3 - 1} = 120. \text{ From this, } 3^n = 81 = 3^4$$

$$\therefore n = 4$$

Ans. : (A)

(84) If the sum of first n natural numbers is $\frac{1}{5}$ times the sum of its squares, then $n = \dots$

(IIT : 1992)

(A) 5

(B) 6

(C) 7

(D) 8

Sol. : Here $\sum n = \frac{1}{5} \sum n^2$

$$\therefore 5 \Sigma n = \Sigma n^2 \quad \text{So, } \frac{5n(n+1)}{2} = \frac{n(n+1)(2n+1)}{6}$$

$$\therefore 2n + 1 = 15$$

$$\therefore n = 7$$

Ans. : (C)

(85) $\log_3 2, \log_6 2, \log_{12} 2$ are in sequence.

(IIT : 1993)

(A) Arithmetic

(B) Geometric

(C) Harmonic

(D) Arithmetico-Geometric

Sol. : $\log_2 6 = \log_2 (3 \times 2) = \log_2 3 + \log_2 2 = 1 + \log_2 3$

and $\log_2 12 = \log_2 (4 \times 3) = \log_2 (2^2 \times 3) = 2 \log_2 2 + \log_2 3 = 2 + \log_2 3.$

$$\begin{aligned} \therefore \log_2 3 + \log_2 12 &= \log_2 3 + 2 + \log_2 3 = 2 + 2 \log_2 3 \\ &= 2(1 + \log_2 3) \\ &= 2 \log_2 6 \end{aligned}$$

$\therefore \log_2 3, \log_2 6, \log_2 12$ are in A.P.

$\therefore \log_3 2, \log_6 2, \log_{12} 2$ are in H.P.

Ans. : (C)

Note : $\log_2 3 + \log_2 12 = \log_2 36 = 2 \log_2 6$. So, $\log_2 3, \log_2 6, \log_2 12$ are in A.P.

(86) In a G.P. of positive real numbers, n th term is a_n . If $\sum_{n=1}^{100} a_{2n} = \alpha$ and $\sum_{n=1}^{100} a_{2n-1} = \beta$, where $\alpha \neq \beta$, then the common ratio of the G.P.

(IIT : 1992)

(A) $\frac{\alpha}{\beta}$

(B) $\frac{\beta}{\alpha}$

(C) $\sqrt{\frac{\alpha}{\beta}}$

(D) $\sqrt{\frac{\beta}{\alpha}}$

Sol. : Suppose the first term of the G.P. is a and the corresponding common ratio is r .

Now, $\alpha = \sum_{n=1}^{100} a_{2n}.$

So, $\alpha = a_2 + a_4 + a_6 + \dots + a_{200}$

$$\therefore \alpha = ar + ar^3 + ar^5 + \dots + ar^{199} = ar(1 + r^2 + r^4 + \dots + r^{198}) \quad (1)$$

$$\beta = \sum_{n=1}^{100} a_{2n-1}$$

$$\begin{aligned} \therefore \beta &= a_1 + a_3 + a_5 + \dots + a_{199} = a + ar^2 + ar^4 + \dots + ar^{198} \\ &= a(1 + r^2 + r^4 + \dots + r^{198}) \end{aligned}$$

$$\therefore \beta = a \left(\frac{\alpha}{ar} \right) \quad \text{(From (1))}$$

$$\therefore \beta = \frac{\alpha}{r} \quad \text{So, } r = \frac{\alpha}{\beta} \quad \text{Ans. : (A)}$$

(87) Out of 4 numbers, first three are in G.P. and the last three numbers are in A.P., whose common difference is 6. If the first and the fourth numbers are same, the first number is

(IIT : 1974)

(A) 2

(B) 4

(C) 6

(D) 8

Sol. : Suppose the numbers are $a, b, b + 6, b + 12$.

$$b^2 = a(b + 6) \text{ and } b + 12 = a$$

$$(a - 12)^2 = a(a - 12 + 6)$$

$$\therefore a^2 - 24a + 144 = a^2 - 6a$$

$$\therefore 18a = 144$$

$$\therefore a = 8$$

Ans. : (D)

(88) If a, b and c are in G.P. and $\frac{1}{a^x} = \frac{1}{b^y} = \frac{1}{c^z}$, then x, y, z are in sequence . (IIT :1970)

(A) Arithmetic

(B) Geometric

(C) Harmonic

(D) Arithmetico-Geometric

Sol. : Here a, b and c are in G.P.

$$\therefore b^2 = ac \quad (1)$$

$$\text{Let } \frac{1}{a^x} = \frac{1}{b^y} = \frac{1}{c^z} = k,$$

$$\therefore a = k^x, b = k^y, c = k^z.$$

$$\therefore \text{From (1), } (k^y)^2 = k^x \cdot k^z.$$

$$\text{So, } k^{2y} = k^{x+z}$$

$$\therefore 2y = x + z.$$

So, x, y, z are in A.P.

Ans. : (A)

(89) If the sum of n terms of a G.P. is S , their product is p and the sum of their reciprocals is R , then $p^2 = \dots\dots$ (IIT : 1981)

(A) $\frac{S}{R}$

(B) $\frac{R}{S}$

(C) $\left(\frac{R}{S}\right)^n$

(D) $\left(\frac{S}{R}\right)^n$

Sol. : Suppose $a, ar, ar^2, \dots, ar^{n-1}$ are n terms of the G.P.

$$\therefore S = a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1} \quad (1)$$

$$\text{Now, } p = (a)(ar)(ar^2) \dots (ar^{n-1}).$$

$$\therefore p = a^n \cdot r^{1+2+3+\dots+(n-1)} = a^n \cdot r^{\frac{n(n-1)}{2}} \quad (2)$$

and the sum of $\frac{1}{a}, \frac{1}{ar}, \frac{1}{ar^2}, \dots, \frac{1}{ar^{n-1}}$ is R .

$$\therefore R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \dots + \frac{1}{ar^{n-1}}$$

$$= \frac{1}{a} \left[1 + \frac{1}{r} + \frac{1}{r^2} + \dots + \frac{1}{r^{n-1}} \right]$$

$$= \frac{1}{a} \left(\frac{1 - (r^{-1})^n}{1 - \frac{1}{r}} \right)$$

$$= \frac{1}{a} \left(\frac{(r^n - 1)r}{r^n(r-1)} \right)$$

$$= \frac{1}{a} \cdot \left(\frac{S}{a} \cdot r^{1-n} \right) \quad \text{(From (1))}$$

$$\therefore a^2 \cdot r^{n-1} = \frac{S}{R} \quad (3)$$

$$\text{Now, } p^2 = \left(a^n \cdot r^{\frac{n(n-1)}{2}} \right)^2 = a^{2n} \cdot r^{n(n-1)} = [a^2 r^{(n-1)}]^n$$

$$\therefore p^2 = \left(\frac{S}{R} \right)^n \quad \text{Ans. : (D)}$$

- (90) The harmonic mean, arithmetic mean and geometric mean of two numbers are 4, A and G respectively. If $2A + G^2 = 27$, then the numbers are (IIT : 1979)

(A) 5, 4 (B) 6, 4 (C) 6, 3 (D) 8, 6

Sol. : Let the numbers be a and b .

$$\text{Here } H = \frac{2ab}{a+b} = 4. \quad \text{So, } a + b = \frac{ab}{2} \quad (1)$$

$$2A = a + b \text{ and } G^2 = ab.$$

$$\text{Also, } 2A + G^2 = 27$$

$$\therefore a + b + ab = 27$$

$$\therefore \frac{ab}{2} + ab = 27. \quad \text{Hence, } \frac{3}{2} ab = 27 \quad \text{(From (1))}$$

$$\therefore ab = 18$$

$$\therefore a + b = 9 \quad \text{(From (1))}$$

(**Note :** Out of the of the given options, only option (C) has numbers whose sum is 9 and product is 18. So, the answer is option C.)

$$\therefore a + \frac{18}{a} = 9 \quad \text{So, } a^2 - 9a + 18 = 0. \quad \text{So, } a = 6 \text{ or } a = 3$$

$$\therefore \text{The two numbers are 6 and 3.} \quad \text{Ans. : (C)}$$

- (91) A function $f(x)$ is a quadratic polynomial $ax^2 + bx + c$. If $f(1) = f(-1)$ and a, b, c are in A.P., then $f(a), f(b)$ and $f(c)$ are in sequence.

(A) Arithmetic (B) Geometric (C) Harmonic (D) Arithmetico-Geometric

Sol. : The quadratic polynomial $f(x) = ax^2 + bx + c$.

$$a, b, c \text{ are in A.P. So, } 2b = a + c$$

$$\text{Given } f(1) = f(-1).$$

$$\therefore f(1) = a + b + c \text{ and } f(-1) = a - b + c$$

$$\therefore a + b + c = a - b + c$$

$$\therefore 2b = 0. \quad \text{So, } b = 0 \text{ and } c = -a$$

$$f(x) = ax^2 - a$$

$$\therefore f'(x) = 2ax$$

$$\therefore f'(a) = 2a^2, f'(b) = 2ab = 0,$$

$$f'(c) = 2ac = -2a^2$$

$$\therefore f'(a), f'(b), f'(c) \text{ are in A.P.}$$

$$\text{Ans. : (A)}$$

- (92) Numbers x_1, x_2, x_3 and y_1, y_2, y_3 are in G.P. and the common ratio in both the sequences is the same. Then points A (x_1, y_1), B (x_2, y_2), C (x_3, y_3) are (AIEEE : 2003)
- (A) collinear (B) cyclic
(C) points on an ellipse (D) vertices of a triangle

Sol. : If r is the common ratio for both the sequences, then $x_2 = rx_1$, $x_3 = r^2x_1$, and $y_2 = ry_1$ and $y_3 = r^2y_1$.

Points A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are (x_1, y_1), (rx_1, ry_1), (r^2x_1, r^2y_1).

$$\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_1r & y_1r & 1 \\ x_1r^2 & y_1r^2 & 1 \end{vmatrix}$$

$$= x_1y_1 \begin{vmatrix} 1 & 1 & 1 \\ r & r & 1 \\ r^2 & r^2 & 1 \end{vmatrix} = 0$$

$\therefore$ A, B, C are collinear points.

Ans. : (A)

- (93) In Δ ABC, if $a \cos^2 \frac{C}{2} + c \cos^2 \frac{A}{2} = \frac{3b}{2}$, then the sides of the triangle

(AIEEE : 2003)

(A) are in A.P. (B) are in G.P. (C) are in H.P. (D) satisfy $a + b = c$

Sol. : Given that $a \cos^2 \frac{C}{2} + c \cos^2 \frac{A}{2} = \frac{3b}{2}$.

$$\therefore a \left(\frac{1+\cos C}{2} \right) + c \left(\frac{1+\cos A}{2} \right) = \frac{3b}{2}$$

$$\therefore \frac{a}{2} + \frac{1}{2} (a \cos C) + \frac{c}{2} + \frac{1}{2} (c \cos A) = \frac{3b}{2}$$

$$\therefore \frac{a}{2} + \frac{c}{2} + \frac{1}{2} (a \cos C) + \frac{1}{2} (c \cos A) = \frac{3b}{2}$$

$$\therefore \frac{a}{2} + \frac{c}{2} + \frac{1}{2} (a \cos C + c \cos A) = \frac{3b}{2}$$

$$\therefore \frac{a+c}{2} + \frac{b}{2} = \frac{3b}{2}$$

(Projection formula)

$$\therefore b = \frac{a+c}{2}$$

$\therefore a, b, c$ are in A.P.

Ans. : (A)

- (94) If the function $f: R \rightarrow R$ satisfies $f(x+y) = f(x) + f(y)$, $\forall x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^n f(r)$ (AIEEE : 2003)

(A) $\frac{7n}{2}$ (B) $\frac{7(n+1)}{2}$ (C) $7n(n+1)$ (D) $\frac{7n(n+1)}{2}$

Sol. : Here $f(1) = 7$. So, $f(1) = 1 \times 7$

$$f(2) = f(1+1) = f(1) + f(1) = 7 + 7 = 14 = 2 \times 7$$

$$f(3) = f(2 + 1) = f(2) + f(1) = 14 + 7 = 21 = 3 \times 7$$

$$(f(x + y) = f(x) + f(y))$$

Proceeding in this way, $f(n) = n \times 7$.

$$\therefore \sum_{r=1}^n f(r) = f(1) + f(2) + f(3) + \dots + f(n) = \frac{7n(n+1)}{2} \quad \text{Ans. : (D)}$$

- (95) The arithmetic mean and the geometric mean of two numbers are 9 and 4 respectively and also the two numbers are roots of a quadratic equation. Then, the quadratic equation is ($k \neq 0$)
(AIEEE : 2004)

(A) $k(x^2 + 18x + 16) = 0$

(B) $k(x^2 - 18x - 16) = 0$

(C) $k(x^2 + 18x - 16) = 0$

(D) $k(x^2 - 18x + 16) = 0$

Sol. : Suppose the numbers a and b are roots of a quadratic equation.

$$\text{Now, } A = \frac{a+b}{2} = 9$$

$$\therefore a + b = 18 = \text{the sum of two roots}$$

$$G = \sqrt{ab} = 4$$

$$\therefore ab = 16 = \text{the product of two roots}$$

$$\therefore \text{The required quadratic equation is } k(x^2 - 18x + 16) = 0, \quad k \neq 0 \quad \text{Ans. : (D)}$$

- (96) In an A.P., the first term is a , common difference is d , r th term is T_r and $T_m = \frac{1}{n}$, $T_n = \frac{1}{m}$ for positive numbers m and n (where $m \neq n$). Then, $a - d = \dots\dots$ (AIEEE : 2004)

(A) 1

(B) 0

(C) $\frac{1}{mn}$

(D) $\frac{1}{m} + \frac{1}{n}$

Sol. : Here $T_n = a + (n-1)d = \frac{1}{m}$ and $T_m = a + (m-1)d = \frac{1}{n}$.

$$\therefore ma + (n-1)md = 1 \text{ and } na + (m-1)nd = 1$$

$$\therefore (m-n)a = [(m-1)n - (n-1)m]d$$

$$\therefore (m-n)a = (m-n)d$$

$$\therefore a = d$$

$$(m \neq n \Rightarrow (m-n) \neq 0)$$

$$\therefore a - d = 0$$

Ans. : (B)

- (97) $\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots = \dots\dots$ (AIEEE : 2004)

(A) $\left(\frac{e^2-1}{2}\right)$

(B) $\frac{e^2-1}{2e}$

(C) $\frac{(e-1)^2}{2e}$

(D) $\frac{e^2-1}{e}$

(Hint : Use $\frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots$)

Sol. : Taking $x = 1$ in $\frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots$

$$\frac{e + \frac{1}{e}}{2} = 1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots$$

$$\therefore \frac{e^2+1}{2e} - 1 = \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots$$

$$\therefore \frac{e^2+1-2e}{2e} = \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots$$

$$\therefore \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} = \frac{(e-1)^2}{2e}$$

Ans. : (C)

- (98) If the lengths of the perpendiculars drawn from the vertices A, B and C to the opposite sides of ΔABC are in H.P., then $\sin A, \sin B, \sin C$ are in progression. (AIEEE : 2005)

(A) Geometric (B) Harmonic (C) Arithmetic (D) Arithmetico-geometric

Sol. : Suppose, $\overline{AD}, \overline{BE}$ and $\overline{CF}$ are the perpendiculars drawn from A, B and C to $\overline{BC}, \overline{AC}$ and $\overline{AB}$ respectively. Suppose their lengths are p_1, p_2 and p_3 , $AB = c, BC = a, AC = b$.

$$\text{Area of } \Delta ABC = \Delta = \frac{1}{2} p_1 \cdot a; \Delta = \frac{1}{2} p_2 \cdot b; \Delta = \frac{1}{2} p_3 \cdot c$$

$$p_1 = \frac{2\Delta}{a}, p_2 = \frac{2\Delta}{b}, p_3 = \frac{2\Delta}{c}. \quad (1)$$

p_1, p_2, p_3 are in H.P.

$$\therefore \frac{1}{p_1}, \frac{1}{p_2}, \frac{1}{p_3} \text{ are in A.P.}$$

$$\therefore \frac{a}{2\Delta}, \frac{b}{2\Delta}, \frac{c}{2\Delta} \text{ are in A.P.}$$

$$\therefore a, b, c \text{ are in A.P.}$$

$$\therefore \frac{a}{2R}, \frac{b}{2R}, \frac{c}{2R} \text{ are in A.P.}$$

$$\therefore \sin A, \sin B, \sin C \text{ are in A.P.}$$

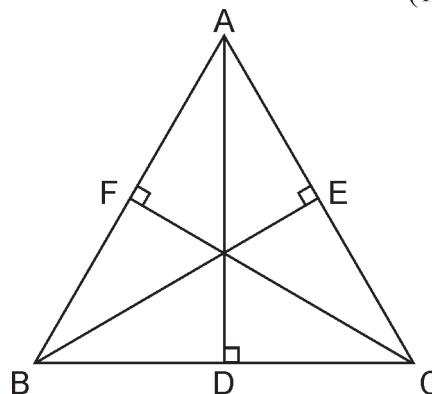


Figure 7.1

Ans. : (C)

- (99) $a_1, a_2, a_3, \dots$ are in A.P. If $\frac{a_1+a_2+a_3+\dots+a_p}{a_1+a_2+a_3+\dots+a_q} = \frac{p^2}{q^2}$, where $p \neq q$, then $\frac{a_6}{a_{21}} = \dots$

(AIEEE : 2006)

(A) $\frac{41}{11}$

(B) $\frac{11}{41}$

(C) $\frac{2}{7}$

(D) $\frac{7}{2}$

Sol. : Given that $\frac{a_1+a_2+a_3+\dots+a_p}{a_1+a_2+a_3+\dots+a_q} = \frac{p^2}{q^2}$.

$$\therefore \frac{S_p}{S_q} = \frac{p^2}{q^2} \quad \text{So,} \quad \frac{\frac{p}{2}[2a_1+(p-1)d]}{\frac{q}{2}[2a_1+(q-1)d]} = \frac{p^2}{q^2}$$

$$\therefore \frac{2a_1+(p-1)d}{2a_1+(q-1)d} = \frac{p}{q} \quad \text{So,} \quad \frac{a_1+\left(\frac{p-1}{2}\right)d}{a_1+\left(\frac{q-1}{2}\right)d} = \frac{p}{q} \quad (1)$$

Now, $\frac{a_6}{a_{21}} = \frac{a_1 + 5d}{a_1 + 20d}$

Let, $\frac{p-1}{2} = 5$ and $\frac{q-1}{2} = 20$. So, $p = 11$ and $q = 41$. So, $\frac{a_6}{a_{21}} = \frac{p}{q} = \frac{11}{41}$ **Ans. : (B)**

(100) $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots = \dots$ (AIEEE 2007)

(A) $\frac{1}{e^2}$ (B) $\frac{1}{e}$ (C) $\frac{1}{\sqrt{e}}$ (D) $\sqrt{e}$

[**Hint :** Use $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$]

Sol. : Taking $x = -1$ in $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$,

$\therefore \frac{1}{e} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots$

$\therefore \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots = \frac{1}{e}$ **Ans. : (B)**

(101) If each term of a G.P. of positive real numbers is the sum of next two terms, then the common ratio of the G.P. is (AIEEE : 2007)

(A) $\frac{1-\sqrt{5}}{2}$ (B) $\frac{\sqrt{5}}{2}$ (C) $\sqrt{5}$ (D) $\frac{\sqrt{5}-1}{2}$

Sol. : Suppose the common ratio of the G.P. $a_1, a_2, a_3, \dots, a_{n-2}, a_{n-1}, a_n$ of positive numbers is $r, r > 0$.

Given that $a_{n-2} = a_{n-1} + a_n$.

$\therefore a_1 \cdot r^{n-3} = a_1 \cdot r^{n-2} + a_1 \cdot r^{n-1}$

$\therefore r^{n-3} = r^{n-3} \cdot r + r^{n-3} \cdot r^2$

$\therefore 1 = r + r^2$

$\therefore r^2 + r - 1 = 0$

$\therefore r = \frac{-1 \pm \sqrt{5}}{2}$. But $r > 0$

$\therefore r = \frac{\sqrt{5}-1}{2}$

Ans. : (D)

(102) In a G.P., the sum of first two terms is 12 and the sum of third and fourth term is 48. If the terms of this G.P. are alternatively negative and positive, then the first term is (AIEEE : 2008)

(A) -4 (B) 4 (C) -12 (D) 12

Sol. : Suppose the first term of the G.P. is a and the common ratio is r . So, $a_2 = ar, a_3 = ar^2, a_4 = ar^3$.

$a + ar = 12$. So, $a(1 + r) = 12$. So, $1 + r = \frac{12}{a}$. (1)

$ar^2 + ar^3 = 48$. So, $ar^2(1 + r) = 48$. So, $1 + r = \frac{48}{ar^2}$. (2)

From (1) and (2), $\frac{12}{a} = \frac{48}{ar^2}$. Hence, $r^2 = 4$

$\therefore r = \pm 2$

Since the terms are alternatively positive and negative, $r = -2$ since $\frac{t_2}{t_1} = \frac{ar}{a} = r < 0$

$\therefore$ Taking $r = -2$, from $a + ar = 12$, $a = -12$

$\therefore$ First term $= -12$

Ans. : (C)

- (103) A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in n minutes. If $a_1 = a_2 = a_3 = \dots = a_{10} = 150$ and $a_{10}, a_{11}, a_{12}, \dots$ are in A.P. with common difference -2 , then the time taken by him to count all the notes is minutes. (AIEEE : 2010)

(A) 34 (B) 125 (C) 24 (D) 135

Sol. : Here $a_1 = a_2 = a_3 = \dots = a_{10} = 150$

$a_{10}, a_{11}, a_{12}, \dots$ are in A.P., where $d = -2$.

$$\therefore a_{11} - a_{10} = -2$$

$$\therefore a_{11} - 150 = -2 \text{ and so, } a_{11} = 148$$

$$\text{and } a_1 + a_2 + a_3 + \dots + a_{10} = 10(150) = 1500$$

$$\therefore \text{The number of notes counted in 10 minutes} = 1500$$

$$\therefore \text{The number of remaining notes} = 4500 - 1500 = 3000$$

$$\therefore a_{11} + a_{12} + a_{13} + \dots + a_n = 3000$$

$$\therefore \frac{(n-10)}{2} [2(a_{11}) + (n-11)(-2)] = 3000$$

$$\therefore \frac{(n-10)}{2} [2(148) - 2(n-11)] = 3000$$

$$\therefore (n-10)(159-n) = 3000$$

$$\therefore n^2 - 169n + 4590 = 0$$

$$\therefore (n-135)(n-34) = 0$$

$$\therefore n = 34 \text{ or } n = 135$$

If $n = 135$, then the n th term will be negative, which is not possible.

$$\therefore n = 34$$

$\therefore$ The person takes 34 minutes to count all the notes.

Ans. : (A)

- (104) If the sum of first $2n$ terms of A.P. 2, 5, 8, 11... and the sum of first n terms of A.P. 57, 59, 61... are equal, then $n = \dots$ (IIT Screening : 2001)

(A) 10 (B) 11 (C) 12 (D) 13

Sol. : For the A.P., 2, 5, 8, 11..., $a = 2$; $d = 3$; $N = 2n$,

$$S_N = \frac{2n}{2} [2(2) + (2n-1)3] = n(6n+1)$$

For the A.P., 57, 59, 61..., $a = 57$, $d = 2$, $N = n$,

$$\begin{aligned} S'_N &= \frac{n}{2} [2(57) + (n-1)2] \\ &= n(56+n) \end{aligned}$$

Given, $S_N = S'_N$.

$$\therefore n(6n+1) = n(56+n). \text{ So, } 6n+1 = 56+n.$$

$$\text{So, } 5n = 55$$

$$\therefore n = 11$$

Ans. : (B)

- (105) If the sum of n terms of an A.P. is cn^2 , then the sum of their squares is (IIT : 2009)

(A) $\frac{n(4n^2-1)c^2}{6}$ (B) $\frac{nc^2(4n^2+1)}{3}$ (C) $\frac{nc^2(4n^2-1)}{3}$ (D) $\frac{nc^2(4n^2+1)}{6}$

Sol. : Given that $S_n = cn^2$.

$$\therefore S_{n-1} = c(n-1)^2$$

$$\therefore t_n = S_n - S_{n-1} = cn^2 - c(n-1)^2 = c(2n-1) \quad n \geq 2, t_1 = c$$

We have to find the sum of squares of these n terms which are,

$$\therefore (c)^2, (3c)^2, (5c)^2, (7c)^2 \dots ((2n-1)c)^2.$$

$$\therefore t_n = (2n-1)^2 \cdot c^2$$

$$\therefore \Sigma t_n = \Sigma c^2 (2n-1)^2$$

$$= c^2 \Sigma (2n-1)^2$$

$$= c^2 \Sigma (4n^2 - 4n + 1)$$

$$= 4c^2 \Sigma n^2 - 4c^2 \Sigma n + c^2 \Sigma 1$$

$$= 4c^2 \left[\frac{n(n+1)(2n+1)}{6} \right] - 4c^2 \frac{n(n+1)}{2} + nc^2$$

$$= nc^2 \left[\frac{2(2n^2+3n+1)}{3} - 2(n+1)+1 \right]$$

$$= nc^2 \left(\frac{4n^2+6n+2-6n-6+3}{3} \right) = \frac{nc^2(4n^2-1)}{3}$$

Ans. : (C)

(106) If $1, \log_9(3^{1-x} + 2), \log_3(4 \cdot 3^x - 1)$ are in A.P., then $x = \dots$ (AIEEE : 2002)

(A) $\log_3 4$

(B) $1 - \log_4 3$

(C) $\log_4 3$

(D) $1 - \log_3 4$

Sol. : Here $1, \log_9(3^{1-x} + 2), \log_3(4 \cdot 3^x - 1)$ are in A.P.

$$\therefore 2 \log_9(3^{1-x} + 2) = \log_3(4 \cdot 3^x - 1) + 1$$

$$\therefore \frac{2 \log_3(3^{1-x} + 2)}{\log_3 9} = \log_3(4 \cdot 3^x - 1) + \log_3 3$$

$$\therefore \frac{2 \log_3(3^{1-x} + 2)}{\log_3 3^2} = \log_3 [3 \cdot (4 \cdot 3^x - 1)]$$

$$\therefore \log_3(3^{1-x} + 2) = \log_3 [3 \cdot (4 \cdot 3^x - 1)]$$

$$\therefore 3^{1-x} + 2 = 12 \cdot 3^x - 3$$

$$\therefore \frac{3}{3^x} + 5 = 12 \cdot 3^x$$

$$\therefore 12(3^x)^2 - 5 \cdot 3^x - 3 = 0$$

$$\therefore (3 \cdot 3^x + 1)(4 \cdot 3^x - 3) = 0$$

$$\therefore 3 \cdot 3^x = -1 \text{ or } 4 \cdot 3^x = 3$$

$$\therefore 3^x = \frac{-1}{3} < 0 \text{ is impossible.}$$

$$\therefore 3^x = \frac{3}{4}$$

$$\therefore x = \log_3 \frac{3}{4} = \log_3 3 - \log_3 4. \text{ So, } x = 1 - \log_3 4$$

Ans. : (D)

(107) A person saves ₹ 200 per month in the first three months and after that his savings increase by ₹ 40 every month. His savings become ₹ 11,040 in months. (AIEEE : 2011)

(A) 18

(B) 19

(C) 20

(D) 21

Sol. : A person saves ₹ 200 per month in the first three months from his salary.

$$\therefore \text{Total savings in the first three months} = ₹ 600$$

After three months, his savings increase by ₹ 40 every month. It will be 240, 280, 320,... which is an A.P.

$$\text{The sum of these terms} = 11040 - 600 = 10440.$$

$$\therefore a = 240, d = 40, S_n = 10440.$$

$$\therefore \text{From } S_n = \frac{n}{2} [2a + (n-1)d]$$

$$10440 = \frac{n}{2} [2(240) + (n-1)40]$$

$$\therefore 10440 = n [240 + 20(n-1)] = 20n^2 + 220n$$

$$\therefore n^2 + 11n - 522 = 0$$

$$\therefore (n+29)(n-18) = 0. \quad n = -29 < 0 \text{ is not possible.}$$

$$\therefore n = 18$$

$\therefore$ The savings will be ₹ 10440 in 18 months.

$\therefore$ After $18 + 3 = 21$ months, ₹ 10440 + ₹ 600 = ₹ 11040 will be the savings.

$$\therefore n = 21$$

Ans. : (D)

(108) If $x = \sum_{n=0}^{\infty} a^n$; $y = \sum_{n=0}^{\infty} b^n$; $z = \sum_{n=0}^{\infty} c^n$ where, a, b, c are in A.P. and $|a| < 1$, $|b| < 1$, $|c| < 1$

then x, y, z will be in

(AIEEE : 2005)

(A) A.P.

(B) G.P.

(C) H.P.

(D) Arithmetico-geometric

Sol. : Here $x = \sum_{n=0}^{\infty} a^n = 1 + a + a^2 + \dots = \frac{1}{1-a}$ ($|a| < 1$)

$$\therefore 1 - a = \frac{1}{x}. \quad \text{So, } a = 1 - \frac{1}{x}. \quad \text{Similarly, } b = 1 - \frac{1}{y}; \quad c = 1 - \frac{1}{z}$$

a, b, c are in A.P.

$\therefore a - 1, b - 1, c - 1$ are in A.P.

$\therefore -\frac{1}{x}, -\frac{1}{y}, -\frac{1}{z}$ are in A.P.

$\therefore \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

$\therefore x, y, z$ are in H.P.

Ans. : (C)

(109) If $a_1, a_2, a_3, \dots, a_n$ are in H.P., then $a_1 a_2 + a_2 a_3 + \dots + a_{n-1} a_n = \dots$

(AIEEE : 2006)

(A) $n(a_1 - a_n)$

(B) $(n-1)(a_1 - a_n)$

(C) $n a_1 a_n$

(D) $(n-1) a_1 a_n$

Sol. : $a_1, a_2, a_3, \dots, a_n$ are in H.P.

$\therefore \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \dots, \frac{1}{a_n}$ are in A.P.

$$\therefore \frac{1}{a_2} - \frac{1}{a_1} = \frac{1}{a_3} - \frac{1}{a_2} = \dots = \frac{1}{a_n} - \frac{1}{a_{n-1}} = d$$

$$\therefore \frac{a_1 - a_2}{d} = a_1 a_2; \frac{a_2 - a_3}{d} = a_2 a_3; \dots, \frac{a_{n-1} - a_n}{d} = a_{n-1} a_n$$

$$\begin{aligned}
\therefore a_1 a_2 + a_2 a_3 + \dots + a_{n-1} a_n &= \frac{a_1 - a_2}{d} + \frac{a_2 - a_3}{d} + \dots + \frac{a_{n-1} - a_n}{d} \\
&= \frac{1}{d} [a_1 - a_2 + a_2 - a_3 + \dots + a_{n-1} - a_n] \\
&= \frac{1}{d} (a_1 - a_n)
\end{aligned} \tag{1}$$

$$\text{Now, } \frac{1}{a_n} = \frac{1}{a_1} + (n-1)d$$

$$\therefore \frac{a_1 - a_n}{a_1 a_n} = (n-1)d$$

$$\therefore d = \frac{a_1 - a_n}{(n-1) a_1 a_n}$$

$$\therefore \text{From (1), } a_1 a_2 + a_2 a_3 + \dots + a_{n-1} a_n = \frac{a_1 - a_n}{\frac{(a_1 - a_n)}{(n-1) a_1 a_n}} = (n-1) a_1 a_n \quad \text{Ans. : (D)}$$

(110) The sum of first 20 terms of $0.7 + 0.77 + 0.777 + \dots$ is (JEE : 2013)

$$(A) \frac{7}{81} (179 - 10^{-20}) \quad (B) \frac{7}{9} (99 - 10^{-20}) \quad (C) \frac{7}{81} (179 + 10^{-20}) \quad (D) \frac{7}{9} (99 + 10^{-20})$$

Sol. : $0.7 + 0.77 + 0.777 + \dots + 0.777\dots 7$ (20 times)

$$= 7 [0.1 + 0.11 + 0.111 + \dots + 0.111\dots 1 \text{ (20 times)}]$$

$$= \frac{7}{9} [(1 - 0.1) + (1 - 0.01) + (1 - 0.001) + \dots + (1 - 0.000\dots 0 \text{ (19 times)})1]$$

$$\begin{aligned}
&= \frac{7}{9} \left[(1 + 1 + 1 + \dots \text{ 20 times}) - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots + \frac{1}{10^{20}} \right) \right] \\
&= \frac{7}{9} \left[20 - \frac{\frac{1}{10} \left(1 - \left(\frac{1}{10} \right)^{20} \right)}{1 - \frac{1}{10}} \right] \quad \left| \quad \begin{aligned} &= \frac{7}{9} \left[20 - \frac{1}{9} (1 - 10^{-20}) \right] \\ &= \frac{7}{9} \left[\frac{179}{9} + \frac{1}{9} \cdot 10^{-20} \right] \\ &= \frac{7}{81} (179 + 10^{-20}) \end{aligned} \right. \quad \text{Ans. : (C)}
\end{aligned}$$

(111) If x, y, z are in A.P. and $\tan^{-1}x, \tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then is true. $0 < x, y, z < 1$ (JEE : 2013)

$$(A) x = y = z \quad (B) 2x = 3y = 6z \quad (C) 6x = 3y = 2z \quad (D) 6x = 4y = 3z$$

Sol. : x, y, z are in A.P.

$$\therefore 2y = x + z \tag{1}$$

$\tan^{-1}x, \tan^{-1}y, \tan^{-1}z$ are in A.P.

$$\therefore 2\tan^{-1}y = \tan^{-1}x + \tan^{-1}z$$

$$\therefore \tan^{-1}y + \tan^{-1}y = \tan^{-1} \left(\frac{x+z}{1-xz} \right)$$

$$\therefore \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$$

$$\therefore \frac{2y}{1-y^2} = \frac{x+z}{1-xz}$$

$$\therefore \frac{x+z}{1-y^2} = \frac{x+z}{1-xz} \quad (\text{From (1)})$$

$$\therefore 1-y^2 = 1-xz \quad \text{So, } y^2 = xz$$

$\therefore x, y, z$ are in G.P.

Since, x, y, z are in A.P. as well as in G.P., $x = y = z$.

Ans. : (A)

- (112) If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $p + q$ is (AIEEE : 2007)

- (A) 2 (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

Sol. : A.M. $\geq$ G.M.

$$\frac{p+q}{2} \geq \sqrt{pq}$$

$$\therefore p^2 + 2pq + q^2 \geq 4pq$$

$$\therefore 1 + 2pq \geq 4pq. \quad \text{So, } 1 \geq 2pq. \quad \text{So, } 2pq \leq 1 \quad (1)$$

$$(p+q)^2 = p^2 + 2pq + q^2 \leq 1 + 1 \quad ((p^2 + q^2 = 1) \text{ and from (1)})$$

$$\therefore p + q \leq \sqrt{2}$$

$\therefore$ Maximum value of $p + q$ is $\sqrt{2}$.

Note : (1) Taking $p = \frac{1}{\sqrt{2}}, q = \frac{1}{\sqrt{2}}$, we get $p^2 + q^2 = 1$ and $p + q = \sqrt{2}$.

(2) Let $p = \cos\theta, q = \sin\theta$. $0 < \theta < \frac{\pi}{2}$, then $p^2 + q^2 = 1$. Range of $\cos\theta + \sin\theta = (0, \sqrt{2}]$.

If $\theta = \frac{\pi}{4}$, maximum is $\sqrt{2}$. **Ans. : (D)**

- (113) If 100 times 100th term of an A.P. is equal to 50 times 50th term, then its 150th term is (AIEEE : 2012)

- (A) -150 (B) 0 (C) 150 (D) 150 times 50th term

Sol. : $100 t_{100} = 50 t_{50}$

$$\therefore 100 [a + 99d] = 50 [a + 49d]$$

$$\therefore 2(a + 99d) = a + 49d$$

$$\therefore a + 149d = 0$$

$$\therefore t_{150} = 0$$

Ans. : (B)

- (114) Three positive numbers form an increasing G.P. If the middle term of this G.P. is doubled, the resulting sequence is A.P. Then the common ratio of the G.P. is (JEE 2014)

- (A) $\sqrt{2} + \sqrt{3}$ (B) $3 + \sqrt{2}$ (C) $2 - \sqrt{3}$ (D) $2 + \sqrt{3}$

Sol. : Suppose the common ratio is r .

a, ar, ar^2 is a G.P.

and $a, 2ar, ar^2$ are in A.P.

$$\therefore 2ar = \frac{a+ar^2}{2}$$

$$\therefore r^2 - 4r + 1 = 0 \quad (a \neq 0)$$

$$\therefore r = \frac{4 \pm \sqrt{16-4(1)(1)}}{2} = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$$

Since, it is an increasing G.P., common ratio is $2 + \sqrt{3}$.

Ans. : (D)

(115) Arithmetic mean of two distinct real numbers l and n ($l, n > 1$) is m and G_1, G_2 and G_3 are three geometric means between these two numbers, then $G_1^4 + 2G_2^4 + G_3^4 = \dots$ (JEE 2015)

(A) $4l^2mn$ (B) $4lm^2n$ (C) $4lmn^2$ (D) $4l^2m^2n^2$

Sol. : Given that the arithmetic mean of real numbers l and n is m .

$$\therefore m = \frac{l+n}{2} \quad (1)$$

Now, G_1, G_2 and G_3 are three geometric means between l and n .

$$\therefore l, G_1, G_2, G_3, n \text{ are in G.P. Let } \frac{G_1}{l} = \frac{G_2}{G_1} = \frac{G_3}{G_2} = \frac{n}{G_3} = r$$

$$\therefore G_1 = lr; G_2 = lr^2, G_3 = lr^3, n = lr^4$$

$$\therefore r^4 = \frac{n}{l}. \quad \text{So, } r = \left(\frac{n}{l}\right)^{\frac{1}{4}} \quad (2)$$

$$G_1^4 + 2G_2^4 + G_3^4 = l^4r^4 + 2(l^4r^8) + l^4r^{12} = l^4r^4(1 + 2r^4 + r^8)$$

$$\therefore G_1^4 + 2G_2^4 + G_3^4 = l^4 \left(\frac{n}{l}\right) \left[1 + 2\left(\frac{n}{l}\right) + \left(\frac{n}{l}\right)^2\right] = nl^3 \left[1 + \frac{n}{l}\right]^2 = nl(n+l)^2 = 4m^2ln \quad \text{Ans. : (B)}$$

(116) The sum of first 9 terms of $1^3 + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \dots$ is (JEE : 2015)

(A) 71 (B) 96 (C) 142 (D) 192

$$\text{Sol. : Here } t_n = \frac{1^3+2^3+3^3+\dots+n^3}{1+3+\dots+(2n-1)} = \frac{\frac{n^2(n+1)^2}{4}}{\frac{2n(n+1)}{2}-n} = \frac{\frac{n^2(n+1)^2}{4}}{n^2} = \frac{(n+1)^2}{4}$$

$$\text{Now, } S_9 = \sum_{n=1}^9 t_n = \sum_{n=1}^9 \frac{(n+1)^2}{4}$$

$$= \frac{1}{4} \sum_{n=1}^9 (n+1)^2$$

$$= \frac{1}{4} [2^2 + 3^2 + \dots + 10^2]$$

$$= \frac{1}{4} [1^2 + 2^2 + 3^2 + \dots + 10^2 - 1^2]$$

$$= \frac{1}{4} \left[\frac{10(11)(21)}{6} - 1 \right] = \frac{1}{4} (384) = 96$$

Ans. : (B)

(117) If $\log_3 2, \log_3 (2^x - 5), \log_3 \left(2^x - \frac{7}{2}\right)$ are in A.P., then $x = \dots$

- (A) 2 (B) 3 (C) 4 (D) 5

Sol. : Here $\log_3 2, \log_3 (2^x - 5), \log_3 \left(2^x - \frac{7}{2}\right)$ are in A.P.

$$\therefore 2 \log_3 (2^x - 5) = \log_3 2 + \log_3 \left(2^x - \frac{7}{2}\right)$$

$$\therefore \log_3 (2^x - 5)^2 = \log_3 \left[2 \cdot \left(2^x - \frac{7}{2}\right)\right]$$

$$\therefore (2^x - 5)^2 = 2 \cdot \left(2^x - \frac{7}{2}\right)$$

$$\therefore (2^x)^2 - 10 \cdot 2^x + 25 = 2 \cdot 2^x - 7$$

$$\therefore (2^x)^2 - 12 \cdot 2^x + 32 = 0$$

$$\therefore (2^x - 8)(2^x - 4) = 0$$

$$\therefore 2^x = 8 \text{ or } 2^x = 4$$

$$\therefore x = 3 \text{ or } x = 2$$

Taking $x = 2$, $\log_3 (2^x - 5) = \log_3 (-1)$ which is not possible since the domain of logarithmic function is $\mathbb{R}^+$.

So, $x = 3$

Ans. : (B)

: Questions with More Than One Correct Answers :

(118) If the H.M. of two numbers $5 - d$ and $5 + 2d$ is 4, then $|2d| = \dots$ (JEE advanced : 2015)

- (A) 2 (B) 3 (C) 4 (D) 5

Sol. : Suppose $a = 5 - d, b = 5 + 2d$.

$$\text{Given } H = \frac{2ab}{a+b} = 4.$$

$$\therefore (5 - d)(5 + 2d) = 2[(5 - d) + (5 + 2d)]$$

$$\therefore 25 + 5d - 2d^2 = 20 + 2d$$

$$\therefore 2d^2 - 3d - 5 = 0$$

$$\therefore (d + 1)(2d - 5) = 0$$

$$\therefore d = -1 \text{ or } d = \frac{5}{2}$$

$$\therefore |2d| = |-2| = 2 \text{ or } |2d| = 5$$

Ans. : (A), (D)

: Questions with Integer Answers :

(119) There is a resthouse outside the city limits. Milestones are to be kept on both sides of the road to the resthouse. One person starts from the resthouse in a loading rickshaw with the stone written 1 km on it, places it at 1 km distance and returns. Then he places the 2 km milestone at the proper place and returns. If he covers distance of 210 km while placing all the milestones at proper places, upto what distance have the milestones been kept ?

- (A) 8 (B) 9 (C) 10 (D) 12

Sol. : Distance covered by the person (on one side) $1 \cdot 2 + 2 \cdot 2 + 3 \cdot 2 + \dots + n \cdot 2$.

and on the other side $1 \cdot 2 + 2 \cdot 2 + 3 \cdot 2 + \dots + (n-1) \cdot 2 + n$.

because he doesn't have to come back after the last stone is placed.

$\therefore$ Total distance = $2(1 + 2 + 3 + \dots + n) + 2(1 + 2 + 3 + \dots + n) - n$

$$210 = 4(1 + 2 + 3 + \dots + n) - n = \frac{4n(n+1)}{2} - n$$

$$= 2n(n+1) - n$$

$$\therefore 210 = 2n^2 + n$$

$$\therefore 2n^2 + n - 210 = 0$$

$$\therefore (2n + 21)(n - 10) = 0$$

$$\therefore n = 10 \text{ or } n = \frac{-21}{2} < 0 \text{ is not possible.}$$

$$\therefore n = 10$$

$\therefore$ The milestones are kept upto 10 km distance on both the sides of the road.

Ans. : (C)

$$(120) \quad 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \dots = \dots$$

(AIEEE-2009)

(A) 6

(B) 3

(C) 2

(D) 4

$$\text{Sol. : Here } S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \dots \infty$$

(1)

$$\therefore \frac{1}{3} S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \dots \infty$$

(2)

Subtracting (2) from (1),

$$\therefore \frac{2}{3} S = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \dots \infty$$

$$= \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \dots \infty$$

$$= \frac{\frac{4}{3}}{1 - \frac{1}{3}} = \frac{\frac{4}{3}}{\frac{2}{3}} = 2. \quad \text{So, } S = 3$$

Ans. : (B)

: Assertion and Reason Type Questions :

Explanation : Each of these questions contains statement 1 (Assertion) and statement 2 (Reason). Each question has four choices, option (A), (B), (C) and (D), which are as follows. You have to select the correct choice.

(A) Statement 1 is true, statement 2 is true. Statement 2 is the correct explanation for statement 1.

(B) Statement 1 is true, statement 2 is true. Statement 2 is not a correct explanation for statement 1.

(C) Statement 1 is true, statement 2 is false.

(D) Statement 1 is false, statement 2 is true.

(121) **Statement 1** : 1, 2, 4, 8 ... is a G.P.

4, 8, 16, 32 ... is a G.P.

1 + 4, 2 + 8, 4 + 16, 8 + 32 ... is also a G.P.

Statement 2 : Suppose the common ratio of a G.P. is r and its general term is T_{k+1} . The general term of a second G.P. with the same ratio is T'_{k+1} . Then $T''_{k+1} = T'_{k+1} + T_{k+1}$ will be the general term of another G.P. with common ratio as r .

Sol. : Suppose $T_{k+1} = ar^k$ and $T'_{k+1} = br^k$.

$$\begin{aligned}\therefore T''_{k+1} &= T'_{k+1} + T_{k+1} \\ &= ar^k + br^k \\ &= (a + b)r^k\end{aligned}$$

$\therefore T''_{k+1}$ is the general term of another G.P.

$\therefore$ Both the statements are true.

Statement 2 is the correct explanation of statement 1.

Ans. : (A)

(122) **Statement 1** : The sum of $1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + \dots + (361 + 380 + 400)$ is 8000. (AIEEE-2012)

Statement 2 : $\sum_{k=1}^n (k^3 - (k-1)^3) = n^3, \forall n \in \mathbb{N}$

Sol. : In the given series, in each bracket, the sum of squares of two numbers and the product of those numbers are given. In the last bracket, it is $19^2 + (19)(20) + (20)^2$.

For 20 terms, the sum is given as $1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + \dots + (361 + 380 + 400) = 8000$.

$$\begin{aligned}\text{It is clear that } k^3 - (k-1)^3 &= (k - (k-1)) (k^2 + k(k-1) + (k-1)^2) \\ &= k^2 + k(k-1) + (k-1)^2\end{aligned}$$

$$\therefore \sum_{k=1}^n (k^3 - (k-1)^3) = \sum_{k=1}^{20} (k^2 + k(k-1) + (k-1)^2) = 20^3 = 8000$$

$\therefore$ Statement 1 is true.

$\therefore \sum_{k=1}^n (k^3 - (k-1)^3) = n^3$ is enough for the statement 1 and statement 2 to be true and

statement 2 is the correct explanation of statement 1.

Ans. : (A)

(123) **Statement 1** : $\forall n, n \geq 2, n \in \mathbb{N}, \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{n}} > \sqrt{n}$ (AIEEE : 2008)

Statement 2 : $\forall n \in \mathbb{N}, n \geq 2, \sqrt{n(n+1)} < n+1$

Sol. : Taking $n = 2$ in $P(n)$: $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{n}} > \sqrt{n}$.

$$P(2) : \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} > \sqrt{2} \quad \text{because} \quad \frac{1}{\sqrt{1}} > \sqrt{2} - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\text{Suppose that } P(k) : \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{k}} > \sqrt{k} \text{ is true. } \forall k \in \mathbb{N}, k \geq 2 \quad (1)$$

$$\therefore \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} \quad (\text{From (1)})$$

$$> \sqrt{k} + \frac{1}{\sqrt{k+1}} = \frac{\sqrt{k^2+k+1}}{\sqrt{k+1}} > \frac{\sqrt{k^2+1}}{\sqrt{k+1}}$$

$$\therefore P(k+1) : \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \frac{k+1}{\sqrt{k+1}} = \sqrt{k+1}$$

Statement 1 is true.

Now, $n < n+1$

$$\therefore n(n+1) < (n+1)^2$$

$$\therefore \sqrt{n(n+1)} < n+1$$

$\therefore$ Statement 2 is true.

$\therefore$ Statement 2 is not a correct explanation of statement 1.

Ans. : (B)

