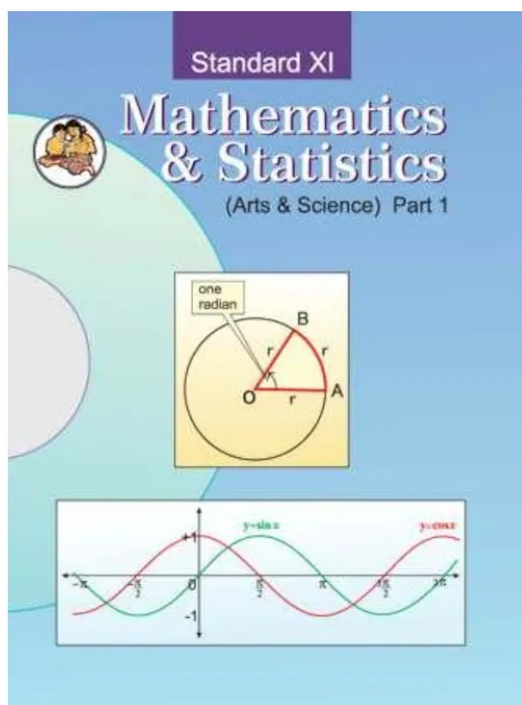


Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 1): Chapter 7- Conic Sections

Class 11 - Chapter 7 Conic Sections



For any clarifications or questions you can write to info@indcareer.com

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Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 1): Chapter 7- Conic Sections

Class 11: Maths Chapter 7 solutions. Complete Class 11 Maths Chapter 1 Notes.

Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 1): Chapter 7- Conic Sections

Maharashtra Board 11th Maths Chapter 7, Class 11 Maths Chapter 7 solutions

Ex 7.1

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Question 1.

Find co-ordinates of focus, equation of directrix, length of latus rectum and the co-ordinates of end points of latus rectum of the parabola:

(i) $5y^2 = 24x$

(ii) $y^2 = -20x$

(iii) $3x^2 = 8y$

(iv) $x^2 = -8y$

(v) $3y^2 = -16x$

Solution:

(i) Given equation of the parabola is $5y^2 = 24x$.

$$\Rightarrow y^2 = \frac{24}{5}x$$

Comparing this equation with $y^2 = 4ax$, we get

$$\Rightarrow 4a = \frac{24}{5}$$

$$\Rightarrow a = \frac{6}{5}$$

Co-ordinates of focus are $S(a, 0)$, i.e., $S(\frac{6}{5}, 0)$ Equation of the directrix is $x + a = 0$.

$$\Rightarrow x + \frac{6}{5} = 0$$

$$\Rightarrow 5x + 6 = 0$$

Length of latus rectum = $4a$

$$= 4(\frac{6}{5})$$

$$= \frac{24}{5}$$

Co-ordinates of end points of latus rectum are $(a, 2a)$ and $(a, -2a)$,

$$\Rightarrow (\frac{6}{5}, \frac{12}{5}) \text{ and } (\frac{6}{5}, -\frac{12}{5})$$

ii) Given equation of the parabola is $y^2 = -20x$.Comparing this equation with $y^2 = -4ax$, we get

$$\Rightarrow 4a = 20$$

$$\Rightarrow a = 5$$

Co-ordinates of focus are $S(-a, 0)$, i.e., $S(-5, 0)$ Equation of the directrix is $x - a = 0$

$$\Rightarrow x - 5 = 0$$

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Length of latus rectum = $4a = 4(5) = 20$

Co-ordinates of end points of latus rectum are $(-a, 2a)$ and $(-a, -2a)$,

$\Rightarrow (-5, 10)$ and $(-5, -10)$.

(iii) Given equation of the parabola is $3x^2 = 8y$

$$\Rightarrow x^2 = \frac{8}{3}y$$

Comparing this equation with $x^2 = 4by$, we get

$$\Rightarrow 4b = \frac{8}{3}$$

$$\Rightarrow b = \frac{2}{3}$$

Co-ordinates of focus are $S(0, b)$, i.e., $S(0, \frac{2}{3})$

Equation of the directrix is $y + b = 0$,

$$\Rightarrow y + \frac{2}{3} = 0$$

$$\Rightarrow 3y + 2 = 0$$

$$\text{Length of latus rectum} = 4b = 4(\frac{2}{3}) = \frac{8}{3}$$

Co-ordinates of end points of latus rectum are $(2b, b)$ and $(-2b, b)$,

$$\Rightarrow (\frac{4}{3}, \frac{2}{3}) \text{ and } (-\frac{4}{3}, \frac{2}{3}).$$

(iv) Given equation of the parabola is $x^2 = -8y$.

Comparing this equation with $x^2 = -4by$, we get

$$\Rightarrow 4b = 8$$

$$\Rightarrow b = 2$$

Co-ordinates of focus are $S(0, -b)$, i.e., $S(0, -2)$

Equation of the directrix is $y - b = 0$, i.e., $y - 2 = 0$

$$\text{Length of latus rectum} = 4b = 4(2) = 8$$

$\therefore$ Co-ordinates of end points of latus rectum are $(2b, -b)$ and $(-2b, -b)$, i.e., $(4, -2)$ and $(-4, -2)$.

(v) Given equation of the parabola is $3y^2 = -16x$.

$$\Rightarrow y^2 = -\frac{16}{3}x$$

Comparing this equation with $y^2 = -4ax$, we get

$$\Rightarrow 4a = \frac{16}{3}$$

$$\Rightarrow a = \frac{4}{3}$$

Co-ordinates of focus are $S(-a, 0)$, i.e., $(-\frac{4}{3}, 0)$

Equation of the directrix is $x - a = 0$,

$$\Rightarrow x - \frac{4}{3} = 0$$

$$\Rightarrow 3x - 4 = 0$$

$$\text{Length of latus rectum} = 4a = 4\left(\frac{4}{3}\right) = \frac{16}{3}$$

Co-ordinates of end points of latus rectum are $(-a, 2a)$ and $(-a, -2a)$,

i.e., $(-\frac{4}{3}, \frac{8}{3})$ and $(-\frac{4}{3}, -\frac{8}{3})$

Question 2.

Find the equation of the parabola with vertex at the origin, the axis along the Y-axis, and passing through the point $(-10, -5)$.

Solution:

Vertex of the parabola is at origin $(0, 0)$ and its axis is along Y-axis.

Equation of the parabola can be either $x^2 = 4by$ or $x^2 = -4by$

Since the parabola passes through $(-10, -5)$, it lies in 3rd quadrant.

Required parabola is $x^2 = -4by$.

Substituting $x = -10$ and $y = -5$ in $x^2 = -4by$, we get

$$\Rightarrow (-10)^2 = -4b(-5)$$

$$\Rightarrow b = \frac{100}{20} = 5$$

$\therefore$ The required equation of the parabola is $x^2 = -4(5)y$, i.e., $x^2 = -20y$.

Question 3.

Find the equation of the parabola with vertex at the origin, the axis along the X-axis, and passing through the point (3, 4).

Solution:

Vertex of the parabola is at the origin (0, 0) and its axis is along X-axis.

Equation of the parabola can be either $y^2 = 4ax$ or $y^2 = -4ax$.

Since the parabola passes through (3, 4), it lies in the 1st quadrant.

Required parabola is $y^2 = 4ax$.

Substituting $x = 3$ and $y = 4$ in $y^2 = 4ax$, we get

$$\Rightarrow (4)^2 = 4a(3)$$

$$\Rightarrow a = \frac{16}{12} = \frac{4}{3}$$

The required equation of the parabola is

$$y^2 = 4\left(\frac{4}{3}\right)x$$

$$\Rightarrow 3y^2 = 16x$$

Question 4.

Find the equation of the parabola whose vertex is O(0, 0) and focus at (-7, 0).

Solution:

Focus of the parabola is S(-7, 0) and vertex is O(0, 0).

Since focus lies on X-axis, it is the axis of the parabola.

Focus S(-7, 0) lies on the left-hand side of the origin.

It is a left-handed parabola.

Required parabola is $y^2 = -4ax$.

Focus is S(-a, 0).

$$a = 7$$

$\therefore$ The required equation of the parabola is $y^2 = -4(7)x$, i.e., $y^2 = -28x$.

Question 5.

Find the equation of the parabola with vertex at the origin, the axis along X-axis, and passing through the point

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(i) (1, -6)

(ii) (2, 3)

Solution:

(i) Vertex of the parabola is at origin (0, 0) and its axis is along X-axis.

Equation of the parabola can be either $y^2 = 4ax$ or $y^2 = -4ax$.

Since the parabola passes through (1, -6), it lies in the 4th quadrant.

Required parabola is $y^2 = 4ax$.

Substituting $x = 1$ and $y = -6$ in $y^2 = 4ax$, we get

$$\Rightarrow (-6)^2 = 4a(1)$$

$$\Rightarrow 36 = 4a$$

$$\Rightarrow a = 9$$

$\therefore$ The required equation of the parabola is $y^2 = 4(9)x$, i.e., $y^2 = 36x$.

(ii) Vertex of the parabola is at origin (0, 0) and its axis is along X-axis.

Equation of the parabola can be either $y^2 = 4ax$ or $y^2 = -4ax$.

Since the parabola passes through (2, 3), it lies in 1st quadrant.

$\therefore$ Required parabola is $y^2 = 4ax$.

Substituting $x = 2$ and $y = 3$ in $y^2 = 4ax$, we get

$$\Rightarrow (3)^2 = 4a(2)$$

$$\Rightarrow 9 = 8a$$

$$\Rightarrow a = \frac{9}{8}$$

The required equation of the parabola is

$$y^2 = 4\left(\frac{9}{8}\right)x$$

$$\Rightarrow y^2 = \frac{9}{2}x$$

$$\Rightarrow 2y^2 = 9x.$$

Question 6.

For the parabola $3y^2 = 16x$, find the parameter of the point:

(i) (3, -4)

(ii) (27, -12)

Solution:

Given the equation of the parabola is $3y^2 = 16x$.

$$\Rightarrow y^2 = \frac{16}{3}x$$

Comparing this equation with $y^2 = 4ax$, we get

$$\Rightarrow 4a = \frac{16}{3}$$

$$\Rightarrow a = \frac{4}{3}$$

If t is the parameter of the point P on the parabola, then

$$P(t) = (at^2, 2at)$$

i.e., $x = at^2$ and $y = 2at$ (i)

(i) Given point is (3, -4)

Substituting $x = 3$, $y = -4$ and $a = \frac{4}{3}$ in (i), we get

$$3 = \frac{4}{3}t^2 \text{ and } -4 = 2\left(\frac{4}{3}\right)t$$

$$t^2 = \frac{9}{4} \text{ and } t = \frac{-3}{2}$$

$$t = \pm \frac{3}{2} \text{ and } t = \frac{-3}{2}$$

$$t = -\frac{3}{2}$$

$\therefore$ The parameter of the given point is $\frac{-3}{2}$

(ii) Given point is (27, -12)

Substituting $x = 27$, $y = -12$ and $a = \frac{4}{3}$ in (i), we get



$$27 = \frac{4}{3}t^2 \text{ and } -12 = 2\left(\frac{4}{3}\right)t$$

$$t^2 = \frac{81}{4} \text{ and } t = \frac{-9}{2}$$

$$t = \pm \frac{9}{2} \text{ and } t = \frac{-9}{2}$$

$$t = \frac{-9}{2}$$

$\therefore$ The parameter of the given point is $\frac{-9}{2}$

Question 7.

Find the focal distance of a point on the parabola $y^2 = 16x$ whose ordinate is 2 times the abscissa.

Solution:

Given the equation of the parabola is $y^2 = 16x$.

Comparing this equation with $y^2 = 4ax$, we get

$$\Rightarrow 4a = 16$$

$$\Rightarrow a = 4$$

Since ordinate is 2 times the abscissa,

$$y = 2x$$

Substituting $y = 2x$ in $y^2 = 16x$, we get

$$\Rightarrow (2x)^2 = 16x$$

$$\Rightarrow 4x^2 = 16x$$

$$\Rightarrow 4x^2 - 16x = 0$$

$$\Rightarrow 4x(x - 4) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 4$$

When $x = 4$,

$$\text{focal distance} = x + a = 4 + 4 = 8$$

When $x = 0$,

$$\text{focal distance} = a = 4$$

$\therefore$ Focal distance is 4 or 8.

Question 8.

Find coordinates of the point on the parabola. Also, find focal distance.

(i) $y^2 = 12x$ whose parameter is $\frac{1}{3}$

(ii) $2y^2 = 7x$ whose parameter is -2

Solution:

(i) Given equation of the parabola is $y^2 = 12x$.

Comparing this equation with $y^2 = 4ax$, we get

$$\Rightarrow 4a = 12$$

$$\Rightarrow a = 3$$

If t is the parameter of the point P on the parabola, then

$$P(t) = (at^2, 2at)$$

i.e., $x = at^2$ and $y = 2at$ (i)

Given, $t = \frac{1}{3}$

Substituting $a = 3$ and $t = \frac{1}{3}$ in (i), we get

$$x = 3\left(\frac{1}{3}\right)^2 \text{ and } y = 2(3)\left(\frac{1}{3}\right)$$

$$x = \frac{1}{3} \text{ and } y = 2$$

The co-ordinates of the point on the parabola are $\left(\frac{1}{3}, 2\right)$

$$\therefore \text{Focal distance} = x + a$$

$$= \frac{1}{3} + 3$$

(ii) Given equation of the parabola is $2y^2 = 7x$.

$$\Rightarrow y^2 = \frac{7}{2}x$$

Comparing this equation with $y^2 = 4ax$, we get

$$\Rightarrow 4a = \frac{7}{2}$$

$$\Rightarrow a = \frac{7}{8}$$

If t is the parameter of the point P on the parabola, then

$$P(t) = (at^2, 2at)$$

i.e., $x = at^2$ and $y = 2at$ (i)

Given, $t = -2$

Substituting $a = \frac{7}{8}$ and $t = -2$ in (i), we get

$$x = \frac{7}{8}(-2)^2 \text{ and } y = 2\left(\frac{7}{8}\right)(-2)$$

$$x = \frac{7}{2} \text{ and } y = \frac{-7}{2}$$

The co-ordinates of the point on the parabola are $\left(\frac{7}{2}, \frac{-7}{2}\right)$

$\therefore$ Focal distance = $x + a$

$$= \frac{7}{2} + \frac{7}{8}$$

$$= \frac{35}{8}$$

Question 9.

For the parabola $y^2 = 4x$, find the coordinates of the point whose focal distance is 17.

Solution:

Given the equation of the parabola is $y^2 = 4x$.

Comparing this equation with $y^2 = 4ax$, we get

$$\Rightarrow 4a = 4$$

$$\Rightarrow a = 1$$

$$\Rightarrow a = 1$$

Focal distance of a point = $x + a$

Given, focal distance = 17

$$\Rightarrow x + 1 = 17$$

$$\Rightarrow x = 16$$

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Substituting $x = 16$ in $y^2 = 4x$, we get

$$\Rightarrow y^2 = 4(16)$$

$$\Rightarrow y^2 = 64$$

$$\Rightarrow y = \pm 8$$

$\therefore$ The co-ordinates of the point on the parabola are $(16, 8)$ or $(16, -8)$.

Question 10.

Find the length of the latus rectum of the parabola $y^2 = 4ax$ passing through the point $(2, -6)$.

Solution:

Given equation of the parabola is $y^2 = 4ax$ and it passes through point $(2, -6)$.

Substituting $x = 2$ and $y = -6$ in $y^2 = 4ax$, we get

$$\Rightarrow (-6)^2 = 4a(2)$$

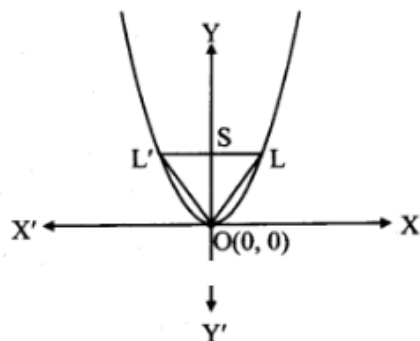
$$\Rightarrow 4a = 18$$

$\therefore$ Length of latus rectum $= 4a = 18$ units

Question 11.

Find the area of the triangle formed by the line joining the vertex of the parabola $x^2 = 12y$ to the endpoints of the latus rectum.

Solution:



Given the equation of the parabola is $x^2 = 12y$.

Comparing this equation with $x^2 = 4by$, we get

$$\Rightarrow 4b = 12$$

$$\Rightarrow b = 3$$

The co-ordinates of focus are $S(0, b)$, i.e., $S(0, 3)$

End points of the latus-rectum are $L(2b, b)$ and $L'(-2b, b)$,

i.e., $L(6, 3)$ and $L'(-6, 3)$

Also $l(LL') = \text{length of latus-rectum} = 4b = 12$

$$l(OS) = b = 3$$

$$\text{Area of } \triangle OLL' = \frac{1}{2} \times l(LL') \times l(OS)$$

$$= \frac{1}{2} \times 12 \times 3$$

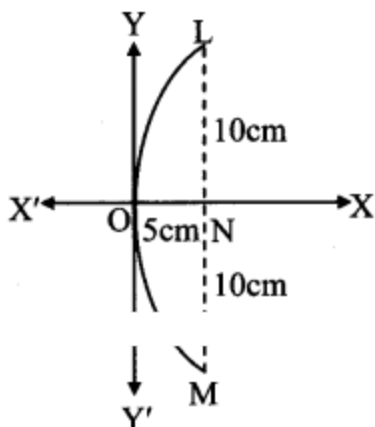
$$\text{Area of } \triangle OLL' = 18 \text{ sq. units}$$

Question 12.

If a parabolic reflector is 20 cm in diameter and 5 cm deep, find its focus.

Solution:

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Let LOM be the parabolic reflector such that LM is the diameter and ON is its depth.

It is given that $ON = 5 \text{ cm}$ and $LM = 20 \text{ cm}$.

$LN = 10 \text{ cm}$

Taking O as the origin, ON along X-axis and a line through O $\perp$ ON as Y-axis.

Let the equation of the reflector be $y^2 = 4ax$ (i)

The point L has the co-ordinates (5, 10) and lies on parabola given by (i).

Substituting $x = 5$ and $y = 10$ in (i), we get

$$\Rightarrow 10^2 = 4a(5)$$

$$\Rightarrow 100 = 20a$$

$$\Rightarrow a = 5$$

Focus is at $(a, 0)$, i.e., $(5, 0)$

Question 13.

Find co-ordinates of focus, vertex, and equation of directrix and the axis of the parabola $y = x^2 - 2x + 3$.

Solution:

Given equation of the parabola is $y = x^2 - 2x + 3$

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$$\Rightarrow y = x^2 - 2x + 1 + 2$$

$$\Rightarrow y - 2 = (x - 1)^2$$

$$\Rightarrow (x - 1)^2 = y - 2$$

Comparing this equation with $X^2 = 4bY$, we get

$$X = x - 1, Y = y - 2$$

$$\Rightarrow 4b = 1$$

$$\Rightarrow 4b = 1$$

$$\Rightarrow b = \frac{1}{4}$$

The co-ordinates of vertex are $(X = 0, Y = 0)$

$$\Rightarrow x - 1 = 0 \text{ and } y - 2 = 0$$

$$\Rightarrow x = 1 \text{ and } y = 2$$

The co-ordinates of vertex are $(1, 2)$.

The co-ordinates of focus are $S(X = 0, Y = b)$

$$\Rightarrow x - 1 = 0 \text{ and } y - 2 = \frac{1}{4}$$

$$\Rightarrow x = 1 \text{ and } y = \frac{9}{4}$$

The co-ordinates of focus are $(1, \frac{9}{4})$

Equation of the axis is $X = 0$

$$x - 1 = 0, \text{ i.e., } x = 1$$

Equation of directrix is $Y + b = 0$

$$\Rightarrow y - 2 + \frac{1}{4} = 0$$

$$\Rightarrow y - \frac{7}{4} = 0$$

$$\Rightarrow 4y - 7 = 0$$

Question 14.

Find the equation of tangent to the parabola

(i) $y^2 = 12x$ from the point (2, 5)

(ii) $y^2 = 36x$ from the point (2, 9)

Solution:

(i) Given equation of the parabola is $y^2 = 12x$.

Comparing this equation with $y^2 = 4ax$, we get

$$\Rightarrow 4a = 12$$

$$\Rightarrow a = 3$$

Equation of tangent to the parabola $y^2 = 4ax$ having slope m is

$$y = mx + \frac{a}{m}$$

Since the tangent passes through the point (2, 5)

$$\Rightarrow 5 = 2m + \frac{3}{m}$$

$$\Rightarrow 5m = 2m^2 + 3$$

$$\Rightarrow 2m^2 - 5m + 3 = 0$$

$$\Rightarrow 2m^2 - 2m - 3m + 3 = 0$$

$$\Rightarrow 2m(m - 1) - 3(m - 1) = 0$$

$$\Rightarrow (m - 1)(2m - 3) = 0$$

$$\Rightarrow m = 1 \text{ or } m = \frac{3}{2}$$

These are the slopes of the required tangents.

By slope point form, $y - y_1 = m(x - x_1)$, the equations of the tangents are

$$\Rightarrow y - 5 = 1(x - 2) \text{ and } y - 5 = \frac{3}{2}(x - 2)$$

$$\Rightarrow y - 5 = x - 2 \text{ and } 2y - 10 = 3x - 6$$

$$\Rightarrow x - y + 3 = 0 \text{ and } 3x - 2y + 4 = 0$$

(ii) Given equation of the parabola is $y^2 = 36x$.

Comparing this equation with $y^2 = 4ax$, we get

$$\Rightarrow 4a = 36$$

$$\Rightarrow a = 9$$

Equation of tangent to the parabola $y^2 = 4ax$ having slope m is

$$y = mx + \frac{a}{m}$$

Since the tangent passes through the point $(2, 9)$,

$$\Rightarrow 9 = 2m + \frac{9}{m}$$

$$\Rightarrow 9m = 2m^2 + 9$$

$$\Rightarrow 2m^2 - 9m + 9 = 0$$

$$\Rightarrow 2m^2 - 6m - 3m + 9 = 0$$

$$\Rightarrow 2m(m - 3) - 3(m - 3) = 0$$

$$\Rightarrow (m - 3)(2m - 3) = 0$$

$$\Rightarrow m = 3 \text{ or } m = \frac{3}{2}$$

These are the slopes of the required tangents.

By slope point form, $y - y_1 = m(x - x_1)$, the equations of the tangents are

$$\Rightarrow y - 9 = 3(x - 2) \text{ and } y - 9 = \frac{3}{2}(x - 2)$$

$$\Rightarrow y - 9 = 3x - 6 \text{ and } 2y - 18 = 3x - 6$$

$$\Rightarrow 3x - y + 3 = 0 \text{ and } 3x - 2y + 12 = 0$$

Question 15.

If the tangents drawn from the point $(-6, 9)$ to the parabola $y^2 = kx$ are perpendicular to each other, find k .

Solution:

Given equation of the parabola is $y^2 = kx$

Comparing this equation with $y^2 = 4ax$, we get

$$\Rightarrow 4a = k$$

$$\Rightarrow a = \frac{k}{4}$$

Equation of tangent to the parabola $y^2 = 4ax$ having slope m is

$$y = mx + \frac{a}{m}$$

Since the tangent passes through the point $(-6, 9)$,

$$\Rightarrow 9 = -6m + \frac{k}{4m}$$

$$\Rightarrow 36m = -24m^2 + k$$

$$\Rightarrow 24m^2 + 36m - k = 0$$

The roots m_1 and m_2 of this quadratic equation are the slopes of the tangents.

$$m_1 m_2 = \frac{-k}{24}$$

Since the tangents are perpendicular to each other,

$$m_1 m_2 = -1$$

$$\Rightarrow \frac{-k}{24} = -1$$

$$\Rightarrow k = 24$$

Alternate method:

We know that, tangents drawn from a point on directrix are perpendicular.

$(-6, 9)$ lies on the directrix $x = -a$.

$$\Rightarrow -6 = -a$$

$$\Rightarrow a = 6$$

Since $4a = k$

$$\Rightarrow k = 4(6) = 24$$

Question 16.

Two tangents to the parabola $y^2 = 8x$ meet the tangents at the vertex in the points P and Q . If

$PQ = 4$, prove that the equation of the locus of the point of intersection of two tangents is $y^2 =$

$8(x + 2)$.

Solution:

Given equation of the parabola is $y^2 = 8x$

Comparing this equation with $y^2 = 4ax$, we get

$$\Rightarrow 4a = 8$$

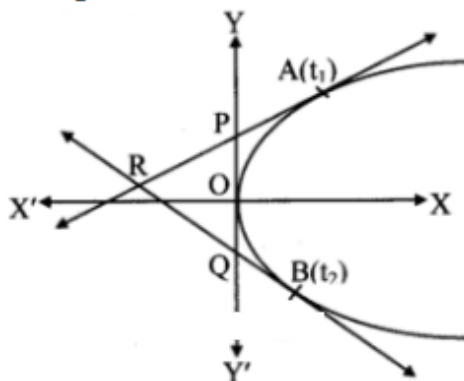
$$\Rightarrow a = 2$$

Equation of tangent to given parabola at $A(t_1)$ is y

$$t_1 = x + 2t_1^2 \dots\dots(i)$$

Equation of tangent to given parabola at $B(t_2)$ is y

$$t_2 = x + 2t_2^2 \dots\dots(ii)$$



A tangent at the vertex is Y-axis whose equation is $x = 0$.

x-coordinate of points P and Q is 0.

Let P be $(0, k_1)$ and Q be $(0, k_2)$.

Then, from (i) and (ii), we get

$$k_1 t_1 = 0 + 2t_1^2 \text{ and } k_2 t_2 = 0 + 2t_2^2$$

$$k_1 = 2t_1 \text{ and } k_2 = 2t_2$$

P is $(0, 2t_1)$ and Q is $(0, 2t_2)$

$$PQ = 2|t_1 - t_2|$$

But PQ is given to be 4.

$$2|t_1 - t_2| = 4$$

Let $R(x_1, y_1)$ be point of intersection of (i) and (ii).

$$y_1 t_1 = x_1 + 2t_1^2 \quad \dots(\text{iv})$$

$$\text{and } y_1 t_2 = x_1 + 2t_2^2$$

Subtracting, we get

$$y_1 t_1 - y_1 t_2 = 2t_1^2 - 2t_2^2$$

$$y_1(t_1 - t_2) = 2(t_1 + t_2)(t_1 - t_2)$$

$$y_1 = 2(t_1 + t_2) \quad \dots(\text{v}) \quad [\because t_1 \neq t_2]$$

$$y_1 t_1 = 2(t_1 + t_2) t_1$$

$$x_1 + 2t_1^2 = 2t_1^2 + 2t_1 t_2 \quad \dots[\text{From (iv)}]$$

$$x_1 = 2t_1 t_2 \quad \dots(\text{vi})$$

To find the equation of locus of $R(x_1, y_1)$, eliminate t_1 and t_2 from the equations (iii), (v) and (vi).

Squaring (v), we get

$$y_1^2 = 4(t_1 + t_2)^2$$

$$= 4[(t_1 - t_2)^2 + 4t_1 t_2]$$

$$= 4[4 + 2x_1] \quad \dots[\text{From (iii) and (vi)}]$$

$$y_1^2 = 8(x_1 + 2)$$

$\therefore$ Equation of locus of R is $y^2 = 8(x + 2)$.

Question 17.

Find the equation of common tangent to the parabolas $y^2 = 4x$ and $x^2 = 32y$.

Solution:

Given equation of the parabola is $y^2 = 4x$

Comparing this equation with $y^2 = 4ax$, we get

$$\Rightarrow 4a = 4$$

$$\Rightarrow a = 1$$

Let the equation of common tangent be

$$y = mx + \frac{1}{m} \quad \dots(\text{i})$$

Substituting $y = mx + \frac{1}{m}$ in $x^2 = 32y$, we get

$$\Rightarrow x^2 = 32\left(mx + \frac{1}{m}\right) = 32mx + \frac{32}{m}$$

$$\Rightarrow mx^2 = 32m^2x + 32$$

$$\Rightarrow mx^2 - 32m^2x - 32 = 0 \dots\dots(ii)$$

Line (i) touches the parabola $x^2 = 32y$.

The quadratic equation (ii) in x has equal roots.

Discriminant = 0

$$\Rightarrow (-32m^2)^2 - 4(m)(-32) = 0$$

$$\Rightarrow 1024m^4 + 128m = 0$$

$$\Rightarrow 128m(8m^3 + 1) = 0$$

$$\Rightarrow 8m^3 + 1 = 0 \dots\dots[\because m \neq 0]$$

$$\Rightarrow m^3 = -\frac{1}{8}$$

$$\Rightarrow m = -\frac{1}{2}$$

Substituting $m = -\frac{1}{2}$ in (i), we get

$$\Rightarrow y = -\frac{1}{2}x + \frac{1}{\left(-\frac{1}{2}\right)}$$

$$\Rightarrow y = -\frac{1}{2}x - 2$$

$\Rightarrow x + 2y + 4 = 0$, which is the equation of the common tangent.

Question 18.

Find the equation of the locus of a point, the tangents from which to the parabola $y^2 = 18x$ are such that sum of their slopes is -3.

Solution:

Given equation of the parabola is $y^2 = 18x$

Comparing this equation with $y^2 = 4ax$, we get

$$\Rightarrow 4a = 18$$

$$\Rightarrow 4a = 18$$

$$\Rightarrow a = \frac{9}{2}$$

Equation of tangent to the parabola $y^2 = 4ax$ having slope m is

$$\Rightarrow y = mx + \frac{a}{m}$$

$$\Rightarrow y = mx + \frac{\frac{9}{2}}{2m}$$

$$\Rightarrow 2ym = 2xm^2 + 9$$

$$\Rightarrow 2xm^2 - 2ym + 9 = 0$$

The roots m_1 and m_2 of this quadratic equation are the slopes of the tangents.

$$m_1 + m_2 = -\frac{(-2y)}{2x} = \frac{y}{x}$$

$$\text{But, } m_1 + m_2 = -3$$

$$\frac{y}{x} = -3$$

$y = -3x$, which is the required equation of locus.

Question 19.

The towers of a bridge, hung in the form of a parabola, have their tops 30 metres above the roadway and are 200 metres apart. If the cable is 5 metres above the roadway at the centre of the bridge, find the length of the vertical supporting cable 30 metres from the centre.

Solution:

Let CAB be the cable of the bridge and X'OX be the roadway.

Let A be the centre of the bridge.

From the figure, vertex of parabola is at A(0, 5).

Let the equation of parabola be

$$x^2 = 4b(y - 5) \dots\dots(i)$$

Since the parabola passes through (100, 30).

Substituting $x = 100$ and $y = 30$ in (i), we get

$$\Rightarrow 100^2 = 4b(30 - 5)$$

$$\Rightarrow 100^2 = 4b(25)$$

$$\Rightarrow 100^2 = 100b$$

$$\Rightarrow b = 100$$

Substituting the value of b in (i), we get

$$x^2 = 400(y - 5) \dots\dots(ii)$$

Let l metres be the length of vertical supporting cable.

Then P(30, l) lies on (ii).

$$\Rightarrow 30^2 = 400(l - 5)$$

$$\Rightarrow 900 = 400(l - 5)$$

$$\Rightarrow \frac{9}{4} = l - 5$$

$$\Rightarrow l = \frac{9}{4} + 5$$

$$\Rightarrow l = \frac{9}{4} \text{ m} = 7.25 \text{ m}$$

The length of the vertical supporting cable is 7.25 m.

Question 20.

A circle whose centre is (4, -1) passes through the focus of the parabola $x^2 + 16y = 0$. Show that the circle touches the directrix of the parabola.

Solution:

Given equation of the parabola is $x^2 + 16y = 0$.

$$\Rightarrow x^2 = -16y$$

Comparing this equation with $x^2 = -4by$, we get

$$\Rightarrow 4b = 16$$

$$\Rightarrow b = 4$$

Focus = S(0, -b) = (0, -4)

Centre of the circle is C(4, -1) and it passes through focus S of the parabola.

Radius = CS

$$= \sqrt{(4-0)^2 + (-1+4)^2}$$

$$= \sqrt{16+9}$$

$$= 5$$

Equation of the directrix is $y - b = 0$, i.e., $y - 4 = 0$

Length of the perpendicular from centre C(4, -1) to the directrix

$$= \left| \frac{0(4)+1(-1)-4}{\sqrt{(0)^2+(1)^2}} \right|$$

$$= \left| \frac{-1-4}{1} \right|$$

$$= 5$$

= radius

∴ The circle touches the directrix of the parabola.

Ex 7.2

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-1-chapter-7-conic-sections/>

Question 1.

Find the

- (i) lengths of the principal axes
- (ii) co-ordinates of the foci
- (iii) equations of directrices
- (iv) length of the latus rectum
- (v) distance between foci
- (vi) distance between directrices of the ellipse:

(a) $\frac{x^2}{25} + \frac{y^2}{9} = 1$

(b) $3x^2 + 4y^2 = 12$

(c) $2x^2 + 6y^2 = 6$

(d) $3x^2 + 4y^2 = 1$

Solution:

(a) Given equation of the ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1$

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$a^2 = 25$ and $b^2 = 9$

$a = 5$ and $b = 3$

Since $a > b$,

X-axis is the major axis and Y-axis is the minor axis.

(i) Length of major axis $= 2a = 2(5) = 10$

Length of minor axis $= 2b = 2(3) = 6$

Lengths of the principal axes are 10 and 6.

(ii) We know that $e = \frac{\sqrt{a^2 - b^2}}{a}$

$= \frac{\sqrt{25 - 9}}{5}$

$= \frac{\sqrt{16}}{5}$

$= \frac{4}{5}$

Co-ordinates of the foci are $S(ae, 0)$ and $S'(-ae, 0)$,

i.e., $S(5(\frac{4}{5}), 0)$ and $S'(-5(\frac{4}{5}), 0)$

i.e., $S(4, 0)$ and $S'(-4, 0)$

(iii) Equations of the directrices are $x = \pm \frac{a}{e}$

$$= \pm \frac{5}{\frac{4}{5}}$$

$$= \pm \frac{25}{4}$$

(iv) Length of latus rectum $= \frac{2b^2}{a} = \frac{2(3)^2}{5} = \frac{18}{5}$

(v) Distance between foci $= 2ae$

$$= 2(5)\left(\frac{4}{5}\right)$$

$$= 8$$

(vi) Distance between directrices $= \frac{2a}{e}$

$$= \frac{2(5)}{\frac{4}{5}}$$

$$= \frac{25}{2}$$

(b) Given equation of the ellipse is $3x^2 + 4y^2 = 12$

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = 4 \text{ and } b^2 = 3$$

$$a = 2 \text{ and } b = \sqrt{3}$$

Since $a > b$,

X-axis is the major axis and Y-axis is the minor axis.

(i) Length of major axis $= 2a = 2(2) = 4$

$$\text{Length of minor axis} = 2b = 2\sqrt{3}$$

Lengths of the principal axes are 4 and $2\sqrt{3}$.

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-1-chapter-7-conic-sections/>

(ii) We know that $e = \frac{\sqrt{a^2 - b^2}}{a}$

$$= \frac{\sqrt{4-3}}{2}$$

$$= \frac{1}{2}$$

Co-ordinates of the foci are $S(ae, 0)$ and $S'(-ae, 0)$,

i.e., $S(2(\frac{1}{2}), 0)$ and $S'(-2(\frac{1}{2}), 0)$

i.e., $S(1, 0)$ and $S'(-1, 0)$

(iii) Equations of the directrices are $x = \pm \frac{a}{e}$

$$= \pm \frac{2}{\frac{1}{2}}$$

$$= \pm 4$$

(iv) Length of latus rectum $= \frac{2b^2}{a} = \frac{2(\sqrt{3})^2}{2} = 3$

(v) Distance between foci $= 2ae = 2(2)(\frac{1}{2}) = 2$

(vi) Distance between directrices $= \frac{2a}{e}$

$$= \frac{2(2)}{\frac{1}{2}}$$

$$= 8$$

(c) Given equation of the ellipse is $2x^2 + 6y^2 = 6$

$$\frac{x^2}{3} + \frac{y^2}{1} = 1$$

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = 3 \text{ and } b^2 = 1$$

$$a = \sqrt{3} \text{ and } b = 1$$

Since $a > b$,

X-axis is the major axis and Y-axis is the minor axis.

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(i) Length of major axis = $2a = 2\sqrt{3}$

Length of minor axis = $2b = 2(1) = 2$

Lengths of the principal axes are $2\sqrt{3}$ and 2.

(ii) We know that $e = \frac{\sqrt{a^2 - b^2}}{a}$

$$= \frac{\sqrt{3-1}}{\sqrt{3}}$$

$$= \frac{\sqrt{2}}{\sqrt{3}}$$

Co-ordinates of the foci are $S(ae, 0)$ and $S'(-ae, 0)$,

i.e., $S(\sqrt{3}(\frac{\sqrt{2}}{\sqrt{3}}), 0)$ and $S'(-\sqrt{3}(\frac{\sqrt{2}}{\sqrt{3}}), 0)$

i.e., $S(\sqrt{2}, 0)$ and $S'(-\sqrt{2}, 0)$

(iii) Equations of the directrices are $x = \pm \frac{a}{e}$,

$$= \pm \frac{\sqrt{3}}{\frac{\sqrt{2}}{\sqrt{3}}}$$

$$= \pm \frac{3}{\sqrt{2}}$$

(iv) Length of latus rectum = $\frac{2b^2}{a} = \frac{2(1)^2}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

(v) Distance between foci = $2ae$

$$= 2(\sqrt{3}) \left(\frac{\sqrt{2}}{\sqrt{3}} \right)$$

$$= 2\sqrt{2}$$

$$= \frac{2\sqrt{3}}{\frac{\sqrt{2}}{\sqrt{3}}}$$

$$= \frac{2 \times 3}{\sqrt{2}}$$

$$= 3\sqrt{2}$$

(vi) Distance between directrices = $\frac{2a}{e}$

(d) Given equation of the ellipse is $3x^2 + 4y = 1$.

$$\frac{x^2}{\frac{1}{3}} + \frac{y^2}{\frac{1}{4}} = 1$$

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = \frac{1}{3} \text{ and } b^2 = \frac{1}{4}$$

$$a = \frac{1}{\sqrt{3}} \text{ and } b = \frac{1}{2}$$

Since $a > b$,

X-axis is the major axis and Y-axis is the minor axis.

$$(i) \text{ Length of major axis} = 2a = 2\left(\frac{1}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}$$

$$\text{Length of minor axis} = 2b = 2\left(\frac{1}{2}\right) = 1$$

Lengths of the principal axes are $\frac{2}{\sqrt{3}}$ and 1.

$$(ii) \text{ We know that } e = \frac{\sqrt{a^2 - b^2}}{a}$$

$$e = \frac{\sqrt{\frac{1}{3} - \frac{1}{4}}}{\frac{1}{\sqrt{3}}} = \frac{\sqrt{\frac{1}{12}}}{\frac{1}{\sqrt{3}}} = \sqrt{\frac{3}{12}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

Co-ordinates of the foci are $S(ae, 0)$ and $S'(-ae, 0)$,

$$\text{i.e., } S\left(\frac{1}{\sqrt{3}}\left(\frac{1}{2}\right), 0\right) \text{ and } S'\left(-\frac{1}{\sqrt{3}}\left(\frac{1}{2}\right), 0\right)$$

$$\text{i.e., } S\left(\frac{1}{2\sqrt{3}}, 0\right) \text{ and } S'\left(-\frac{1}{2\sqrt{3}}, 0\right)$$

(iii) Equations of the directrices are $x = \pm \frac{a}{e}$,

$$= \pm \frac{\frac{1}{\sqrt{3}}}{\frac{1}{2}}$$

$$\begin{aligned}\text{(iv) Length of latus rectum} &= \frac{2b^2}{a} \\ &= \frac{2\left(\frac{1}{2}\right)^2}{\frac{1}{\sqrt{3}}} \\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

$$\begin{aligned}\text{(v) Distance between foci} &= 2ae \\ &= 2\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{2}\right) \\ &= \frac{1}{\sqrt{3}}\end{aligned}$$

$$\begin{aligned}\text{(vi) Distance between directrices} &= \frac{2a}{e} \\ &= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{\frac{1}{2}} \\ &= \frac{4}{\sqrt{3}}\end{aligned}$$

Question 2.

Find the equation of the ellipse in standard form if

- (i) eccentricity = $\frac{3}{8}$ and distance between its foci = 6.
- (ii) the length of the major axis is 10 and the distance between foci is 8.
- (iii) distance between directrices is 18 and eccentricity is $\frac{1}{3}$.
- (iv) minor axis is 16 and eccentricity is $\frac{1}{3}$.
- (v) the distance between foci is 6 and the distance between directrices is $\frac{50}{3}$.
- (vi) the latus rectum has length 6 and foci are $(\pm 2, 0)$.

(vii) passing through the points $(-3, 1)$ and $(2, -2)$.

(viii) the distance between its directrices is 10 and which passes through $(-\sqrt{5}, 2)$.

(ix) eccentricity is $\frac{2}{3}$ and passes through $(2, \frac{-5}{3})$.

Solution:

(i) Let the required equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$.

Given, eccentricity $(e) = \frac{3}{8}$

Distance between foci = $2ae$

Given, distance between foci = 6

$$2ae = 6$$

$$2a\left(\frac{3}{8}\right) = 6$$

$$\frac{3a}{4} = 6$$

$$a = \frac{6 \times 4}{3} = 8$$

$$a^2 = 64$$

$$\text{Now, } b^2 = a^2(1 - e^2)$$

$$= 64 \left[1 - \left(\frac{3}{8}\right)^2 \right]$$

$$= 64 \left(1 - \frac{9}{64} \right) = 64 \left(\frac{55}{64} \right) = 55$$

The required equation of ellipse is $\frac{x^2}{64} + \frac{y^2}{55} = 1$.

(ii) Let the required equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$.

Length of major axis = $2a$

Given, length of major axis = 10

$$2a = 10$$

$$a^2 = 25$$

Distance between foci = $2ae$

Given, distance between foci = 8

$$2ae = 8$$

$$2(5)e = 8$$

$$e = \frac{8}{10} = \frac{4}{5}$$

$$\text{Now, } b^2 = a^2 (1 - e^2)$$

$$= 25 \left[1 - \left(\frac{4}{5} \right)^2 \right]$$

$$= 25 \left(1 - \frac{16}{25} \right) = 25 \left(\frac{9}{25} \right) = 9$$

The required equation of ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

(iii) Let the required equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$.

Given, eccentricity (e) = $\frac{1}{3}$

Distance between directrices = $\frac{2a}{e}$

Given, distance between directrices = 18

$$\frac{2a}{e} = 18$$

$$\frac{2a}{\frac{1}{3}} = 18$$

$$6a = 18$$

$$a = \frac{18}{6} = 3$$

$$a^2 = 9$$

$$\text{Now, } b^2 = a^2 (1 - e^2)$$

$$= 9 \left[1 - \left(\frac{1}{3} \right)^2 \right]$$

$$= 9 \left(1 - \frac{1}{9} \right) = 9 \left(\frac{8}{9} \right) = 8$$

The required equation of ellipse is $\frac{x^2}{9} + \frac{y^2}{8} = 1$

(iv) Let the required equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$.

Length of minor axis = $2b$

Given, length of minor axis = 16

$$2b = 16$$

$$b = 8$$

$$b = 8$$

$$b^2 = 64$$

$$\text{Given, eccentricity (e)} = \frac{1}{3}$$

$$\text{Now, } b^2 = a^2 (1 - e^2)$$

$$64 = a^2 \left[1 - \left(\frac{1}{3} \right)^2 \right]$$

$$64 = a^2 \left(1 - \frac{1}{9} \right)$$

$$64 = a^2 \left(\frac{8}{9} \right)$$

$$a^2 = \frac{64 \times 9}{8} = 72$$

The required equation of ellipse is $\frac{x^2}{72} + \frac{y^2}{64} = 1$.

(v) Let the required equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$.

Distance between foci = $2ae$

Given, distance between foci = 6

$$2ae = 6$$

$$ae = 3$$

$$a = \frac{3}{e} \dots\dots(i)$$

Distance between directrices = $\frac{2a}{e}$

Given, distance between directrices = $\frac{50}{3}$

$$\frac{2a}{e} = \frac{50}{3}$$

$$\frac{a}{e} = \frac{25}{3}$$

$$\frac{3}{e} = \frac{25}{3}$$

...[From (i)]

$$\frac{3}{e^2} = \frac{25}{3}$$

$$e^2 = \frac{9}{25}$$

$$e = \frac{3}{5} \quad \dots[\because 0 < e < 1]$$

Substituting $e = \frac{3}{5}$ in (i), we get

$$a = \frac{3}{\frac{3}{5}}$$

$$a = 5$$

$$a^2 = 25$$

$$\text{Now, } b^2 = a^2 (1 - e^2)$$

$$\begin{aligned} &= 25 \left[1 - \left(\frac{3}{5} \right)^2 \right] \\ &= 25 \left(1 - \frac{9}{25} \right) = 25 \left(\frac{16}{25} \right) = 16 \end{aligned}$$

The required equation of ellipse is $\frac{x^2}{25} + \frac{y^2}{16} = 1$.

(vi) Given, the length of the latus rectum is 6, and co-ordinates of foci are $(\pm 2, 0)$.

The foci of the ellipse are on the X-axis.

Let the required equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$.

Length of latus rectum = $\frac{2b^2}{a}$

$$\frac{2b^2}{a} = 6$$

$$b^2 = 3a \dots\dots(i)$$

Co-ordinates of foci are $(\pm ae, 0)$

$$ae = 2$$

$$a^2 e^2 = 4 \dots\dots(ii)$$

$$\text{Now, } b^2 = a^2 (1 - e^2)$$

$$b^2 = a^2 - a^2 e^2$$

$$3a = a^2 - 4 \dots\dots[\text{From (i) and (ii)}]$$

$$a^2 - 3a - 4 = 0$$

$$a^2 - 4a + a - 4 = 0$$

$$a(a - 4) + 1(a - 4) = 0$$

$$(a - 4)(a + 1) = 0$$

$$a - 4 = 0 \text{ or } a + 1 = 0$$

$$a - 4 = 0 \text{ or } a + 1 = 0$$

$$a = 4 \text{ or } a = -1$$

Since $a = -1$ is not possible,

$$a = 4$$

$$a^2 = 16$$

Substituting $a = 4$ in (i), we get

$$b^2 = 3(4) = 12$$

The required equation of ellipse is $\frac{x^2}{16} + \frac{y^2}{12} = 1$.

(vii) Let the required equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$.

The ellipse passes through the points $(-3, 1)$ and $(2, -2)$.

Substituting $x = -3$ and $y = 1$ in equation of ellipse, we get

$$\frac{(-3)^2}{a^2} + \frac{1^2}{b^2} = 1$$

$$\frac{9}{a^2} + \frac{1}{b^2} = 1 \quad \dots(i)$$

Substituting $x = 2$ and $y = -2$ in equation of ellipse, we get

$$\frac{2^2}{a^2} + \frac{(-2)^2}{b^2} = 1$$

$$\frac{4}{a^2} + \frac{4}{b^2} = 1 \quad \dots(ii)$$

$$\text{Let } \frac{1}{a^2} = A \text{ and } \frac{1}{b^2} = B$$

Equations (i) and (ii) become

$$9A + B = 1 \quad \dots(iii)$$

$$4A + 4B = 1 \quad \dots(iv)$$

$$4A + 4B = 1 \dots (iv)$$

Multiplying (iii) by 4, we get

$$36A + 4B = 4 \dots (v)$$

Subtracting (iv) from (v), we get

$$32A = 3$$

$$A = \frac{3}{32}$$

Substituting $A = \frac{3}{32}$ in (iv), we get

$$4\left(\frac{3}{32}\right) + 4B = 1$$

$$\frac{3}{8} + 4B = 1$$

$$4B = 1 - \frac{3}{8}$$

$$4B = \frac{5}{8}$$

$$B = \frac{5}{32}$$

Since $\frac{1}{a^2} = A$ and $\frac{1}{b^2} = B$,

$$\frac{1}{a^2} = \frac{3}{32} \text{ and } \frac{1}{b^2} = \frac{5}{32}$$

$$a^2 = \frac{32}{3} \text{ and } b^2 = \frac{32}{5}$$

The required equation of ellipse is

$$\frac{x^2}{\left(\frac{32}{3}\right)} + \frac{y^2}{\left(\frac{32}{5}\right)}, \text{ i.e., } 3x^2 + 5y^2 = 32.$$

(viii) Let the required equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$.

$$\text{Distance between directrices} = \frac{2a}{e}$$

Given, distance between directrices = 10

$$\frac{2a}{e} = 10$$

$$a = 5e \dots\dots(i)$$

The ellipse passes through $(-\sqrt{5}, 2)$.

Substituting $x = -\sqrt{5}$ and $y = 2$ in equation of ellipse, we get

$$\frac{(-\sqrt{5})^2}{a^2} + \frac{2^2}{b^2} = 1$$

$$\frac{5}{a^2} + \frac{4}{b^2} = 1$$

$$\frac{5}{a^2} + \frac{4}{a^2(1-e^2)} = 1 \quad \dots[\because b^2 = a^2(1-e^2)]$$

Multiplying throughout by a^2 , we get

$$5 + \frac{4}{1-e^2} = a^2$$

$$5 + \frac{4}{1-e^2} = (5e)^2 \quad \dots[\text{From (i)}]$$

$$5(1-e^2) + 4 = 25e^2(1-e^2)$$

$$5 - 5e^2 + 4 = 25e^2 - 25e^4$$

$$25e^4 - 30e^2 + 9 = 0$$

$$(5e^2 - 3)^2 = 0$$

$$5e^2 - 3 = 0$$

$$e^2 = \frac{3}{5}$$

$$\text{From (i), } a = 5e$$

$$a^2 = 25e^2 = 25 \times \frac{3}{5}$$

$$a^2 = 15$$

We know that,

$$b^2 = a^2(1-e^2)$$

$$b^2 = 15 \left(1 - \frac{3}{5}\right)$$

$$b^2 = 15\left(\frac{2}{5}\right)$$

$$b^2 = 6$$

The required equation of ellipse is $\frac{x^2}{15} + \frac{y^2}{6} = 1$.

(ix) Let the required equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$.

Given, eccentricity $(e) = \frac{2}{3}$

The ellipse passes through $(2, \frac{-5}{3})$.

Substituting $x = 2$ and $y = \frac{-5}{3}$ in equation of ellipse, we get

$$\frac{2^2}{a^2} + \frac{\left(\frac{-5}{3}\right)^2}{b^2} = 1$$

$$\frac{4}{a^2} + \frac{25}{9b^2} = 1$$

$$\frac{4}{a^2} + \frac{25}{9a^2(1-e^2)} = 1 \quad \dots [\because b^2 = a^2(1-e^2)]$$

Multiplying throughout by a^2 , we get

$$4 + \frac{25}{9(1-e^2)} = a^2$$

$$4 + \frac{25}{9\left[1-\left(\frac{2}{3}\right)^2\right]} = a^2$$

$$4 + \frac{25}{9\left(1-\frac{4}{9}\right)} = a^2$$

$$4 + \frac{25}{5} = a^2$$

$$4 + 5 = a^2$$

$$a^2 = 9$$

$$\text{Now, } b^2 = a^2 (1 - e^2)$$

$$= 9 \left[1 - \left(\frac{2}{3} \right)^2 \right]$$

$$= 9 \left(1 - \frac{4}{9} \right) = 9 \left(\frac{5}{9} \right) = 5$$

The required equation of ellipse is $\frac{x^2}{9} + \frac{y^2}{5} = 1$.

Question 3.

Find the eccentricity of an ellipse, if the length of its latus rectum is one-third of its minor axis.

Solution:

Let the equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$.

$$\text{Length of latus rectum} = \frac{2b^2}{a}$$

$$\text{Length of minor axis} = 2b$$

According to the given condition,

$$\text{Length of latus rectum} = \frac{1}{3} (\text{Minor axis})$$

$$\frac{2b^2}{a} = \frac{1}{3} (2b)$$

$$\frac{b}{a} = \frac{1}{3}$$

$$b = \frac{1}{3} a$$

$$b^2 = \frac{1}{9} a^2 \quad \dots(i)$$

$$\text{Now, } b^2 = a^2 (1 - e^2)$$

$$\frac{1}{9} a^2 = a^2 (1 - e^2) \quad \dots[\text{From (i)}]$$

$$\frac{1}{9} = 1 - e^2$$

$$e^2 = 1 - \frac{1}{9}$$

$$e^2 = \frac{8}{9}$$

$$e = \frac{2\sqrt{2}}{3} \quad \dots[\because 0 < e < 1]$$

Question 4.

Find the eccentricity of an ellipse, if the distance between its directrices is three times the distance between its foci.

Solution:

Let the required equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$.

$$\text{Distance between directrices} = \frac{2a}{e}$$

$$\text{Distance between foci} = 2ae$$

According to the given condition,

$$\text{distance between directrices} = 3(\text{distance between foci})$$

$$\frac{2a}{e} = 3(2ae)$$

$$\frac{1}{e} = 3e$$

$$\frac{1}{e} = 3e$$

$$\frac{1}{3} = e^2$$

$$e = \frac{1}{\sqrt{3}} \dots\dots [\because 0 < e < 1]$$

Eccentricity of the ellipse is $\frac{1}{\sqrt{3}}$

Question 5.

Show that the product of the lengths of the perpendicular segments drawn from the foci to any tangent line to the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is equal to 16.

Solution:

Given equation of the ellipse is $\frac{x^2}{25} + \frac{y^2}{16} = 1$.

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = 25, b^2 = 16$$

$$a = 5, b = 4$$

We know that $e = \frac{\sqrt{a^2 - b^2}}{a}$

$$e = \frac{\sqrt{25 - 16}}{5} = \frac{\sqrt{9}}{5} = \frac{3}{5}$$

$$ae = 5 \left(\frac{3}{5} \right) = 3$$

Co-ordinates of foci are $S(ae, 0)$ and $S'(-ae, 0)$,

i.e., $S(3, 0)$ and $S'(-3, 0)$

Equations of tangents to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ having slope } m \text{ are}$$

$$y = mx \pm \sqrt{a^2 m^2 + b^2}$$

Equation of one of the tangents to the ellipse is

$$y = mx + \sqrt{25m^2 + 16}$$

$$mx - y + \sqrt{25m^2 + 16} = 0 \quad \dots(i)$$

p_1 = length of perpendicular segment from $S(3, 0)$ to the tangent (i)

$$= \left| \frac{m(3) - 0 + \sqrt{25m^2 + 16}}{\sqrt{m^2 + 1}} \right|$$

$$p_1 = \left| \frac{3m + \sqrt{25m^2 + 16}}{\sqrt{m^2 + 1}} \right|$$

p_2 = length of perpendicular segment from $S'(-3, 0)$ to the tangent (i)

$$= \left| \frac{m(-3) - 0 + \sqrt{25m^2 + 16}}{\sqrt{m^2 + 1}} \right|$$

$$p_2 = \left| \frac{-3m + \sqrt{25m^2 + 16}}{\sqrt{m^2 + 1}} \right|$$

$$\begin{aligned} p_1 p_2 &= \left| \frac{3m + \sqrt{25m^2 + 16}}{\sqrt{m^2 + 1}} \right| \left| \frac{-3m + \sqrt{25m^2 + 16}}{\sqrt{m^2 + 1}} \right| \\ &= \frac{(25m^2 + 16) - 9m^2}{m^2 + 1} = \frac{16(m^2 + 1)}{m^2 + 1} = 16 \end{aligned}$$

Question 6.

A tangent having slope $\left(-\frac{1}{2}\right)$ the ellipse $3x^2 + 4y^2 = 12$ intersects the X and Y axes in the points A and B respectively. If O is the origin, find the area of the triangle AOB.

Solution:

Given equation of the ellipse is $3x^2 + 4y^2 = 12$.

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = 4, b^2 = 3$$

Equations of tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ having slope m are

$$y = mx \pm \sqrt{a^2 m^2 + b^2}$$

$$\text{Here, } m = -\frac{1}{2}$$

Equations of the tangents are

$$y = \frac{-1}{2}x \pm \sqrt{4\left(-\frac{1}{2}\right)^2 + 3} = \frac{-1}{2}x \pm 2$$

$$2y = -x \pm 4$$

$$x + 2y \pm 4 = 0$$

Consider the tangent $x + 2y - 4 = 0$

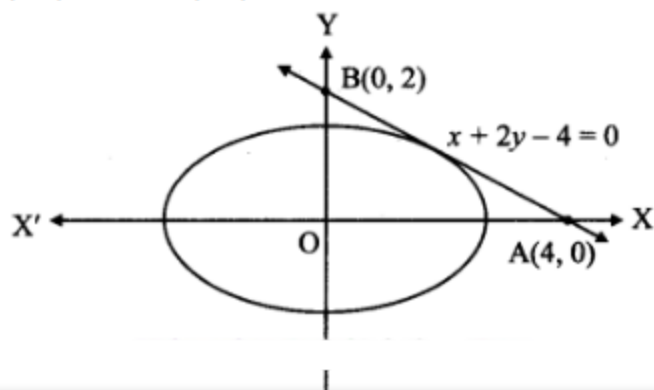
Let this tangent intersect the X-axis at $A(x_1, 0)$ and Y-axis at $B(0, y_1)$.

$$x_1 + 0 - 4 = 0 \text{ and } 0 + 2y_1 - 4 = 0$$

$$x_1 = 4 \text{ and } y_1 = 2$$

$$A = (4, 0) \text{ and } B = (0, 2)$$

$$l(OA) = 4 \text{ and } l(OB) = 2$$



$$\begin{aligned}\text{Area of } \triangle AOB &= \frac{1}{2} \times l(OA) \times l(OB) \\ &= \frac{1}{2} \times 4 \times 2 \\ &= 4 \text{ sq.units}\end{aligned}$$

Question 7.

Show that the line $x - y = 5$ is a tangent to the ellipse $9x^2 + 16y^2 = 144$. Find the point of contact.

Solution:

Given equation of the ellipse is $9x^2 + 16y^2 = 144$

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = 16 \text{ and } b^2 = 9$$

Given equation of line is $x - y = 5$, i.e., $y = x - 5$

$$c^2 = a^2 m^2 + b^2$$

Comparing this equation with $y = mx + c$, we get

$$m = 1 \text{ and } c = -5$$

For the line $y = mx + c$ to be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we must have

$$c^2 = a^2 m^2 + b^2$$

$$c^2 = (-5)^2 = 25$$

$$a^2 m^2 + b^2 = 16(1)^2 + 9 = 16 + 9 = 25 = c^2$$

The given line is a tangent to the given ellipse and point of contact

$$\begin{aligned}&= \left(\frac{-a^2 m}{c}, \frac{b^2}{c} \right) \\ &= \left(\frac{(-16)(1)}{-5}, \frac{9}{-5} \right) \\ &= \left(\frac{16}{5}, -\frac{9}{5} \right)\end{aligned}$$

Question 8.

Show that the line $8y + x = 17$ touches the ellipse $x^2 + 4y^2 = 17$. Find the point of contact.

Solution:

Given equation of the ellipse is $x^2 + 4y^2 = 17$.

$$\frac{x^2}{17} + \frac{y^2}{\frac{17}{4}} = 1$$

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = 17 \text{ and } b^2 = \frac{17}{4}$$

Given equation of line is $8y + x = 17$,

$$y = \frac{-1}{8}x + \frac{17}{8}$$

Comparing this equation with $y = mx + c$, we get

$$m = \frac{-1}{8} \text{ and } c = \frac{17}{8}$$

For the line $y = mx + c$ to be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we must have

$$c^2 = a^2 m^2 + b^2$$

$$c^2 = \left(\frac{17}{8}\right)^2 = \frac{289}{64}$$

$$a^2 m^2 + b^2 = 17 \left(\frac{-1}{8}\right)^2 + \frac{17}{4} = \frac{17}{64} + \frac{17}{4} = \frac{289}{64} = c^2$$

The given line touches the given ellipse and point of contact is

$$\begin{aligned} \left(\frac{-a^2 m}{c}, \frac{b^2}{c}\right) &= \left(\frac{-17\left(\frac{-1}{8}\right)}{\frac{17}{8}}, \frac{\frac{17}{4}}{\frac{17}{8}}\right) \\ &= (1, 2) \end{aligned}$$

Question 9.

Determine whether the line $x + 3y\sqrt{2} = 9$ is a tangent to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. If so, find the co-ordinates of the point of contact.

Solution:

Given equation of the ellipse is $\frac{x^2}{9} + \frac{y^2}{4} = 1$

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = 9 \text{ and } b^2 = 4$$

Given equation of line is $x + 3y\sqrt{2} = 9$,

$$\text{i.e., } y = \frac{-1}{3\sqrt{2}}x + \frac{3}{\sqrt{2}}$$

Comparing this equation with $y = mx + c$, we get

$$m = \frac{-1}{3\sqrt{2}} \text{ and } c = \frac{3}{\sqrt{2}}$$

For the line $y = mx + c$ to be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we must have

$$c^2 = a^2 m^2 + b^2$$

$$c^2 = \left(\frac{3}{\sqrt{2}}\right)^2 = \frac{9}{2}$$

$$a^2 m^2 + b^2 = 9 \left(\frac{-1}{3\sqrt{2}}\right)^2 + 4 = \frac{1}{2} + 4 = \frac{9}{2} = c^2$$

The given line is a tangent to the given ellipse and point of contact is

$$\left(\frac{-a^2 m}{c}, \frac{b^2}{c}\right) = \left(\frac{(-9)\left(\frac{-1}{3\sqrt{2}}\right)}{\frac{3}{\sqrt{2}}}, \frac{4}{\frac{3}{\sqrt{2}}}\right) = \left(1, \frac{4\sqrt{2}}{3}\right)$$



Question 10.

Find k , if the line $3x + 4y + k = 0$ touches $9x^2 + 16y^2 = 144$.

Solution:

Given equation of the ellipse is $9x^2 + 16y^2 = 144$.

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = 16 \text{ and } b^2 = 9$$

Given equation of line is $3x + 4y + k = 0$,

$$\text{i.e., } y = -\frac{3}{4}x - \frac{k}{4}$$

Comparing this equation with $y = mx + c$, we get

$$m = -\frac{3}{4} \text{ and } c = -\frac{k}{4}$$

For the line $y = mx + c$ to be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we must have

$$c^2 = a^2 m^2 + b^2$$

$$\left(-\frac{k}{4}\right)^2 = 16\left(-\frac{3}{4}\right)^2 + 9$$

$$\frac{k^2}{16} = 9 + 9$$

$$\frac{k^2}{16} = 18$$

$$k^2 = 288$$

$$k = \pm 12\sqrt{2}$$

Question 11.

Find the equations of the tangents to the ellipse:

(i) $\frac{x^2}{5} + \frac{y^2}{4} = 1$ passing through the point $(2, -2)$.

(ii) $4x^2 + 7y^2 = 28$ from the point (3, -2).

(iii) $2x^2 + y^2 = 6$ from the point (2, 1).

(iv) $x^2 + 4y^2 = 9$ which are parallel to the line $2x + 3y - 5 = 0$.

(v) $\frac{x^2}{25} + \frac{y^2}{4} = 1$ which are parallel to the line $x + y + 1 = 0$.

(vi) $5x^2 + 9y^2 = 45$ which are $\perp$ to the line $3x + 2y + 1 = 0$.

(vii) $x^2 + 4y^2 = 20$ which are $\perp$ to the line $4x + 3y = 7$.

Solution:

(i) Given equation of the ellipse is $\frac{x^2}{5} + \frac{y^2}{4} = 1$.

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = 5 \text{ and } b^2 = 4$$

Equations of tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ having slope m are

$$y = mx \pm \sqrt{a^2 m^2 + b^2}$$

Since (2, -2) lies on both the tangents,

$$-2 = 2m \pm \sqrt{5m^2 + 4}$$

$$-2 - 2m = \pm \sqrt{5m^2 + 4}$$

Squaring both the sides, we get

$$4m^2 + 8m + 4 = 5m^2 + 4$$

$$m^2 - 8m = 0$$

$$m(m - 8) = 0$$

$$m = 0 \text{ or } m = 8$$

These are the slopes of the required tangents.

By slope point form $y - y_1 = m(x - x_1)$,

the equations of the tangents are

$$y + 2 = 0(x - 2) \text{ and } y + 2 = 8(x - 2)$$

$$y + 2 = 0 \text{ and } y + 2 = 8x - 16$$

$$y + 2 = 0 \text{ and } 8x - y - 18 = 0$$

(ii) Given equation of the ellipse is $4x^2 + 7y^2 = 28$.

$$\frac{x^2}{7} + \frac{y^2}{4} = 1$$

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = 7 \text{ and } b^2 = 4$$

Equations of tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ having slope m are

$$y = mx \pm \sqrt{a^2 m^2 + b^2}$$

Since $(3, -2)$ lies on both the tangents,

$$-2 = 3m \pm \sqrt{7m^2 + 4}$$

$$-2 - 3m = \pm \sqrt{7m^2 + 4}$$

Squaring both the sides, we get

$$9m^2 + 12m + 4 = 7m^2 + 4$$

$$2m^2 + 12m = 0$$

$$2m(m + 6) = 0$$

$$m = 0 \text{ or } m = -6$$

These are the slopes of the required tangents.

By slope point form $y - y_1 = m(x - x_1)$,

the equations of the tangents are

$$y + 2 = 0(x - 3) \text{ and } y + 2 = -6(x - 3)$$

$$y + 2 = 0 \text{ and } y + 2 = -6x + 18$$

$$y + 2 = 0 \text{ and } 6x + y - 16 = 0$$

(iii) Given equation of the ellipse is $2x^2 + y^2 = 6$.

$$\frac{x^2}{3} + \frac{y^2}{6} = 1$$

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = 3 \text{ and } b^2 = 6$$

Equations of tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ having slope m are

$$y = mx \pm \sqrt{a^2m^2 + b^2}$$

Since $(2, 1)$ lies on both the tangents,

$$1 = 2m \pm \sqrt{3m^2 + 6}$$

$$1 - 2m = \pm \sqrt{3m^2 + 6}$$

Squaring both the sides, we get

$$1 - 4m + 4m^2 = 3m^2 + 6$$

$$m^2 - 4m - 5 = 0$$

$$(m - 5)(m + 1) = 0$$

$$m = 5 \text{ or } m = -1$$

These are the slopes of the required tangents.

By slope point form $y - y_1 = m(x - x_1)$,

the equations of the tangents are

$$y - 1 = 5(x - 2) \text{ and } y - 1 = -1(x - 2)$$

$$y - 1 = 5x - 10 \text{ and } y - 1 = -x + 2$$

$$5x - y - 9 = 0 \text{ and } x + y - 3 = 0$$

(iv) Given equation of the ellipse is $x^2 + 4y^2 = 9$.

$$\frac{x^2}{9} + \frac{y^2}{\frac{9}{4}} = 1$$

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = 9 \text{ and } b^2 = \frac{9}{4}$$

Slope of the line $2x + 3y - 5 = 0$ is $-\frac{2}{3}$.

Since the given line is parallel to the required tangents, slope of the required tangents is

$$m = -\frac{2}{3}$$

Equations of tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ having slope m are

$$y = mx \pm \sqrt{a^2 m^2 + b^2}$$

$$y = -\frac{2x}{3} \pm \sqrt{9\left(-\frac{2}{3}\right)^2 + \frac{9}{4}}$$

$$y = -\frac{2x}{3} \pm \sqrt{4 + \frac{9}{4}}$$

$$y = -\frac{2x}{3} \pm \sqrt{\frac{25}{4}}$$

$$y = -\frac{2x}{3} \pm \frac{5}{2}$$

$$4x + 6y = \pm 15$$

(v) Given equation of the ellipse is $\frac{x^2}{25} + \frac{y^2}{4} = 1$.

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = 25 \text{ and } b^2 = 4$$

Slope of the given line $x + y + 1 = 0$ is -1 .

Since the given line is parallel to the required tangents, the slope of the required tangents is $m = -1$.

Equations of tangents to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ having slope } m \text{ are}$$

$$y = mx \pm \sqrt{a^2 m^2 + b^2}$$

$$y = -x \pm \sqrt{25(-1)^2 + 4}$$

$$y = -x \pm \sqrt{29}$$

$$x + y = \pm \sqrt{29}$$

(vi) Given equation of the ellipse is $5x^2 + 9y^2 = 45$.

$$\frac{x^2}{9} + \frac{y^2}{5} = 1$$

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = 9 \text{ and } b^2 = 5$$

Slope of the given line $3x + 2y + 1 = 0$ is $-\frac{3}{2}$

Since the given line is perpendicular to the required tangents, slope of the required tangents is

$$m = \frac{2}{3}$$

Equations of tangents to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ having slope } m \text{ are}$$

$$y = mx \pm \sqrt{a^2 m^2 + b^2}$$

$$y = \frac{2}{3}x \pm \sqrt{9\left(\frac{2}{3}\right)^2 + 5} = \frac{2}{3}x \pm \sqrt{9\left(\frac{4}{9}\right) + 5}$$

$$y = \frac{2}{3}x \pm 3$$

$$2x - 3y = \pm 9$$

(vii) Given equation of the ellipse is $x^2 + 4y^2 = 20$.

$$\frac{x^2}{20} + \frac{y^2}{5} = 1$$

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = 20 \text{ and } b^2 = 5$$

Slope of the given line $4x + 3y = 7$ is $-\frac{4}{3}$.

Since the given line is perpendicular to the required tangents,

slope of the required tangents is $m = \frac{3}{4}$.

Equations of tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ having slope m are

$$y = mx \pm \sqrt{a^2 m^2 + b^2}$$

$$y = \frac{3}{4}x \pm \sqrt{20\left(\frac{3}{4}\right)^2 + 5}$$

$$y = \frac{3}{4}x \pm \sqrt{\frac{45}{4} + 5}$$

$$y = \frac{3}{4}x \pm \frac{\sqrt{65}}{2}$$

$$4y = 3x \pm 2\sqrt{65}$$

$$3x - 4y = \pm 2\sqrt{65}$$

Question 12.

Find the equation of the locus of a point, the tangents from which to the ellipse $3x^2 + 5y^2 = 15$ are at right angles.

Solution:

Given equation of the ellipse is $3x^2 + 5y^2 = 15$.

Given equation of the ellipse is $3x^2 + 5y^2 = 15$.

$$\frac{x^2}{5} + \frac{y^2}{3} = 1$$

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = 5 \text{ and } b^2 = 3$$

Equations of tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ having slope m are

$$y = mx \pm \sqrt{a^2 m^2 + b^2}$$

$$y = mx \pm \sqrt{5m^2 + 3}$$

$$y - mx = \pm \sqrt{5m^2 + 3}$$

Squaring both the sides, we get

$$y^2 - 2mxy + m^2x^2 = 5m^2 + 3$$

$$(x^2 - 5)m^2 - 2xym + (y^2 - 3) = 0$$

The roots m_1 and m_2 of this quadratic equation are the slopes of the tangents.

$$m_1 m_2 = \frac{y^2 - 3}{x^2 - 5}$$

Since the tangents are at right angles,

$$m_1 m_2 = -1$$

$$\frac{y^2 - 3}{x^2 - 5} = -1$$

$$y^2 - 3 = -x^2 + 5$$

$$x^2 + y^2 = 8, \text{ which is the required equation of the locus.}$$

Alternate method:

The locus of the point of intersection of perpendicular tangents is the director circle of an ellipse.

The equation of the director circle of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $x^2 + y^2 = a^2 + b^2$

Here, $a^2 = 5$ and $b^2 = 3$

$$x^2 + y^2 = 5 + 3$$

Question 13.

Tangents are drawn through a point P to the ellipse $4x^2 + 5y^2 = 20$ having inclinations θ_1 and θ_2 such that $\tan \theta_1 + \tan \theta_2 = 2$. Find the equation of the locus of P.

Solution:

Given equation of the ellipse is $4x^2 + 5y^2 = 20$.

$$\frac{x^2}{5} + \frac{y^2}{4} = 1$$

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = 5 \text{ and } b^2 = 4$$

Since inclinations of tangents are θ_1 and θ_2 ,

$$m_1 = \tan \theta_1 \text{ and } m_2 = \tan \theta_2$$

Equation of tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ having slope m are

$$y = mx \pm \sqrt{a^2 m^2 + b^2}$$

$$y = mx \pm \sqrt{5 m^2 + 4}$$

$$y - mx = \pm \sqrt{5 m^2 + 4}$$

Squaring both the sides, we get

$$y^2 - 2mxy + m^2 x^2 = 5m^2 + 4$$

$$(x^2 - 5)m^2 - 2xym + (y^2 - 4) = 0$$

The roots m_1 and m_2 of this quadratic equation are the slopes of the tangents.

$$m_1 + m_2 = \frac{-(-2xy)}{x^2 - 5} = \frac{2xy}{x^2 - 5}$$

$$\text{Given, } \tan \theta_1 + \tan \theta_2 = 2$$

$$m_1 + m_2 = 2$$

$$\frac{2xy}{x^2 - 5} = 2$$

$$xy = x^2 - 5$$

$$x^2 - xy - 5 = 0, \text{ which is the required equation of the locus of P.}$$

Question 14.

Show that the locus of the point of intersection of tangents at two points on an ellipse, whose eccentric angles differ by a constant, is an ellipse.

Solution:

Let $P(\theta_1)$ and $Q(\theta_2)$ be any two points on the given ellipse such that $\theta_1 - \theta_2 = k$, where k is a constant.

The equation of the tangent at point $P(\theta_1)$ is

$$\frac{x \cos \theta_1}{a} + \frac{y \sin \theta_1}{b} = 1 \dots\dots(i)$$

The equation of the tangent at point $Q(\theta_2)$ is

$$\frac{x \cos \theta_2}{a} + \frac{y \sin \theta_2}{b} = 1 \dots\dots(ii)$$

Multiplying equation (i) by $\cos \theta_2$ and equation (ii) by $\cos \theta_1$ and subtracting, we get

$$\frac{y}{b} (\sin \theta_1 \cos \theta_2 - \sin \theta_2 \cos \theta_1) = \cos \theta_2 - \cos \theta_1$$

$$\frac{y}{b} [\sin (\theta_1 - \theta_2)] = \cos \theta_2 - \cos \theta_1$$

$$\frac{y}{b} \sin k = \cos \theta_2 - \cos \theta_1$$

$$\frac{y}{b} = \frac{\cos \theta_2 - \cos \theta_1}{\sin k} \dots\dots(iii)$$

Similarly,

Multiplying equation (i) by $\sin \theta_2$ and equation (ii) by $\sin \theta_1$ and subtracting, we get

$$\frac{x}{a} (\cos \theta_1 \sin \theta_2 - \cos \theta_2 \sin \theta_1) = \sin \theta_2 - \sin \theta_1$$

$$-\frac{x}{a} [\sin (\theta_1 - \theta_2)] = \sin \theta_2 - \sin \theta_1$$

$$-\frac{x}{a} \sin k = \sin \theta_2 - \sin \theta_1$$

$$-\frac{x}{a} = \frac{\sin \theta_2 - \sin \theta_1}{\sin k} \quad \dots (iv)$$

Squaring (iii) and (iv) and adding, we get

$$\begin{aligned} \frac{x^2}{a^2} + \frac{y^2}{b^2} &= \frac{(\cos \theta_2 - \cos \theta_1)^2 + (\sin \theta_2 - \sin \theta_1)^2}{\sin^2 k} \\ &= \frac{1}{\sin^2 k} [\cos^2 \theta_2 + \sin^2 \theta_2 + \cos^2 \theta_1 + \sin^2 \theta_1 \\ &\quad - 2(\sin \theta_1 \sin \theta_2 + \cos \theta_1 \cos \theta_2)] \\ \frac{x^2}{a^2} + \frac{y^2}{b^2} &= \frac{2 - 2 \cos (\theta_1 - \theta_2)}{\sin^2 k} \\ &= \frac{2 - 2 \cos k}{\sin^2 k} \\ &= \frac{2(1 - \cos k)}{\sin^2 k} = \frac{4 \sin^2 \frac{k}{2}}{4 \sin^2 \frac{k}{2} \cos^2 \frac{k}{2}} \end{aligned}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \sec^2 \frac{k}{2}, \text{ which is an ellipse.}$$

Question 15.

P and Q are two points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with eccentric angles θ_1 and θ_2 . Find the equation of the locus of the point of intersection of the tangents at P and Q if $\theta_1 + \theta_2 = \frac{\pi}{2}$.

Solution:

Given equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

θ_1 and θ_2 are the eccentric angles of a tangent.

Equation of tangent at point P is

$$\frac{x}{a} \cos \theta_1 + \frac{y}{b} \sin \theta_1 = 1 \dots\dots(i)$$

Equation of tangent at point Q is

$$\frac{x}{a} \cos \theta_2 + \frac{y}{b} \sin \theta_2 = 1 \dots\dots(ii)$$

$$\theta_1 + \theta_2 = \frac{\pi}{2} \dots\dots[\text{Given}]$$

$$\theta_2 = \frac{\pi}{2} - \theta_1$$

$$\frac{x}{a} \cos \left(\frac{\pi}{2} - \theta_1 \right) + \frac{y}{b} \sin \left(\frac{\pi}{2} - \theta_1 \right) = 1$$

$$\frac{x}{a} \sin \theta_1 + \frac{y}{b} \cos \theta_1 = 1 \dots(ii)$$

From (i) and (ii), we get

$$\frac{x}{a} \cos \theta_1 + \frac{y}{b} \sin \theta_1 = \frac{x}{a} \sin \theta_1 + \frac{y}{b} \cos \theta_1$$

Let $M(x_1, y_1)$ be the point of intersection of the tangents.

$$\frac{x_1}{a} \cos \theta_1 + \frac{y_1}{b} \sin \theta_1 = \frac{x_1}{a} \sin \theta_1 + \frac{y_1}{b} \cos \theta_1$$

$$\frac{x_1}{a} (\cos \theta_1 - \sin \theta_1) = \frac{y_1}{b} (\cos \theta_1 - \sin \theta_1) \dots(iii)$$

If $\cos \theta_1 - \sin \theta_1 = 0$,

$$\cos \theta_1 = \sin \theta_1$$

$$\tan \theta_1 = 1$$

$$\theta_1 = \frac{\pi}{4}$$

$$\text{Since } \theta_1 + \theta_2 = \frac{\pi}{2}, \theta_2 = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

i.e., points P and Q coincide, which is not possible, as P and Q are two different points.

$$\cos \theta_1 - \sin \theta_1 \neq 0$$

Dividing equation (iii) by $(\cos \theta_1 - \sin \theta_1)$, we get

$$\frac{x_1}{a} = \frac{y_1}{b}$$

$$bx_1 - ay_1 = 0$$

$bx - ay = 0$, which is the required equation of locus of point M.

Question 16.

The eccentric angles of two points P and Q of the ellipse $4x^2 + y^2 = 4$ differ by $\frac{2\pi}{3}$. Show that the locus of the point of intersection of the tangents at P and Q is the ellipse $4x^2 + y^2 = 16$.

Solution:

Given equation of the ellipse is $4x^2 + y^2 = 4$.

$$\frac{x^2}{1} + \frac{y^2}{4} = 1$$

Let $P(\theta_1)$ and $Q(\theta_2)$ be any two points on the given ellipse such that

$$\theta_1 - \theta_2 = \frac{2\pi}{3}$$

The equation of the tangent at point $P(\theta_1)$ is

$$\frac{x \cos \theta_1}{1} + \frac{y \sin \theta_1}{2} = 1 \dots\dots(i)$$

The equation of the tangent at point $Q(\theta_2)$ is

$$\frac{x \cos \theta_2}{1} + \frac{y \sin \theta_2}{2} = 1$$

Multiplying equation (i) by $\cos \theta_2$ and equation (ii) by $\cos \theta_1$ and subtracting, we get

$$\frac{y}{2} (\sin \theta_1 \cos \theta_2 - \sin \theta_2 \cos \theta_1) = \cos \theta_2 - \cos \theta_1$$

$$\frac{y}{2} [\sin (\theta_1 - \theta_2)] = \cos \theta_2 - \cos \theta_1$$

$$\frac{y}{2} \left[\sin \left(\frac{2\pi}{3} \right) \right] = \cos \theta_2 - \cos \theta_1$$

$$\frac{y}{2} \sin \left(\pi - \frac{\pi}{3} \right) = \cos \theta_2 - \cos \theta_1$$

$$\frac{y}{2} \sin \left(\frac{\pi}{3} \right) = \cos \theta_2 - \cos \theta_1$$

$$\frac{y}{2} \left(\frac{\sqrt{3}}{2} \right) = \cos \theta_2 - \cos \theta_1$$

$$\frac{\sqrt{3} y}{4} = \cos \theta_2 - \cos \theta_1 \quad \dots(iii)$$

Multiplying equation (i) by $\sin \theta_2$ and equation (ii) by $\sin \theta_1$ and subtracting, we get

$$x(\sin \theta_2 \cos \theta_1 - \cos \theta_2 \sin \theta_1) = \sin \theta_2 - \sin \theta_1$$

$$-x \sin (\theta_1 - \theta_2) = \sin \theta_2 - \sin \theta_1$$

$$-x \sin \left(\frac{2\pi}{3} \right) = \sin \theta_2 - \sin \theta_1$$

$$-x \sin \left(\pi - \frac{\pi}{3} \right) = \sin \theta_2 - \sin \theta_1$$

$$-x \sin \frac{\pi}{3} = \sin \theta_2 - \sin \theta_1$$

$$-\frac{\sqrt{3}}{2} x = \sin \theta_2 - \sin \theta_1 \quad \dots(iv)$$

Squaring (iii) and (iv) and adding, we get

$$\begin{aligned} \frac{3x^2}{4} + \frac{3y^2}{16} &= \sin^2 \theta_2 - 2\sin \theta_2 \sin \theta_1 + \sin^2 \theta_1 \\ &\quad + \cos^2 \theta_2 - 2\cos \theta_2 \cos \theta_1 + \cos^2 \theta_1 \\ &= (\cos^2 \theta_2 + \sin^2 \theta_2) + (\cos^2 \theta_1 + \sin^2 \theta_1) \\ &\quad - 2\cos \theta_2 \cos \theta_1 - 2\sin \theta_2 \sin \theta_1 \\ &= 1 + 1 - 2(\cos \theta_2 \cos \theta_1 + \sin \theta_2 \sin \theta_1) \\ &= 2 - 2[\cos (\theta_1 - \theta_2)] \end{aligned}$$

$$= 2 - 2 [\cos (\theta_1 - \theta_2)]$$

$$= 2 - 2 \cos \left(\frac{2\pi}{3} \right)$$

$$= 2 - 2 \left(\frac{-1}{2} \right)$$

$$= 2 + 1$$

$$\frac{3x^2}{4} + \frac{3y^2}{16} = 3$$

$$\frac{x^2}{4} + \frac{y^2}{16} = 1$$

$4x^2 + y^2 = 16$, which is the required equation of locus.

Question 17.

Find the equations of the tangents to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, making equal intercepts on co-ordinate axes.

Solution:

Given equation of the ellipse is $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = 16 \text{ and } b^2 = 9$$

Since the tangents make equal intercepts on the co-ordinate axes,

$$m = -1.$$

Equations of tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ having slope m are

$$y = mx \pm \sqrt{a^2 m^2 + b^2}$$

$$\begin{aligned} \bar{y} &= -x \pm \sqrt{16(-1)^2 + 9} \\ y &= -x \pm \sqrt{25} \\ x + y &= \pm 5 \end{aligned}$$

Question 18.

A tangent having slope $\left(-\frac{1}{2}\right)$ to the ellipse $3x^2 + 4y^2 = 12$ intersects the X and Y axes in the points A and B respectively. If O is the origin, find the area of the triangle AOB.

Solution:

The equation of the ellipse is $3x^2 + 4y^2 = 12$

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

Comparing with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a^2 = 4, b^2 = 3$$

The equation of tangent with slope m is

$$y = mx \pm \sqrt{a^2m^2 + b^2}$$

$$\text{i.e., } y = mx \pm \sqrt{4m^2 + 3} \quad \dots [\because a^2 = 4, b^2 = 3]$$

$$\therefore y = -\frac{1}{2}x \pm \sqrt{4\left(\frac{1}{4}\right) + 3} \quad \dots \left[\because m = -\frac{1}{2}\right]$$

$$\therefore y = -\frac{x}{2} \pm 2$$

$$\therefore x + 2y \pm 4 = 0 \quad \dots(1)$$

It meets X axis at A

$\therefore$ for A, put $y = 0$ in equation (1), we get,

$$x = \pm 4$$

$$\therefore A = (\pm 4, 0)$$

Similarly, $B = (0, \pm 2)$

$$\therefore OA = 4, OB = 2$$

$$\therefore \text{Area of } \triangle OAB = \frac{1}{2} \times OA \times OB$$

$$= \frac{1}{2} \times 4 \times 2$$

$$= 4 \text{ sq. units}$$

Ex 7.3

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Question 1.

Find the length of the transverse axis, length of conjugate axis, the eccentricity, the co-ordinates of foci, equations of directrices, and the length of the latus rectum of the hyperbolae.

(i) $\frac{x^2}{25} - \frac{y^2}{16} = 1$

(ii) $\frac{x^2}{25} - \frac{y^2}{16} = -1$

(iii) $16x^2 - 9y^2 = 144$

(iv) $21x^2 - 4y^2 = 84$

(v) $3x^2 - y^2 = 4$

(vi) $x^2 - y^2 = 16$

(vii) $\frac{y^2}{25} - \frac{x^2}{9} = 1$

(viii) $\frac{y^2}{25} - \frac{x^2}{144} = 1$

(ix) $\frac{x^2}{100} - \frac{y^2}{25} = 1$

(x) $x = 2 \sec \theta, y = 2\sqrt{3} \tan \theta$

Solution:

(i) Given equation of the hyperbola is $\frac{x^2}{25} - \frac{y^2}{16} = 1$

Comparing this equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get

$$a^2 = 25 \text{ and } b^2 = 16$$

$$\Rightarrow a = 5 \text{ and } b = 4$$

$$\text{Length of transverse axis} = 2a = 2(5) = 10$$

$$\text{Length of conjugate axis} = 2b = 2(4) = 8$$

We know that

$$e = \frac{\sqrt{a^2 + b^2}}{a} = \frac{\sqrt{25+16}}{5} = \frac{\sqrt{41}}{5}$$

Co-ordinates of foci are $S(ae, 0)$ and $S'(-ae, 0)$,

$$\text{i.e., } S\left(5\left(\frac{\sqrt{41}}{5}\right), 0\right) \text{ and } S'\left(-5\left(\frac{\sqrt{41}}{5}\right), 0\right),$$

$$\text{i.e., } S(\sqrt{41}, 0) \text{ and } S'(-\sqrt{41}, 0)$$

Equations of the directrices are $x = \pm \frac{a}{e}$.

$$x = \pm \frac{5}{\left(\frac{\sqrt{41}}{5}\right)}$$

$$x = \pm \frac{25}{\sqrt{41}}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2(16)}{5} = \frac{32}{5}$$

(ii) Given equation of the hyperbola is $\frac{x^2}{25} - \frac{y^2}{16} = -1$

$$\frac{y^2}{16} - \frac{x^2}{25} = 1$$

Comparing this equation with $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, we get

$$b^2 = 16 \text{ and } a^2 = 25$$

$$\Rightarrow b = 4 \text{ and } a = 5$$

$$\text{Length of transverse axis} = 2b = 2(4) = 8$$

$$\text{Length of conjugate axis} = 2a = 2(5) = 10$$

Co-ordinates of vertices are B(0, b) and B' (0, -b)

i.e., B(0, 4) and B'(0, -4)

We know that

$$e = \frac{\sqrt{b^2 + a^2}}{b} = \frac{\sqrt{16 + 25}}{4} = \frac{\sqrt{41}}{4}$$

Co-ordinates of foci are S(0, be) and S'(0, -be),

$$\text{i.e., } S\left(0, 4\left(\frac{\sqrt{41}}{4}\right)\right) \text{ and } S'\left(0, -4\left(\frac{\sqrt{41}}{4}\right)\right),$$

$$\text{i.e., } S(0, \sqrt{41}) \text{ and } S'(0, -\sqrt{41})$$

$$\text{Equations of the directrices are } y = \pm \frac{b}{e}.$$

$$y = \pm \frac{4}{\frac{\sqrt{41}}{4}}$$

$$y = \pm \frac{16}{\sqrt{41}}$$

$$\text{Length of latus-rectum} = \frac{2a^2}{b} = \frac{2(25)}{4} = \frac{25}{2}$$

(iii) Given equation of the hyperbola is $16x^2 - 9y^2 = 144$.

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

Comparing this equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get

$$a^2 = 9 \text{ and } b^2 = 16$$

$$\Rightarrow a = 3 \text{ and } b = 4$$

$$\text{Length of transverse axis} = 2a = 2(3) = 6$$

$$\text{Length of conjugate axis} = 2b = 2(4) = 8$$

We know that

$$e = \frac{\sqrt{a^2 + b^2}}{a} = \frac{\sqrt{9+16}}{3} = \frac{\sqrt{25}}{3} = \frac{5}{3}$$

Co-ordinates of foci are $S(ae, 0)$ and $S'(-ae, 0)$,

i.e., $S(3(\frac{5}{3}), 0)$ and $S'(-3(\frac{5}{3}), 0)$

i.e., $S(5, 0)$ and $S'(-5, 0)$

Equations of the directrices are $x = \pm \frac{a}{e}$

$$= \pm \frac{3}{\left(\frac{5}{3}\right)}$$

$$= \pm \frac{9}{5}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2(16)}{3} = \frac{32}{3}$$

(iv) Given equation of the hyperbola is $21x^2 - 4y^2 = 84$.

$$\frac{x^2}{4} - \frac{y^2}{21} = 1$$

Comparing this equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get

$$a^2 = 4 \text{ and } b^2 = 21$$

$$\Rightarrow a = 2 \text{ and } b = \sqrt{21}$$

$$\text{Length of transverse axis} = 2a = 2(2) = 4$$

$$\text{Length of conjugate axis} = 2b = 2\sqrt{21}$$

We know that

$$e = \frac{\sqrt{a^2 + b^2}}{a} = \frac{\sqrt{4 + 21}}{2} = \frac{\sqrt{25}}{2} = \frac{5}{2}$$

Co-ordinates of foci are $S(ae, 0)$ and $S'(-ae, 0)$,

$$\text{i.e., } S\left(2\left(\frac{5}{2}\right), 0\right) \text{ and } S'\left(-2\left(\frac{5}{2}\right), 0\right),$$

$$\text{i.e., } S(5, 0) \text{ and } S'(-5, 0)$$

Equations of the directrices are $x = \pm \frac{a}{e}$.

$$x = \pm \frac{2}{\left(\frac{5}{2}\right)}$$

$$x = \pm \frac{4}{5}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2(21)}{2} = 21$$

(v) Given equation of the hyperbola is $3x^2 - y^2 = 4$.

$$\frac{x^2}{\left(\frac{4}{3}\right)} - \frac{y^2}{4} = 1$$

Comparing this equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get

$$a^2 = \frac{4}{3} \text{ and } b^2 = 4$$

$$a = \frac{2}{\sqrt{3}} \text{ and } b = 2$$

$$\text{Length of transverse axis} = 2a = 2\left(\frac{2}{\sqrt{3}}\right) = \frac{4}{\sqrt{3}}$$

$$\text{Length of conjugate axis} = 2b = 2(2) = 4$$

We know that

$$e = \frac{\sqrt{a^2 + b^2}}{a} = \frac{\sqrt{\frac{4}{3} + 4}}{\left(\frac{2}{\sqrt{3}}\right)} = \frac{\sqrt{\frac{16}{3}}}{\left(\frac{2}{\sqrt{3}}\right)} = \frac{4}{2} = 2$$

Co-ordinates of foci are $S(ae, 0)$ and $S'(-ae, 0)$,

$$\text{i.e., } S\left(\frac{2}{\sqrt{3}}(2), 0\right) \text{ and } S'\left(-\frac{2}{\sqrt{3}}(2), 0\right),$$

$$\text{i.e., } S\left(\frac{4}{\sqrt{3}}, 0\right) \text{ and } S'\left(-\frac{4}{\sqrt{3}}, 0\right)$$

Equations of the directrices are $x = \pm \frac{a}{e}$.

$$x = \pm \frac{\left(\frac{2}{\sqrt{3}}\right)}{2}$$

$$x = \pm \frac{1}{\sqrt{3}}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2(4)}{\left(\frac{2}{\sqrt{3}}\right)} = 4\sqrt{3}$$

(vi) Given equation of the hyperbola is $x^2 - y^2 = 16$.

$$\frac{x^2}{16} - \frac{y^2}{16} = 1$$

Comparing this equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get

$$a^2 = 16 \text{ and } b^2 = 16$$

$$\Rightarrow a = 4 \text{ and } b = 4$$

$$\text{Length of transverse axis} = 2a = 2(4) = 8$$

$$\text{Length of conjugate axis} = 2b = 2(4) = 8$$

We know that

$$e = \frac{\sqrt{a^2 + b^2}}{a} = \frac{\sqrt{16 + 16}}{4} = \frac{\sqrt{32}}{4} = \frac{4\sqrt{2}}{4} = \sqrt{2}$$

Co-ordinates of foci are $S(ae, 0)$ and $S'(-ae, 0)$,

i.e., $S(4\sqrt{2}, 0)$ and $S'(-4\sqrt{2}, 0)$

Equations of the directrices are $x = \pm \frac{a}{e}$

$$\Rightarrow x = \pm \frac{4}{\sqrt{2}}$$

$$\Rightarrow x = \pm 2\sqrt{2}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2(16)}{4} = 8$$

(vii) Given equation of the hyperbola is $\frac{y^2}{25} - \frac{x^2}{9} = 1$.

Comparing this equation with $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, we get

$$b^2 = 25 \text{ and } a^2 = 9$$

$$\Rightarrow b = 5 \text{ and } a = 3$$

$$\text{Length of transverse axis} = 2b = 2(5) = 10$$

$$\text{Length of conjugate axis} = 2a = 2(3) = 6$$

Co-ordinates of vertices are $B(0, b)$ and $B'(0, -b)$,

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i.e., $B(0, 5)$ and $B'(0, -5)$

We know that

$$e = \frac{\sqrt{b^2 + a^2}}{b} = \frac{\sqrt{25+9}}{5} = \frac{\sqrt{34}}{5}$$

Co-ordinates of foci are $S(0, be)$ and $S'(0, -be)$,

$$\text{i.e., } S\left(0, 5\left(\frac{\sqrt{34}}{5}\right)\right) \text{ and } S'\left(0, -5\left(\frac{\sqrt{34}}{5}\right)\right),$$

$$\text{i.e., } S(0, \sqrt{34}) \text{ and } S'(0, -\sqrt{34})$$

Equations of the directrices are $y = \pm \frac{b}{e}$.

$$y = \pm \frac{5}{\left(\frac{\sqrt{34}}{5}\right)}$$

$$y = \pm \frac{25}{\sqrt{34}}$$

$$\text{Length of latus-rectum} = \frac{2a^2}{b} = \frac{2(9)}{5} = \frac{18}{5}$$

(viii) Given equation of the hyperbola is $\frac{y^2}{25} - \frac{x^2}{144} = 1$.

Comparing this equation with $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, we get

$$b^2 = 25 \text{ and } a^2 = 144$$

$$\Rightarrow b = 5 \text{ and } a = 12$$

$$\text{Length of transverse axis} = 2b = 2(5) = 10$$

$$\text{Length of conjugate axis} = 2a = 2(12) = 24$$

Co-ordinates of vertices are $B(0, b)$ and $B'(0, -b)$,

i.e., $B(0, 5)$ and $B'(0, -5)$

We know that

$$e = \frac{\sqrt{b^2 + a^2}}{b} = \frac{\sqrt{25 + 144}}{5} = \frac{\sqrt{169}}{5} = \frac{13}{5}$$

Co-ordinates of foci are $S(0, be)$ and $S'(0, -be)$,

$$\text{i.e., } S\left(0, 5\left(\frac{13}{5}\right)\right) \text{ and } S'\left(0, -5\left(\frac{13}{5}\right)\right),$$

i.e., $S(0, 13)$ and $S'(0, -13)$

Equations of the directrices are $y = \pm \frac{b}{e}$.

$$y = \pm \frac{5}{\left(\frac{13}{5}\right)}$$

$$y = \pm \frac{25}{13}$$

$$\text{Length of latus-rectum} = \frac{2a^2}{b} = \frac{2(144)}{5} = \frac{288}{5}$$

(ix) Given equation of the hyperbola is $\frac{x^2}{100} - \frac{y^2}{25} = 1$

Comparing this equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get

$$a^2 = 100 \text{ and } b^2 = 25$$

$$\Rightarrow a = 10 \text{ and } b = 5$$

$$\text{Length of transverse axis} = 2a = 2(10) = 20$$

$$\text{Length of conjugate axis} = 2b = 2(5) = 10$$

We know that

$$\begin{aligned} e &= \frac{\sqrt{a^2 + b^2}}{a} = \frac{\sqrt{100 + 25}}{10} \\ &= \frac{\sqrt{125}}{10} \\ &= \frac{5\sqrt{5}}{10} = \frac{\sqrt{5}}{2} \end{aligned}$$

Co-ordinates of foci are $S(ae, 0)$ and $S'(-ae, 0)$,

$$\text{i.e., } S\left(10\left(\frac{\sqrt{5}}{2}\right), 0\right) \text{ and } S'\left(-10\left(\frac{\sqrt{5}}{2}\right), 0\right),$$

$$\text{i.e., } S(5\sqrt{5}, 0) \text{ and } S'(-5\sqrt{5}, 0)$$

Equations of the directrices are $x = \pm \frac{a}{e}$.

$$x = \pm \frac{10}{\left(\frac{\sqrt{5}}{2}\right)}$$

$$x = \pm \frac{20}{\sqrt{5}}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2(25)}{10} = 5$$

(x) Given equation of the hyperbola is $x = 2 \sec \theta$, $y = 2\sqrt{3} \tan \theta$.

Since $\sec^2 \theta - \tan^2 \theta = 1$,

$$\left(\frac{x}{2}\right)^2 - \left(\frac{y}{2\sqrt{3}}\right)^2 = 1$$

$$\frac{x^2}{4} - \frac{y^2}{12} = 1$$

Comparing this equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get

$$a^2 = 4 \text{ and } b^2 = 12$$

$$\Rightarrow a = 2 \text{ and } b = 2\sqrt{3}$$

$$\text{Length of transverse axis} = 2a = 2(2) = 4$$

$$\text{Length of conjugate axis} = 2b = 2(2\sqrt{3}) = 4\sqrt{3}$$

We know that

$$e = \frac{\sqrt{a^2 + b^2}}{a} = \frac{\sqrt{4 + 12}}{2} = 2$$

Co-ordinates of foci are $S(ae, 0)$ and $S'(-ae, 0)$,

i.e., $S(2(2), 0)$ and $S'(-2(2), 0)$,

i.e., $S(4, 0)$ and $S'(-4, 0)$

Equations of the directrices are $x = \pm \frac{a}{e}$.

$$\Rightarrow x = \pm \frac{2}{2}$$

$$\Rightarrow x = \pm 1$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2(12)}{2} = 12$$

Question 2.

Find the equation of the hyperbola with centre at the origin, length of the conjugate axis as 10, and one of the foci as $(-7, 0)$.

Solution:

Given, one of the foci of the hyperbola is $(-7, 0)$.

Since this focus lies on the X-axis, it is a standard hyperbola.

Let the required equation of hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Length of conjugate axis = $2b$

Given, length of conjugate axis = 10

$$\Rightarrow 2b = 10$$

$$\Rightarrow b = 5$$

$$\Rightarrow b^2 = 25$$

Co-ordinates of focus are $(-ae, 0)$

$$ae = 7$$

$$\Rightarrow a^2e^2 = 49$$

Equations of the directrices are $x = \pm \frac{a}{e}$.

$$\Rightarrow x = \pm \frac{2}{2}$$

$$\Rightarrow x = \pm 1$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2(12)}{2} = 12$$

Question 2.

Find the equation of the hyperbola with centre at the origin, length of the conjugate axis as 10, and one of the foci as $(-7, 0)$.

Solution:

Given, one of the foci of the hyperbola is $(-7, 0)$.

Since this focus lies on the X-axis, it is a standard hyperbola.

Let the required equation of hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Length of conjugate axis = $2b$

Given, length of conjugate axis = 10

$$\Rightarrow 2b = 10$$

$$\Rightarrow b = 5$$

$$\Rightarrow b^2 = 25$$

Co-ordinates of focus are $(-ae, 0)$

$$ae = 7$$

$$\Rightarrow a^2e^2 = 49$$

$$\Rightarrow 25 = 49 - a^2$$

$$\Rightarrow a^2 = 49 - 25 = 24$$

The required equation of hyperbola is $\frac{x^2}{24} - \frac{y^2}{25} = 1$

Question 3.

Find the eccentricity of the hyperbola, which is conjugate to the hyperbola $x^2 - 3y^2 = 3$

Solution:

Given, equation of hyperbola is $x^2 - 3y^2 = 3$.

$$\frac{x^2}{3} - \frac{y^2}{1} = 1$$

Equation of the hyperbola conjugate to the above hyperbola is $\frac{y^2}{1} - \frac{x^2}{3} = 1$

Comparing this equation with $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, we get

$$b^2 = 1 \text{ and } a^2 = 3$$

$$\text{Now, } a^2 = b^2(e^2 - 1)$$

$$\Rightarrow 3 = 1(e^2 - 1)$$

$$\Rightarrow 3 = e^2 - 1$$

$$\Rightarrow e^2 = 4$$

$$\Rightarrow e = 2 \dots [\because e > 1]$$

Question 4.

If e and e' are the eccentricities of a hyperbola and its conjugate hyperbola respectively, prove

$$\text{that } \frac{1}{e^2} + \frac{1}{(e')^2} = 1.$$

Solution:

Let e be the eccentricity of a hyperbola

Also, e' is the eccentricity of conjugate

$$\text{hyperbola } \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

$$e' = \frac{\sqrt{b^2 + a^2}}{b}$$

$$(e')^2 = \frac{b^2 + a^2}{b^2}$$

$$\frac{1}{(e')^2} = \frac{b^2}{b^2 + a^2} \quad \dots(ii)$$

Adding (i) and (ii), we get

$$\frac{1}{e^2} + \frac{1}{(e')^2} = \frac{a^2}{a^2 + b^2} + \frac{b^2}{a^2 + b^2} = \frac{a^2 + b^2}{a^2 + b^2}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

$$e = \frac{\sqrt{a^2 + b^2}}{a}$$

$$e^2 = \frac{a^2 + b^2}{a^2}$$

$$\frac{1}{e^2} = \frac{a^2}{a^2 + b^2}$$

...(i)

$$\frac{1}{e^2} + \frac{1}{(e')^2} = 1$$

Question 5.

Find the equation of the hyperbola referred to its principal axes:

- (i) whose distance between foci is 10 and eccentricity is $\frac{5}{2}$
- (ii) whose distance between foci is 10 and length of the conjugate axis is 6.
- (iii) whose distance between directrices is $\frac{8}{3}$ and eccentricity is $\frac{3}{2}$.
- (iv) whose length of conjugate axis = 12 and passing through (1, -2).
- (v) which passes through the points (6, 9) and (3, 0).
- (vi) whose vertices are $(\pm 7, 0)$ and endpoints of the conjugate axis are $(0, \pm 3)$.
- (vii) whose foci are at $(\pm 2, 0)$ and eccentricity is $\frac{3}{2}$.
- (viii) whose lengths of transverse and conjugate axes are 6 and 9 respectively.
- (ix) whose length of transverse axis is 8 and distance between foci is 10.

Solution:

(i) Let the required equation of hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Given, eccentricity $(e) = \frac{5}{2}$

Distance between foci = $2ae$

Given, distance between foci = 10

$$\Rightarrow 2ae = 10$$

$$\Rightarrow ae = 5$$

$$\Rightarrow a\left(\frac{5}{2}\right) = 5$$

$$\Rightarrow a = 2$$

$$\Rightarrow a^2 = 4$$

$$\text{Now, } b^2 = a^2(e^2 - 1)$$

$$b^2 = 4 \left[\left(\frac{5}{2} \right)^2 - 1 \right]$$

$$= 4 \left(\frac{25}{4} - 1 \right)$$

$$= 4 \left(\frac{21}{4} \right)$$

$$b^2 = 21$$

The required equation of hyperbola is

$$\frac{x^2}{4} - \frac{y^2}{21} = 1.$$

(ii) Let the required equation of hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Length of conjugate axis = $2b$

Given, length of conjugate axis = 6

$$\Rightarrow 2b = 6$$

$$\Rightarrow b = 3$$

$$\Rightarrow b^2 = 9$$

Distance between foci = $2ae$

Given, distance between foci = 10

$$\Rightarrow 2ae = 10$$

$$\Rightarrow ae = 5$$

$$\Rightarrow a^2 e^2 = 25$$

$$\text{Now, } b^2 = a^2 (e^2 - 1)$$

$$\Rightarrow b^2 = a^2 e^2 - a^2$$

$$\Rightarrow 9 = 25 - a^2$$

$$\Rightarrow a^2 = 25 - 9$$

$$\Rightarrow a^2 = 16$$

The required equation of hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = 1$

(iii) Let the required equation of hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Given, eccentricity (e) = $\frac{3}{2}$

Distance between directrices = $\frac{2a}{e}$

Given, distance between directrices = $\frac{8}{3}$

$$\frac{2a}{e} = \frac{8}{3}$$

$$\frac{2a}{\frac{3}{2}} = \frac{8}{3}$$

$$\frac{4a}{3} = \frac{8}{3}$$

$$a = 2$$

$$a^2 = 4$$

$$\text{Now, } b^2 = a^2 (e^2 - 1)$$

$$b^2 = 4 \left[\left(\frac{3}{2} \right)^2 - 1 \right]$$

$$= 4 \left(\frac{9}{4} - 1 \right)$$

$$= 4 \left(\frac{5}{4} \right)$$

$$b^2 = 5$$

The required equation of hyperbola is

$$\frac{x^2}{4} - \frac{y^2}{5} = 1.$$

(iv) Let the required equation of hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \dots\dots(i)$$

Length of conjugate axis = $2b$

Given, length of conjugate axis = 12

$$\Rightarrow 2b = 12$$

$$\Rightarrow b = 6 \dots\dots(ii)$$

$$\Rightarrow b^2 = 36$$

The hyperbola passes through (1, -2)

Substituting $x = 1$ and $y = -2$ in (i), we get

$$\frac{1^2}{a^2} - \frac{(-2)^2}{b^2} = 1$$

$$\frac{1}{a^2} - \frac{4}{b^2} = 1$$

$$\frac{1}{a^2} - \frac{4}{6^2} = 1 \quad \dots[\text{From(ii)}]$$

$$\frac{1}{a^2} - \frac{4}{36} = 1$$

$$\frac{1}{a^2} = 1 + \frac{1}{9}$$

$$\frac{1}{a^2} = \frac{10}{9}$$

$$a^2 = \frac{9}{10}$$

The required equation of hyperbola is

$$\frac{x^2}{\frac{9}{10}} - \frac{y^2}{36} = 1, \text{ i.e., } \frac{10x^2}{9} - \frac{y^2}{36} = 1.$$

(v) Let the required equation of hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \dots\dots(i)$$

The hyperbola passes through the points (6, 9) and (3, 0).

Substituting $x = 6$ and $y = 9$ in (i), we get

$$\frac{6^2}{a^2} - \frac{9^2}{b^2} = 1$$

$$\frac{36}{a^2} - \frac{81}{b^2} = 1 \dots(ii)$$

Substituting $x = 3$ and $y = 0$ in (i), we get

$$\frac{3^2}{a^2} - \frac{0^2}{b^2} = 1$$

$$\frac{9}{a^2} - 0 = 1$$

$$a^2 = 9$$

Substituting $a^2 = 9$ in (ii), we get

$$\frac{36}{9} - \frac{81}{b^2} = 1$$

$$\frac{81}{b^2} = \frac{36}{9} - 1$$

$$\frac{81}{b^2} = 4 - 1 = 3$$

$$b^2 = \frac{81}{3} = 27$$

The required equation of hyperbola is

$$\frac{x^2}{9} - \frac{y^2}{27} = 1.$$

(vi) Let the required equation of hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Co-ordinates of vertices are $(\pm a, 0)$.

Given that, co-ordinates of vertices are $(\pm 7, 0)$

$$\therefore a = 7$$

Endpoints of the conjugate axis are $(0, b)$ and $(0, -b)$.

Given, the endpoints of the conjugate axis are $(0, \pm 3)$.

$$\therefore b = 3$$

The required equation of hyperbola is $\frac{x^2}{7^2} - \frac{y^2}{3^2} = 1$

$$\text{i.e., } \frac{x^2}{49} - \frac{y^2}{9} = 1$$

(vii) Let the required equation of hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \dots\dots(i)$$

Given, eccentricity (e) = $\frac{3}{2}$

Co-ordinates of foci are $(\pm ae, 0)$.

Given co-ordinates of foci are $(\pm 2, 0)$

$$ae = 2$$

$$\Rightarrow a\left(\frac{3}{2}\right) = 2$$

$$\Rightarrow a = \frac{4}{3}$$

$$\Rightarrow a^2 = \frac{16}{9}$$

(viii) Let the required equation of hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Length of transverse axis = $2a$

Given, length of transverse axis = 6

$$\Rightarrow 2a = 6$$

$$\Rightarrow a = 3$$

$$\Rightarrow a^2 = 9$$

Length of conjugate axis = $2b$

Given, length of conjugate axis = 9

$$\Rightarrow 2b = 9$$

$$\Rightarrow b = \frac{9}{2}$$

$$\Rightarrow b^2 = \frac{81}{4}$$

The required equation of hyperbola is

$$\frac{x^2}{9} - \frac{y^2}{\left(\frac{81}{4}\right)} = 1$$

$$\text{i.e., } \frac{x^2}{9} - \frac{4y^2}{81} = 1$$

(ix) Let the required equation of hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Length of transverse axis = $2a$

Given, length of transverse axis = 8

$$\Rightarrow 2a = 8$$

$$\Rightarrow a = 4$$

$$\Rightarrow a^2 = 16$$

Distance between foci = $2ae$

Given, distance between foci = 10

$$\Rightarrow 2ae = 10$$

$$\Rightarrow ae = 5$$

$$\Rightarrow a^2 e^2 = 25$$

$$\text{Now, } b^2 = a^2 (e^2 - 1)$$

$$\Rightarrow b^2 = a^2 e^2 - a^2$$

$$\Rightarrow b^2 = 25 - 16 = 9$$

The required equation of hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = 1$

Question 6.

Find the equation of the tangent to the hyperbola.

(i) $3x^2 - y^2 = 4$ at the point $(2, 2\sqrt{2})$.

(ii) $3x^2 - y^2 = 12$ at the point $(4, 6)$

(iii) $\frac{x^2}{144} - \frac{y^2}{25} = 1$ at the point whose eccentric angle is $\frac{\pi}{3}$.

(iv) $\frac{x^2}{16} - \frac{y^2}{9} = 1$ at the point in a first quadrant whose ordinate is 3.

(v) $9x^2 - 16y^2 = 144$ at the point L of the latus rectum in the first quadrant.

Solution:

i. Given equation of the hyperbola is $3x^2 - y^2 = 4$.

$$\therefore \frac{x^2}{\left(\frac{4}{3}\right)} - \frac{y^2}{4} = 1$$

Comparing this equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$,

we get

$$a^2 = \frac{4}{3} \text{ and } b^2 = 4$$

Equation of the tangent to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ at } (x_1, y_1) \text{ is}$$

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$

$\therefore$ Equation of the tangent at $(2, 2\sqrt{2})$ is

$$\frac{2x}{\left(\frac{4}{3}\right)} - \frac{2\sqrt{2}y}{4} = 1$$

$$\therefore \frac{3x}{2} - \frac{\sqrt{2}y}{2} = 1$$

$$\therefore 3x - \sqrt{2}y = 2$$

ii. Given equation of the hyperbola is $3x^2 - y^2 = 12$.

$$\therefore \frac{x^2}{4} - \frac{y^2}{12} = 1$$

Comparing this equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$,

we get

$$a^2 = 4 \text{ and } b^2 = 12$$

Equation of the tangent to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ at } (x_1, y_1) \text{ is}$$

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$

Equation of the tangent at (4, 6) is

$$\frac{4x}{4} - \frac{6y}{12} = 1$$

$$x - \frac{y}{2} = 1$$

$$2x - y = 2$$

iii. Given equation of the hyperbola is

$$\frac{x^2}{144} - \frac{y^2}{25} = 1.$$

Comparing this equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$,

we get

$$a^2 = 144 \text{ and } b^2 = 25$$

$$\therefore a = 12 \text{ and } b = 5$$

Equation of the tangent to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ at } P(\theta) \text{ is}$$

$$\frac{x \sec \theta}{a} - \frac{y \tan \theta}{b} = 1$$

$$\text{Eccentric angle } (\theta) = \frac{\pi}{3}$$

$\therefore$ Equation of the tangent at $P\left(\frac{\pi}{3}\right)$ is

$$\frac{x \sec \frac{\pi}{3}}{12} - \frac{y \tan \frac{\pi}{3}}{5} = 1$$

$$\therefore \frac{2x}{12} - \frac{\sqrt{3}y}{5} = 1$$

$$\therefore \frac{x}{6} - \frac{\sqrt{3}y}{5} = 1$$

$$\therefore 5x - 6\sqrt{3}y = 30$$

iv. Given equation of the hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

Comparing this equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$,

we get

$$a^2 = 16 \text{ and } b^2 = 9$$

Let $P(x_1, 3)$ be the point on the hyperbola in the first quadrant at which the tangent is drawn.

Substituting $x = x_1$ and $y = 3$ in equation of hyperbola, we get

$$\frac{x_1^2}{16} - \frac{3^2}{9} = 1$$

$$\therefore \frac{x_1^2}{16} - 1 = 1$$

$$\therefore \frac{x_1^2}{16} = 2$$

$$\therefore x_1^2 = 32$$

$$\therefore x_1 = \pm 4\sqrt{2}$$

Since P lies in the first quadrant,

$$P \equiv (4\sqrt{2}, 3)$$

Equation of the tangent to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ at } (x_1, y_1) \text{ is}$$

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$

Equation of the tangent at $P(4\sqrt{2}, 3)$ is

$$\frac{4\sqrt{2}x}{16} - \frac{3y}{9} = 1$$

$$\therefore \frac{\sqrt{2}}{4}x - \frac{y}{3} = 1$$

$$\therefore 3\sqrt{2}x - 4y = 12$$

v. Given equation of the hyperbola is

$$9x^2 - 16y^2 = 144.$$

$$\therefore \frac{x^2}{16} - \frac{y^2}{9} = 1$$

Comparing this equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$,

we get

$$a^2 = 16 \text{ and } b^2 = 9$$

$$\therefore a = 4 \text{ and } b = 3$$

Since the point L lies in the first quadrant,

Latus rectum of the hyperbola is

$$L \left(ae, \frac{b^2}{a} \right)$$

$$\text{Now, } b^2 = a^2 (e^2 - 1)$$

$$\therefore 9 = 16 (e^2 - 1)$$

$$\therefore \frac{9}{16} = e^2 - 1$$

$$\therefore e^2 = \frac{9}{16} + 1 = \frac{25}{16}$$

$$\therefore e = \frac{5}{4} \quad \dots [\because e > 1]$$

$$\therefore ae = 4 \left(\frac{5}{4} \right) = 5$$

$$\therefore L \left(ae, \frac{b^2}{a} \right) = L \left(5, \frac{9}{4} \right)$$

Equation of the tangent to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ at } (x_1, y_1) \text{ is}$$

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$

$$\therefore \text{Equation of the tangent at } L \left(5, \frac{9}{4} \right) \text{ is}$$

$$\frac{5x}{16} - \frac{\frac{9}{4}y}{9} = 1$$

$$\therefore \frac{5x}{16} - \frac{y}{4} = 1$$

$$\therefore 5x - 4y = 16$$

Question 7.

Show that the line $3x - 4y + 10 = 0$ is a tangent to the hyperbola $x^2 - 4y^2 = 20$. Also, find the point of contact.

Solution:

Given equation of the hyperbola is $x^2 - 4y^2 = 20$

$$\frac{x^2}{20} - \frac{y^2}{5} = 1$$

Comparing this equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get

$$a^2 = 20 \text{ and } b^2 = 5$$

Given equation of line is $3x - 4y + 10 = 0$.

$$y = \frac{3x}{4} + \frac{5}{2}$$

Comparing this equation with $y = mx + c$, we get

$$m = \frac{3}{4} \text{ and } c = \frac{5}{2}$$

For the line $y = mx + c$ to be a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we must have

$$c^2 = a^2 m^2 - b^2$$

$$c^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

$$a^2 m^2 - b^2 = 20 \left(\frac{3}{4}\right)^2 - 5$$

$$= 20 \left(\frac{9}{16}\right) - 5 = \frac{45}{4} - 5 = \frac{25}{4} = c^2$$

The given line is a tangent to the given hyperbola and point of contact

$$= \left(\frac{-a^2 m}{c}, \frac{-b^2}{c} \right)$$

$$= \left(\frac{-20 \left(\frac{3}{4}\right)}{\left(\frac{5}{2}\right)}, \frac{-5}{\left(\frac{5}{2}\right)} \right) = (-6, -2)$$

Question 8.

If the line $3x - 4y = k$ touches the hyperbola $\frac{x^2}{5} - \frac{4y^2}{5} = 1$, then find the value of k .

Solution:

Given equation of the hyperbola is

$$\frac{x^2}{5} - \frac{4y^2}{5} = 1$$

$$\frac{x^2}{5} - \frac{y^2}{\frac{5}{4}} = 1$$

Comparing this equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get

$$a^2 = 5, b^2 = \frac{5}{4}$$

Given equation of line is $3x - 4y = k$

$$y = \frac{3}{4}x - \frac{k}{4}$$

Comparing this equation with $y = mx + c$, we get

$$m = \frac{3}{4}, c = -\frac{k}{4}$$

For the line $y = mx + c$ to be a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we must have

$$c^2 = a^2 m^2 - b^2$$

$$\Rightarrow \left(-\frac{k}{4}\right)^2 = 5\left(\frac{3}{4}\right)^2 - \frac{5}{4}$$

$$\Rightarrow \frac{k^2}{16} = \frac{5}{16}(9 - 4)$$

$$\Rightarrow \frac{k^2}{16} = \frac{5}{16}(5)$$

$$\Rightarrow k^2 = 25$$

$$\Rightarrow k = \pm 5$$

Alternate method:

Given equation of the hyperbola is

$$\frac{x^2}{5} - \frac{4y^2}{5} = 1 \dots\dots(i)$$

Given equation of the line is $3x - 4y = k$

$$y = \frac{3x-k}{4}$$

Substituting this value of y in (i), we get

$$\frac{x^2}{5} - \frac{4}{5} \left(\frac{3x-k}{4} \right)^2 = 1$$

$$\Rightarrow \frac{x^2}{5} - \frac{4}{5} \left(\frac{9x^2 - 6kx + k^2}{16} \right) = 1$$

$$\Rightarrow 4x^2 - (9x^2 - 6kx + k^2) = 20$$

$$\Rightarrow 4x^2 - 9x^2 + 6kx - k^2 = 20$$

$$\Rightarrow -5x^2 + 6kx - k^2 = 20$$

$$\Rightarrow 5x^2 - 6kx + (k^2 + 20) = 0 \dots\dots(ii)$$

Since, the given line touches the given hyperbola.

The quadratic equation (ii) in x has equal roots.

$$(-6k)^2 - 4(5)(k^2 + 20) = 0$$

$$\Rightarrow 36k^2 - 20k^2 - 400 = 0$$

$$\Rightarrow 16k^2 = 400$$

$$\Rightarrow k^2 = 25$$

$$\Rightarrow k = \pm 5$$

Question 9.

Find the equations of the tangents to the hyperbola $\frac{x^2}{25} - \frac{y^2}{9} = 1$ making equal intercepts on

the co-ordinate axes.

Solution:

Given equation of the hyperbola is $\frac{x^2}{25} - \frac{y^2}{9} = 1$.

Comparing this equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get

$$a^2 = 25 \text{ and } b^2 = 9$$

Since the tangents make equal intercepts on the co-ordinate axes,

$$\therefore m = -1$$

Equations of tangents to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having slope m are

$$y = mx \pm \sqrt{a^2m^2 - b^2}$$

$$\Rightarrow y = -x \pm \sqrt{25(-1)^2 - 9}$$

$$\Rightarrow y = -x \pm \sqrt{16}$$

$$\Rightarrow x + y = \pm 4$$

Question 10.

Find the equations of the tangents to the hyperbola $5x^2 - 4y^2 = 20$ which are parallel to the line $3x + 2y + 12 = 0$.

Solution:

Given equation of the hyperbola is $5x^2 - 4y^2 = 20$

$$\frac{x^2}{4} - \frac{y^2}{5} = 1$$

Comparing this equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get

$$a^2 = 4 \text{ and } b^2 = 5$$

Slope of the line $3x + 2y + 12 = 0$ is $-\frac{3}{2}$

Since the given line is parallel to the tangents,

Slope of the required tangents (m) = $-\frac{3}{2}$

Equations of tangents to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having slope m are

$$y = mx \pm \sqrt{a^2 m^2 - b^2}$$

$$y = -\frac{3}{2}x \pm \sqrt{4\left(-\frac{3}{2}\right)^2 - 5}$$

$$y = -\frac{3}{2}x \pm \sqrt{4\left(\frac{9}{4}\right) - 5}$$

$$y = -\frac{3}{2}x \pm \sqrt{4}$$

$$y = -\frac{3}{2}x \pm 2$$

$$3x + 2y = \pm 4$$



Maharashtra Board Solutions

Class 11 Arts & Science Maths

(Part 1)

- Chapter 1- Angle and its Measurement
- Chapter 2- Trigonometry – I
- Chapter 3- Trigonometry – II
- Chapter 4- Determinants and Matrices
- Chapter 5- Straight Line
- Chapter 6- Circle
- Chapter 7- Conic Sections
- Chapter 8- Measures of Dispersion
- Chapter 9- Probability

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Who developed the Maharashtra State board books?

As of now, the MSCERT and Balbharti are responsible for the syllabus and textbooks of Classes 1 to 8, while Classes 9 and 10 are under the Maharashtra State Board of Secondary and Higher Secondary Education (MSBSHSE).

How many state boards are there in Maharashtra?

The Maharashtra State Board of Secondary & Higher Secondary Education, conducts the HSC and SSC Examinations in the state of Maharashtra through its nine Divisional Boards located at Pune, Mumbai, Aurangabad, Nasik, Kolhapur, Amravati, Latur, Nagpur and Ratnagiri.

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