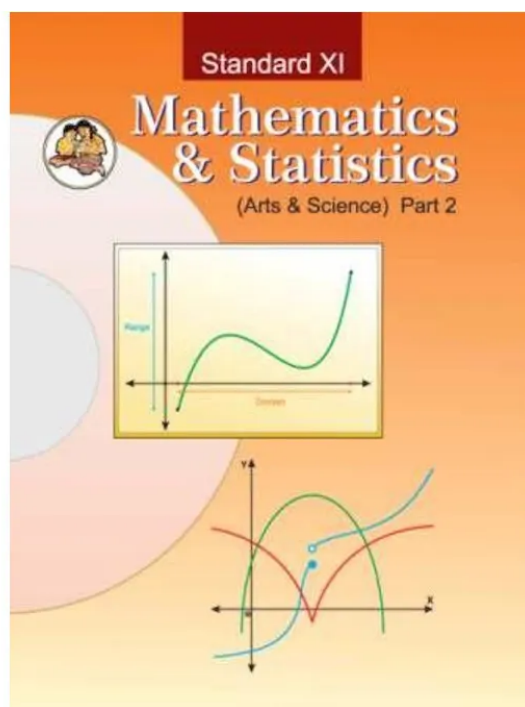


Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 6- Functions

Class 11 - Chapter 6 Functions



For any clarifications or questions you can write to info@indcareer.com

Postal Address

IndCareer.com, 52, Shilpa Nagar, Somalwada Nagpur - 440015
Maharashtra, India

WhatsApp: +91 9561 204 888, **Website:** <https://www.indcareer.com>

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-6-functions/>



Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 6- Functions

Class 11: Maths Chapter 5 solutions. Complete Class 11 Maths Chapter 5 Notes.

Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 6- Functions

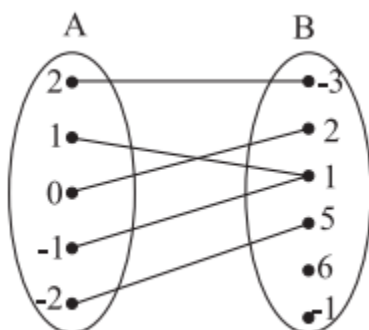
Maharashtra Board 11th Maths Chapter 5, Class 11 Maths Chapter 5 solutions

Ex 6.1

Question 1.

Check if the following relations are functions.

(a)



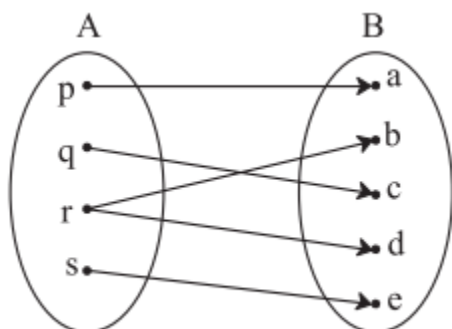
Solution:

Yes.

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p2-art-2-chapter-6-functions/>

Reason: Every element of set A has been assigned a unique element in set B.

(b)

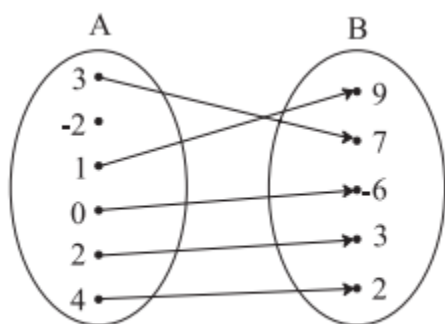


Solution:

No.

Reason: An element of set A has been assigned more than one element from set B.

(c)



Solution:

No.

Reason:

Not every element of set A has been assigned an image from set B.

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-2-chapter-6-functions/>

Question 2.

Which sets of ordered pairs represent functions from $A = \{1, 2, 3, 4\}$ to $B = \{-1, 0, 1, 2, 3\}$? Justify.

(i) $\{(1, 0), (3, 3), (2, -1), (4, 1), (2, 2)\}$

(ii) $\{(1, 2), (2, -1), (3, 1), (4, 3)\}$

(iii) $\{(1, 3), (4, 1), (2, 2)\}$

(iv) $\{(1, 1), (2, 1), (3, 1), (4, 1)\}$

Solution:

(i) $\{(1, 0), (3, 3), (2, -1), (4, 1), (2, 2)\}$ does not represent a function.

Reason: $(2, -1), (2, 2)$, show that element $2 \in A$ has been assigned two images -1 and 2 from set B .

(ii) $\{(1, 2), (2, -1), (3, 1), (4, 3)\}$ represents a function.

Reason: Every element of set A has been assigned a unique image in set B .

(iii) $\{(1, 3), (4, 1), (2, 2)\}$ does not represent a function.

Reason:

$3 \in A$ does not have an image in set B .

(iv) $\{(1, 1), (2, 1), (3, 1), (4, 1)\}$ represents a function

Reason: Every element of set A has been assigned a unique image in set B .

Question 3.

Check if the relation given by the equation represents y as function of x .

(i) $2x + 3y = 12$

(ii) $x + y^2 = 9$

(iii) $x^2 - y = 25$

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-6-functions/>

(iv) $2y + 10 = 0$

(v) $3x - 6 = 21$

Solution:

	Relation	Reflexive	Symmetric	Transitive
i.	$R = \{(a, b) : a, b \in \mathbb{Z}, a - b \text{ is an integer}\}$	✓	✓	✓
ii.	$R = \{(a, b) : a, b \in \mathbb{N}, a + b \text{ is even}\}$	✓	✓	✓
iii.	$R = \{(a, b) : a, b \in \mathbb{N}, a \text{ divides } b\}$	✓	✗	✓
iv.	$R = \{(a, b) : a, b \in \mathbb{N}, a^2 - 4ab + 3b^2 = 0\}$	✓	✗	✗
v.	$R = \{(a, b) : a \text{ is sister of } b \text{ and } a, b \in G = \text{Set of girls}\}$	✗	✓	✓
vi.	$R = \{(a, b) : \text{Line } a \text{ is perpendicular to line } b \text{ in a plane}\}$	✗	✓	✗
vii.	$R = \{(a, b) : a, b \in \mathbb{R}, a < b\}$	✗	✗	✓
viii.	$R = \{(a, b) : a, b \in \mathbb{R}, a \leq b^3\}$	✗	✗	✗

(iii) $x^2 - y = 25$

$\therefore y = x^2 - 25$

$\therefore$ For every value of x , there is a unique value of y .

$\therefore y$ is a function of x .

(iv) $2y + 10 = 0$

$\therefore y = -5$

$\therefore$ For every value of x , there is a unique value of y .

$\therefore y$ is a function of x .

(v) $3x - 6 = 21$

$\therefore x = 9$

$\therefore x = 9$ represents a point on the X -axis.

There is no y involved in the equation.

So the given equation does not represent a function.

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-6-functions/>

Question 4.

If $f(m) = m^2 - 3m + 1$, find

(i) $f(0)$

(ii) $f(-3)$

(iii) $f\left(\frac{1}{2}\right)$

(iv) $f(x + 1)$

(v) $f(-x)$

(vi) $\left(\frac{f(2+h)-f(2)}{h}\right), h \neq 0.$

Solution:

$$f(m) = m^2 - 3m + 1$$

(i) $f(0) = 0^2 - 3(0) + 1 = 1$

(ii) $f(-3) = (-3)^2 - 3(-3) + 1$

$$= 9 + 9 + 1$$

$$= 19$$

(iii) $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 1$

$$= \frac{1}{4} - \frac{3}{2} + 1$$

$$= \frac{1-6+4}{4}$$

$$= -\frac{1}{4}$$

(iv) $f(x + 1) = (x + 1)^2 - 3(x + 1) + 1$

$$= x^2 + 2x + 1 - 3x - 3 + 1$$

$$= x^2 - x - 1$$

$$(v) f(-x) = (-x)^2 - 3(-x) + 1 = x^2 + 3x + 1$$

$$\begin{aligned} (vi) & \left(\frac{f(2+h) - f(2)}{h} \right) \\ &= \frac{(2+h)^2 - 3(2+h) + 1 - (2^2 - 3(2) + 1)}{h} \\ &= \frac{h^2 + h}{h} \\ &= h + 1 \end{aligned}$$

Question 5.

Find x, if $g(x) = 0$ where

$$(i) g(x) = \frac{5x-6}{7}$$

$$(ii) g(x) = \frac{18-2x^2}{7}$$

$$(iii) g(x) = 6x^2 + x - 2$$

$$(iv) g(x) = x^3 - 2x^2 - 5x + 6$$

Solution:

$$(i) g(x) = \frac{5x-6}{7}$$

$$g(x) = 0$$

$$\therefore \frac{5x-6}{7} = 0$$

$$\therefore x = \frac{6}{5}$$

$$(ii) g(x) = \frac{18-2x^2}{7}$$

$$g(x) = 0$$

$$\frac{18-2x^2}{7} = 0$$

$$\therefore 18 - 2x^2 = 0$$

$$\therefore x^2 = 9$$

$$\therefore x = \pm 3$$

$$(iii) g(x) = 6x^2 + x - 2$$

$$g(x) = 0$$

$$\therefore 6x^2 + x - 2 = 0$$

$$\therefore (2x - 1)(3x + 2) = 0$$

$$\therefore 2x - 1 = 0 \text{ or } 3x + 2 = 0$$

$$\therefore x = \frac{1}{2} \text{ or } x = -\frac{2}{3}$$

$$(iv) g(x) = x^3 - 2x^2 - 5x + 6$$

$$= (x - 1)(x^2 - x - 6)$$

$$= (x - 1)(x + 2)(x - 3)$$

$$g(x) = 0$$

$$\therefore (x - 1)(x + 2)(x - 3) = 0$$

$$\therefore x - 1 = 0 \text{ or } x + 2 = 0 \text{ or } x - 3 = 0$$

$$\therefore x = 1, -2, 3$$

Question 6.

Find x, if $f(x) = g(x)$ where

$$(i) f(x) = x^4 + 2x^2, g(x) = 11x^2$$

$$(ii) f(x) = \sqrt{x} - 3, g(x) = 5 - x$$

Solution:

$$(i) f(x) = x^4 + 2x^2, g(x) = 11x^2$$

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-6-functions/>

$$f(x) = g(x)$$

$$\therefore x^4 + 2x^2 = 11x^2$$

$$\therefore x^4 - 9x^2 = 0$$

$$\therefore x^2 (x^2 - 9) = 0$$

$$\therefore x^2 = 0 \text{ or } x^2 - 9 = 0$$

$$\therefore x = 0 \text{ or } x^2 = 9$$

$$\therefore x = 0, \pm 3$$

$$(ii) f(x) = \sqrt{x} - 3, g(x) = 5 - x$$

$$f(x) = g(x)$$

$$\therefore \sqrt{x} - 3 = 5 - x$$

$$\therefore \sqrt{x} = 5 - x + 3$$

$$\therefore \sqrt{x} = 8 - x$$

on squaring, we get

on squaring, we get

$$x = 64 + x^2 - 16x$$

$$\therefore x^2 - 17x + 64 = 0$$

$$\therefore x = \frac{17 \pm \sqrt{(-17)^2 - 4(64)}}{2}$$

$$\therefore x = \frac{17 \pm \sqrt{289 - 256}}{2}$$

$$\therefore x = \frac{17 \pm \sqrt{33}}{2}$$

Question 7.

If $f(x) = \frac{a-x}{b-x}$, $f(2)$ is undefined, and $f(3) = 5$, find a and b .

Solution:

$$f(x) = \frac{a-x}{b-x}$$

Given that,

$f(2)$ is undefined

$$b - 2 = 0$$

$$\therefore b = 2 \dots\dots(i)$$

$$f(3) = 5$$

$$\therefore \frac{a-3}{b-3} = 5$$

$$\therefore \frac{a-3}{2-3} = 5 \dots\dots [\text{From (i)}]$$

$$\therefore a - 3 = -5$$

$$\therefore a = -2$$

$$\therefore a = -2, b = 2$$

Question 8.

Find the domain and range of the following functions.

(i) $f(x) = 7x^2 + 4x - 1$

Solution:

$$f(x) = 7x^2 + 4x - 1$$

f is defined for all x .

$\therefore$ Domain of $f = \mathbb{R}$ (i.e., the set of real numbers)

$$7x^2 + 4x - 1 = 7\left(x^2 + \frac{4}{7}x\right) - 1$$

$$\begin{aligned} &= 7\left(x^2 + \frac{4}{7}x\right) + \frac{4}{7} - 1 - \frac{4}{7} \\ &= 7\left(x^2 + \frac{4}{7}x + \frac{4}{49}\right) - 1 - \frac{4}{7} \\ &= 7\left(x + \frac{2}{7}\right)^2 - \frac{11}{7} \geq -\frac{11}{7} \end{aligned}$$

$\therefore$ Range of $f = \left[-\frac{11}{7}, \infty\right)$

(ii) $g(x) = \frac{x+4}{x-2}$

Solution:

$$g(x) = \frac{x+4}{x-2}$$

Function g is defined everywhere except at $x = 2$.

$\therefore$ Domain of $g = \mathbb{R} - \{2\}$

$$\text{Let } y = g(x) = \frac{x+4}{x-2}$$

$$\therefore (x-2)y = x+4$$

$$\therefore x(y-1) = 4+2y$$

$\therefore$ For every y , we can find x , except for $y = 1$.

$\therefore y = 1 \notin$ range of function g

$\therefore$ Range of $g = \mathbb{R} - \{1\}$

(iii) $h(x) = \frac{\sqrt{x+5}}{5+x}$

Solution:

$$h(x) = \frac{\sqrt{x+5}}{5+x} = \frac{1}{\sqrt{x+5}}, x \neq -5$$

For $x = -5$, function h is not defined.

$\therefore x + 5 > 0$ for function h to be well defined.

$$\therefore x > -5$$

$$\therefore \text{Domain of } h = (-5, \infty)$$

$$\text{Let } y = \frac{1}{\sqrt{x+5}}$$

$$\therefore y > 0$$

$$\text{Range of } h = (0, \infty) \text{ or } \mathbb{R}^+$$

$$(iv) f(x) = \sqrt[3]{x+1}$$

Solution:

$$f(x) = \sqrt[3]{x+1}$$

f is defined for all real x and the values of $f(x) \in \mathbb{R}$

$$\therefore \text{Domain of } f = \mathbb{R}, \text{ Range of } f = \mathbb{R}$$

$$(v) f(x) = \sqrt{(x-2)(5-x)}$$

Solution:

$$f(x) = \sqrt{(x-2)(5-x)}$$

For f to be defined,

$$(x-2)(5-x) \geq 0$$

$$\therefore (x-2)(x-5) \leq 0$$



$\therefore 2 \leq x \leq 5$ [$\because$ The solution of $(x - a)(x - b) \leq 0$ is $a \leq x \leq b$, for $a < b$]

$\therefore$ Domain of $f = [2, 5]$

$$(x - 2)(5 - x) = -x^2 + 7x - 10$$

$$= -\left(x - \frac{7}{2}\right)^2 + \frac{49}{4} - 10$$

$$= \frac{9}{4} - \left(x - \frac{7}{2}\right)^2 \leq \frac{9}{4}$$

$$\therefore \sqrt{(x - 2)(5 - x)} \leq \sqrt{\frac{9}{4}} \leq \frac{3}{2}$$

Range of $f = [0, \frac{3}{2}]$

$$(vi) f(x) = \sqrt{\frac{x-3}{7-x}}$$

Solution:

$$f(x) = \sqrt{\frac{x-3}{7-x}}$$

For f to be defined,

$$\sqrt{\frac{x-3}{7-x}} \geq 0, 7 - x \neq 0$$

$$\therefore \sqrt{\frac{x-3}{7-x}} \leq 0 \text{ and } x \neq 7$$

$$\therefore 3 \leq x < 7$$

Let $a < b$, $\frac{x-a}{x-b} \leq 0 \Rightarrow a \leq x < b$

$\therefore$ Domain of $f = [3, 7)$

$f(x) \geq 0$... [$\because$ The value of square root function is non-negative]

$\therefore$ Range of $f = [0, \infty)$

$$(vii) f(x) = \sqrt{16 - x^2}$$

Solution:

$$f(x) = \sqrt{16 - x^2}$$

For f to be defined,

$$16 - x^2 \geq 0$$

$$\therefore x^2 \leq 16$$

$$\therefore -4 \leq x \leq 4$$

$$\therefore \text{Domain of } f = [-4, 4]$$

Clearly, $f(x) \geq 0$ and the value of $f(x)$ would be maximum when the quantity subtracted from 16 is minimum i.e. $x = 0$

$$\therefore \text{Maximum value of } f(x) = \sqrt{16} = 4$$

$$\therefore \text{Range of } f = [0, 4]$$

Question 9.

Express the area A of a square as a function of its

(a) side s

(b) perimeter P

Solution:

$$(a) \text{ area } (A) = s^2$$

$$(b) \text{ perimeter } (P) = 4s$$

$$\therefore s = \frac{P}{4}$$

$$\text{Area } (A) = s^2 = \left(\frac{P}{4}\right)^2$$

$$\therefore A = \frac{P^2}{16}$$

Question 10.

Express the area A of a circle as a function of its

(i) radius r

(ii) diameter d

(iii) circumference C

Solution:

(i) Area (A) = πr^2

(ii) Diameter (d) = $2r$

$$\therefore r = \frac{d}{2}$$

$$\therefore \text{Area (A)} = \pi r^2 = \frac{\pi d^2}{4}$$

(iii) Circumference (C) = $2\pi r$

$$\therefore r = \frac{C}{2\pi}$$

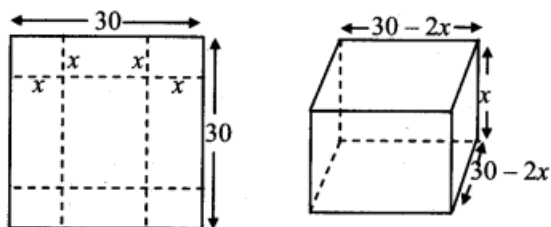
$$\text{Area (A)} = \pi r^2 = \pi \left(\frac{C}{2\pi} \right)^2$$

$$\therefore A = \frac{C^2}{4\pi}$$

Question 11.

An open box is made from a square of cardboard of 30 cms side, by cutting squares of length x centimeters from each corner and folding the sides up. Express the volume of the box as a function of x. Also, find its domain.

Solution:



Length of the box = $30 - 2x$

Breadth of the box = $30 - 2x$

Height of the box = x

Volume = $(30 - 2x)^2 x$, $x < 15$, $x \neq 15$, $x > 0$

= $4x(15 - x)^2$, $x \neq 15$, $x > 0$

Domain $(0, 15)$

Question 12.

Let f be a subset of $Z \times Z$ defined by $f = \{(ab, a + b) : a, b \in Z\}$. Is f a function from Z to Z ? Justify?

Solution:

$f = \{(ab, a + b) : a, b \in Z\}$

Let $a = 1$, $b = 1$. Then, $ab = 1$, $a + b = 2$

$\therefore (1, 2) \in f$

Let $a = -1$, $b = -1$. Then, $ab = 1$, $a + b = -2$

$\therefore (1, -2) \in f$

Since $(1, 2) \in f$ and $(1, -2) \in f$,

f is not a function as element 1 does not have a unique image.

Question 13.

Check the injectivity and surjectivity of the following functions.

(i) $f: \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = x^2$

Solution:

$f: \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = x^2$

Let $f(x_1) = f(x_2)$, $x_1, x_2 \in \mathbb{N}$

$$\therefore x_1^2 = x_2^2$$

$$\therefore x_1^2 - x_2^2 = 0$$

$$\therefore (x_1 - x_2) \underbrace{(x_1 + x_2)}_{\text{for } x_1, x_2 \in \mathbb{N}} = 0$$

$$\therefore x_1 = x_2$$

$\therefore f$ is injective.

For every $y = x^2 \in \mathbb{N}$, there does not exist $x \in \mathbb{N}$.

Example: $7 \in \mathbb{N}$ (codomain) for which there is no x in domain $\mathbb{N}$ such that $x^2 = 7$

$\therefore f$ is not surjective.

(ii) $f: \mathbb{Z} \rightarrow \mathbb{Z}$ given by $f(x) = x^2$

Solution:

$f: \mathbb{Z} \rightarrow \mathbb{Z}$ given by $f(x) = x^2$

Let $f(x_1) = f(x_2)$, $x_1, x_2 \in \mathbb{Z}$

$$\therefore x_1^2 = x_2^2$$

$$\therefore x_1^2 - x_2^2 = 0$$

$$\therefore (x_1 - x_2)(x_1 + x_2) = 0$$

$$\therefore x_1 = x_2 \text{ or } x_1 = -x_2$$

$\therefore f$ is not injective.

(Example: $f(-2) = 4 = f(2)$. So, -2, 2 have the same image. So, f is not injective.)

Since $x^2 \geq 0$,

$f(x) \geq 0$

Therefore all negative integers of codomain are not images under f .

$\therefore f$ is not surjective.

(iii) $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^2$

Solution:

$f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^2$

Let $f(x_1) = f(x_2)$, $x_1, x_2 \in \mathbb{R}$

$$\therefore x_1^2 = x_2^2$$

$$\therefore x_1^2 - x_2^2 = 0$$

$$\therefore (x_1 - x_2)(x_1 + x_2) = 0$$

$$\therefore x_1 = x_2 \quad \text{or} \quad x_1 = -x_2$$

$\therefore f$ is not injective.

$f(x) = x^2 \geq 0$

Therefore all negative integers of codomain are not images under f .

$\therefore f$ is not surjective.

(iv) $f: \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = x^3$

Solution:

$f: \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = x^3$

Let $f(x_1) = f(x_2)$, $x_1, x_2 \in \mathbb{N}$

$$\therefore x_1^3 = x_2^3$$

$$\therefore x_1^3 - x_2^3 = 0$$

$$\therefore (x_1 - x_2) \underbrace{(x_1^2 + x_1 x_2 + x_2^2)}_{> 0 \text{ for all } x_1, x_2 \text{ as it's discriminant } < 0} = 0$$

$$\therefore x_1 = x_2$$

$\therefore f$ is injective.

Numbers from codomain which are not cubes of natural numbers are not images under f .

$\therefore f$ is not surjective.

(v) $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^3$

Solution:

$f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^3$

Let $x_1^3 = x_2^3$, $x_1, x_2 \in \mathbb{R}$

$$\therefore x_1^3 - x_2^3 = 0$$

$$\therefore (x_1 - x_2) \underbrace{(x_1^2 + x_1 x_2 + x_2^2)}_{> 0 \text{ for all } x_1, x_2 \text{ as it's discriminant } < 0} = 0$$

$$\therefore x_1 = x_2$$

$\therefore f$ is injective.

Let $y = x^3$

$$\therefore x = y^{\frac{1}{3}}$$

$\therefore$ For every $y \in \mathbb{R}$, there is some $x \in \mathbb{R}$.

$\therefore f$ is surjective.

Question 14.

Show that if $f : A \rightarrow B$ and $g : B \rightarrow C$ are one-one, then $g \circ f$ is also one-one.

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-6-functions/>

Solution:

f is a one-one function.

$$\text{Let } f(x_1) = f(x_2)$$

$$\text{Then, } x_1 = x_2 \text{ for all } x_1, x_2 \dots (i)$$

g is a one-one function.

$$\text{Let } g(y_1) = g(y_2)$$

$$\text{Then, } y_1 = y_2 \text{ for all } y_1, y_2 \dots (ii)$$

$$\text{Let } (g \circ f)(x_1) = (g \circ f)(x_2)$$

$$\therefore g(f(x_1)) = g(f(x_2))$$

$$\therefore g(y_1) = g(y_2),$$

$$\text{where } y_1 = f(x_1), y_2 = f(x_2) \in B$$

$$\therefore y_1 = y_2 \dots [\text{From (ii)}]$$

$$\text{i.e., } f(x_1) = f(x_2)$$

$$\therefore x_1 = x_2 \dots [\text{From (i)}]$$

$$\therefore g \circ f \text{ is one-one.}$$

Question 15.

Show that if $f : A \rightarrow B$ and $g : B \rightarrow C$ are onto, then $g \circ f$ is also onto.

Solution:

Since g is surjective (onto),

there exists $y \in B$ for every $z \in C$ such that

$$g(y) = z \dots (i)$$

Since f is surjective,

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-6-functions/>

there exists $x \in A$ for every $y \in B$ such that

$$f(x) = y \dots\dots(ii)$$

$$(g \circ f) x = g(f(x))$$

$$= g(y) \dots\dots[From (ii)]$$

$$= z \dots\dots[From(i)]$$

i.e., for every $z \in C$, there is x in A such that $(g \circ f) x = z$

$\therefore$ $g \circ f$ is surjective (onto).

Question 16.

If $f(x) = 3(4^{x+1})$, find $f(-3)$.

Solution:

$$f(x) = 3(4^{x+1})$$

$$\therefore f(-3) = 3(4^{-3+1})$$

$$= 3(4^{-2})$$

$$= \frac{3}{16}$$

Question 17.

Express the following exponential equations in logarithmic form:

(i) $2^5 = 32$

(ii) $54^0 = 1$

(iii) $23^1 = 23$

(iv) $9^{\frac{3}{2}} = 27$

(v) $3^{-4} = \frac{1}{81}$

(vi) $10^{-2} = 0.01$

(vii) $e^2 = 7.3890$

(viii) $e^{\frac{1}{2}} = 1.6487$

(ix) $e^{-x} = 6$

Solution:

i. $2^5 = 32$

i. $2^5 = 32$

$\therefore 5 = \log_2 32$...[By definition of logarithm]

i.e., $\log_2 32 = 5$

ii. $54^0 = 1$
 $\therefore 0 = \log_{54} 1$...[By definition of logarithm]
i.e., $\log_{54} 1 = 0$

iii. $23^1 = 23$
 $\therefore 1 = \log_{23} 23$...[By definition of logarithm]
i.e., $\log_{23} 23 = 1$

iv. $9^{\frac{3}{2}} = 27$
 $\therefore \frac{3}{2} = \log_9 27$...[By definition of logarithm]
i.e., $\log_9 27 = \frac{3}{2}$

v. $3^{-4} = \frac{1}{81}$
 $\therefore -4 = \log_3 \left(\frac{1}{81} \right)$...[By definition of logarithm]
i.e., $\log_3 \left(\frac{1}{81} \right) = -4$

vi. $10^{-2} = 0.01$

$\therefore -2 = \log_{10}(0.01) \dots [\text{By definition of logarithm}]$
i.e., $\log_{10}(0.01) = -2$

vii. $e^2 = 7.3890$

$\therefore 2 = \log_e(7.3890) \dots [\text{By definition of logarithm}]$
i.e., $\log_e(7.3890) = 2$
(e is a mathematical constant, whose value is approximately 2.71828)

viii. $e^{\frac{1}{2}} = 1.6487$

$\therefore \frac{1}{2} = \log_e(1.6487) \dots [\text{By definition of logarithm}]$
i.e., $\log_e(1.6487) = \frac{1}{2}$

ix. $e^{-x} = 6$

$\therefore -x = \log_e 6 \dots [\text{By definition of logarithm}]$
i.e., $\log_e 6 = -x$

Question 18.

Express the following logarithmic equations in exponential form:

(i) $\log_2 64 = 6$

(ii) $\log_5 \frac{1}{25} = -2$

(iii) $\log_{10} 0.001 = -3$

(iv) $\log_{\frac{1}{2}} (8) = -3$

(v) $\ln 1 = 0$

(vi) $\ln e = 1$

(vii) $\ln \frac{1}{2} = -0.693$

Solution:

(i) $\log_2 64 = 6$

$\therefore 64 = 2^6$, i.e., $2^6 = 64$

ii. $\log_5 \left(\frac{1}{25} \right) = -2$

$\therefore \frac{1}{25} = 5^{-2}$, i.e., $5^{-2} = \frac{1}{25}$

iii. $\log_{10} (0.001) = -3$

$\therefore 0.001 = 10^{-3}$, i.e., $10^{-3} = 0.001$

iv. $\log_{\frac{1}{2}} (8) = -3$

$\therefore 8 = \left(\frac{1}{2} \right)^{-3}$, i.e., $\left(\frac{1}{2} \right)^{-3} = 8$

$$\begin{aligned}\text{v.} \quad & \ln 1 = 0 \\ \therefore & 1 = e^0, \text{i.e., } e^0 = 1\end{aligned}$$

$$\begin{aligned}\text{vi.} \quad & \ln e = 1 \\ \therefore & e = e^1, \text{i.e., } e^1 = e\end{aligned}$$

$$\begin{aligned}\text{vii.} \quad & \ln \left(\frac{1}{2} \right) = -0.693 \\ \therefore & \frac{1}{2} = e^{-0.693}, \text{i.e., } e^{-0.693} = \frac{1}{2}\end{aligned}$$

Question 19.

Find the domain of

(i) $f(x) = \ln(x - 5)$

(ii) $f(x) = \log_{10}(x^2 - 5x + 6)$

Solution:

(i) $f(x) = \ln(x - 5)$

f is defined, when $x - 5 > 0$

$$\therefore x > 5$$

$$\therefore \text{Domain of } f = (5, \infty)$$

(ii) $f(x) = \log_{10}(x^2 - 5x + 6)$

$$x^2 - 5x + 6 = (x - 2)(x - 3)$$

f is defined, when $(x - 2)(x - 3) > 0$

$$\therefore x < 2 \text{ or } x > 3$$

Solution of $(x - a)(x - b) > 0$ is $x < a$ or $x > b$ where $a < b$

$$\therefore \text{Domain of } f = (-\infty, 2) \cup (3, \infty)$$

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-6-functions/>

Question 20.

Write the following expressions as sum or difference of logarithms:

(a) $\log\left(\frac{pq}{rs}\right)$

Solution:

$$\begin{aligned}\log\left(\frac{pq}{rs}\right) &= \log(pq) - \log(rs) \\ &\dots \left[\log\frac{m}{n} = \log m - \log n \right] \\ &= \log p + \log q - (\log r + \log s) \\ &\dots \left[\log mn = \log m + \log n \right] \\ &= \log p + \log q - \log r - \log s\end{aligned}$$

(b) $\log(\sqrt{x}\sqrt[3]{y})$

Solution:

$$\begin{aligned}\log(\sqrt{x}\sqrt[3]{y}) &= \log(\sqrt{x}) + \log(\sqrt[3]{y}) \\ &\dots \left[\log mn = \log m + \log n \right] \\ &= \log x^{\frac{1}{2}} + \log y^{\frac{1}{3}} \\ &= \frac{1}{2} \log x + \frac{1}{3} \log y \\ &\dots [\log m^n = n \log m]\end{aligned}$$

(c) $\ln\left(\frac{a^3(a-2)^2}{\sqrt{b^2+5}}\right)$

Solution:

$$\begin{aligned} & \ln \left(\frac{a^3 (a-2)^2}{\sqrt{b^2+5}} \right) \\ &= \ln (a^3 (a-2)^2) - \ln \sqrt{b^2+5} \\ & \quad \dots \left[\log \frac{m}{n} = \log m - \log n \right] \\ &= \ln a^3 + \ln (a-2)^2 - \ln (b^2+5)^{\frac{1}{2}} \\ & \quad \dots [\log mn = \log m + \log n] \\ &= 3 \ln a + 2 \ln (a-2) - \frac{1}{2} \ln (b^2+5) \\ & \quad \dots [\log m^n = n \log m] \end{aligned}$$

$$(d) \ln \left[\frac{\sqrt[3]{x-2}(2x+1)^4}{(x+4)\sqrt{2x+4}} \right]^2$$

Solution:

$$\begin{aligned} & \ln \left[\frac{\sqrt[3]{x-2}(2x+1)^4}{(x+4)\sqrt{2x+4}} \right]^2 \\ &= 2 \ln \left[\frac{\sqrt[3]{x-2}(2x+1)^4}{(x+4)\sqrt{2x+4}} \right] \quad \dots [\log m^n = n \log m] \\ &= 2 \left[\ln \sqrt[3]{x-2}(2x+1)^4 - \ln (x+4)\sqrt{2x+4} \right] \\ &= 2 \left[\ln \sqrt[3]{x-2} + \ln (2x+1)^4 - (\ln (x+4) + \ln \sqrt{2x+4}) \right] \\ &= 2 \left[\frac{1}{3} \ln (x-2) + 4 \ln (2x+1) \right. \\ & \quad \left. - \ln (x+4) - \frac{1}{2} \ln (2x+4) \right] \end{aligned}$$

Question 21.

Write the following expressions as a single logarithm.

(i) $5 \log x + 7 \log y - \log z$

Solution:

$$\begin{aligned} & 5 \log x + 7 \log y - \log z \\ &= \log (x^5) + \log (y^7) - \log z \\ & \quad \dots [n \log m = \log m^n] \end{aligned}$$

$$= \log (x^5 y^7) - \log z$$

$$\dots [\log m + \log n = \log mn]$$

$$= \log \left(\frac{x^5 y^7}{z} \right) \quad \dots \left[\log m - \log n = \log \frac{m}{n} \right]$$

(ii) $\frac{1}{3} \log(x-1) + \frac{1}{2} \log(x)$

Solution:

$$\frac{1}{3} \log(x-1) + \frac{1}{2} \log x$$

$$= \log \left((x-1)^{\frac{1}{3}} \right) + \log \left(x^{\frac{1}{2}} \right)$$

$$\dots [n \log m = \log m^n]$$

$$= \log \left(\sqrt[3]{x-1} \sqrt{x} \right)$$

$$\dots [\log m + \log n = \log mn]$$

(iii) $\ln(x+2) + \ln(x-2) - 3 \ln(x+5)$

Solution:

$$\ln(x+2) + \ln(x-2) - 3 \ln(x+5)$$

$$= \ln[(x+2)(x-2)] - \ln(x+5)^3$$

$$\dots \left[\begin{array}{l} \log m + \log n = \log mn \\ n \log m = \log m^n \end{array} \right]$$

$$= \ln(x^2 - 4) - \ln(x+5)^3$$

$$= \ln \left(\frac{x^2 - 4}{(x + 5)^3} \right) \quad \dots \left[\log m - \log n = \log \frac{m}{n} \right]$$

Question 22.

Given that $\log 2 = a$ and $\log 3 = b$, write $\log \sqrt{96}$ terms of a and b .

Solution:

$$\log 2 = a \text{ and } \log 3 = b$$

$$\log \sqrt{96} = \frac{1}{2} \log (96)$$

$$= \frac{1}{2} \log (2^5 \times 3)$$

$$= \frac{1}{2} (\log 2^5 + \log 3) \dots [\because \log mn = \log m + \log n]$$

$$= \frac{1}{2} (5 \log 2 + \log 3) \dots [\because \log m^n = n \log m]$$

$$= \frac{5a+b}{2}$$

Question 23.

Prove that:

$$(a) \ b^{\log_b a} = a$$

Solution:

$$\text{We have to prove that } b^{\log_b a} = a$$

$$\text{i.e., to prove that } (\log_b a) (\log_b b) = \log_b a$$

(Taking log on both sides with base b)

$$\begin{aligned}\text{L.H.S.} &= (\log_b a) (\log_b b) \\ &= \log_b a \dots [\because \log_b b = 1] \\ &= \text{R.H.S.}\end{aligned}$$

$$(b) \log_{b^m} a = \frac{1}{m} \log_b a$$

Solution:

$$\begin{aligned}\log_{b^m} a &= \frac{1}{m} \log_b a \\ \text{L.H.S.} &= \log_{b^m} a \\ &= \frac{\log a}{\log b^m} \dots \left[\log_y x = \frac{\log x}{\log y} \right] \\ &= \frac{\log a}{m \log b} = \frac{1}{m} \log_b a = \text{R.H.S.}\end{aligned}$$

$$(c) a^{\log_c b} = b^{\log_c a}$$

Solution:

$$\begin{aligned}a^{\log_c b} &= b^{\log_c a} \\ \text{L.H.S.} &= a^{\log_c b} \\ &= (e^{\log a})^{\log_c b} \dots [x = e^{\log x}] \\ &= (e^{\log a})^{\frac{\log b}{\log c}} = (e^{\log b})^{\frac{\log a}{\log c}} \\ &= (e^{\log b})^{\log_c a} = b^{\log_c a} = \text{R.H.S.}\end{aligned}$$

Question 24.

If $f(x) = ax^2 - bx + 6$ and $f(2) = 3$ and $f(4) = 30$, find a and b .

Solution:

$$f(x) = ax^2 - bx + 6$$

$$f(2) = 3$$

$$\therefore a(2)^2 - b(2) + 6 = 3$$

$$\therefore 4a - 2b + 6 = 3$$

$$\therefore 4a - 2b + 3 = 0 \dots\dots(i)$$

$$f(4) = 30$$

$$\therefore a(4)^2 - b(4) + 6 = 30$$

$$\therefore 16a - 4b + 6 = 30$$

$$\therefore 16a - 4b - 24 = 0 \dots\dots(ii)$$

By (ii) $- 2 \times$ (i), we get

$$8a - 30 = 0$$

$$\therefore a = \frac{30}{8} = \frac{15}{4}$$

Substituting $a = \frac{15}{4}$ in (i), we get

$$4\left(\frac{15}{4}\right) - 2b + 3 = 0$$

$$\therefore 2b = 18$$

$$\therefore b = 9$$

$$\therefore a = \frac{15}{4}, b = 9$$

Question 25.

Solve for x :

$$(i) \log 2 + \log (x + 3) - \log (3x - 5) = \log 3$$

Solution:

$$\log 2 + \log (x + 3) - \log (3x - 5) = \log 3$$

$$\therefore \log 2(x + 3) - \log(3x - 5) = \log 3 \dots\dots[\because \log m + \log n = \log mn]$$

$$\therefore \log \frac{2(x+3)}{3x-5} = \log 3 \dots\dots[\because \log m - \log n = \log \frac{m}{n}]$$

$$\therefore \frac{2(x+3)}{3x-5} = 3$$

$$\therefore 2x + 6 = 9x - 15$$

$$\therefore 7x = 21$$

$$\therefore x = 3$$

Check:

If $x = 3$ satisfies the given condition, then our answer is correct.

$$\text{L.H.S.} = \log 2 + \log (x + 3) - \log (3x - 5)$$

$$= \log 2 + \log (3 + 3) - \log (9 - 5)$$

$$= \log 2 + \log 6 - \log 4$$

$$= \log (2 \times 6) - \log 4$$

$$= \log \frac{12}{4}$$

$$= \log 3$$

$$= \text{R.H.S.}$$

Thus, our answer is correct.

$$(ii) 2 \log_{10} x = 1 + \log_{10} \left(x + \frac{11}{10} \right)$$

Solution:

$$2 \log_{10} x = 1 + \log_{10} \left(x + \frac{11}{10} \right)$$

$$\log_{10} x^2 - \log_{10} \left(x + \frac{11}{10} \right) = 1$$

$$\dots [n \log m = \log m^n]$$

$$\therefore \log_{10} x^2 - \log_{10} \left(\frac{10x+11}{10} \right) = 1$$

$$\therefore \log_{10} \left(\frac{x^2}{\frac{10x+11}{10}} \right) = 1 \dots \left[\log m - \log n = \log \frac{m}{n} \right]$$

$$\therefore \log_{10} \left(\frac{10x^2}{10x+11} \right) = \log_{10} 10 \dots [\log_a a = 1]$$

$$\therefore \frac{10x^2}{10x+11} = 10$$

$$\therefore \frac{x^2}{10x+11} = 1$$

$$\therefore x^2 = 10x + 11$$

$$\therefore x^2 - 10x - 11 = 0$$

$$\therefore (x-11)(x+1) = 0$$

$$\therefore x = 11 \text{ or } x = -1$$

But log of a negative numbers does not exist

$$\therefore x \neq -1$$

$$\therefore x = 11$$

$$(iii) \log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$$

Solution:

$$\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$$

$$\therefore \frac{\log x}{\log 2} + \frac{\log x}{\log 4} + \frac{\log x}{\log 16} = \frac{21}{4} \quad \dots \left[\log_y x = \frac{\log x}{\log y} \right]$$

$$\therefore \frac{\log x}{\log 2} + \frac{\log x}{\log(2)^2} + \frac{\log x}{\log(2)^4} = \frac{21}{4}$$

$$\therefore \frac{\log x}{\log 2} + \frac{\log x}{2 \cdot \log 2} + \frac{\log x}{4 \cdot \log 2} = \frac{21}{4}$$

$$\dots [\log m^n = n \log m]$$

$$\therefore \frac{\log x}{\log 2} \left(1 + \frac{1}{2} + \frac{1}{4} \right) = \frac{21}{4}$$

$$\therefore \frac{\log x}{\log 2} \times \left(\frac{7}{4} \right) = \frac{21}{4}$$

$$\therefore \frac{\log x}{\log 2} = 3$$

$$\therefore \log x = 3 \log 2$$

$$\therefore \log x = \log 2^3$$

$$\therefore x = 2^3$$

$$\therefore x = 8$$

$$(iv) x + \log_{10} (1 + 2^x) = x \log_{10} 5 + \log_{10} 6$$

Solution:

$$\begin{aligned} x + \log_{10} (1 + 2^x) &= x \log_{10} 5 + \log_{10} 6 \\ \therefore x \log_{10} 10 + \log_{10} (1 + 2^x) &= x \log_{10} 5 + \log_{10} 6 \\ &\quad \dots [\log_a a = 1] \\ \therefore \log_{10} 10^x + \log_{10} (1 + 2^x) &= \log_{10} 5^x + \log_{10} 6 \\ &\quad \dots [n \log m = \log m^n] \\ \therefore \log_{10} [10^x (1 + 2^x)] &= \log_{10} (6 \times 5^x) \\ &\quad \dots [\log m + \log n = \log mn] \\ \therefore 10^x (1 + 2^x) &= 6 \times 5^x \\ \therefore 2^x \times 5^x (1 + 2^x) &= 6 \times 5^x \\ \therefore 2^x (1 + 2^x) &= 6 \\ \text{Let } 2^x &= a \\ \therefore a(1 + a) &= 6 \end{aligned}$$

$$\therefore a + a^2 = 6$$

$$\therefore a^2 + a - 6 = 0$$

$$\therefore (a + 3)(a - 2) = 0$$

$$\therefore a + 3 = 0 \text{ or } a - 2 = 0$$

$$\therefore a = -3 \text{ or } a = 2$$

Since $2^x = -3$ is not possible,

$$2^x = 2 = 2^1$$

$$\therefore x = 1$$

Question 26.

If $\log \left(\frac{x+y}{3} \right) = \frac{1}{2} \log x + \frac{1}{2} \log y$, show that $\frac{x}{y} + \frac{y}{x} = 7$.

Solution:

$$\log \left(\frac{x+y}{3} \right) = \frac{1}{2} \log x + \frac{1}{2} \log y$$

Multiplying throughout by 2, we get

$$2 \log \left(\frac{x+y}{3} \right) = \log x + \log y$$

$$\therefore 2 \log \left(\frac{x+y}{3} \right) = \log xy$$

$$\dots [\log m + \log n = \log mn]$$

$$\therefore \log \left(\frac{x+y}{3} \right)^2 = \log xy \quad \dots [n \log m = \log m^n]$$

$$\therefore \frac{(x+y)^2}{9} = xy$$

$$\therefore x^2 + 2xy + y^2 = 9xy$$

$$\therefore x^2 + y^2 = 7xy$$

Dividing throughout by xy , we get

$$\frac{x}{y} + \frac{y}{x} = 7$$

Question 27.

If $\log \left(\frac{x-y}{4} \right) = \log \sqrt{x} + \log \sqrt{y}$, show that $(x+y)^2 = 20xy$.

Solution:

$$\log\left(\frac{x-y}{4}\right) = \log\sqrt{x} + \log\sqrt{y}$$

$$\therefore \log\left(\frac{x-y}{4}\right) = \log(\sqrt{x}\cdot\sqrt{y})$$

$$\dots[\log m + \log n = \log mn]$$

$$\therefore \log\left(\frac{x-y}{4}\right) = \log\sqrt{xy}$$

$$\therefore \frac{x-y}{4} = \sqrt{xy}$$

Squaring on both sides, we get

$$\frac{(x-y)^2}{16} = xy$$

$$\therefore x^2 - 2xy + y^2 = 16xy$$

Adding $4xy$ on both sides, we get

$$x^2 + 2xy + y^2 = 20xy$$

$$\therefore (x+y)^2 = 20xy$$

Question 28.

If $x = \log_a bc$, $y = \log_b ca$, $z = \log_c ab$, then prove that $\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z} = 1$.

Solution:

$$x = \log_a (bc), y = \log_b (ca), z = \log_c (ab)$$

$$\text{L.H.S.} = \frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$$

$$\begin{aligned}
 &= \frac{1}{1 + \log_a(bc)} + \frac{1}{1 + \log_b(ca)} + \frac{1}{1 + \log_c(ab)} \\
 &= \frac{1}{\log_a a + \log_a(bc)} + \frac{1}{\log_b b + \log_b(ca)} \\
 &\quad + \frac{1}{\log_c c + \log_c(ab)} \\
 &\quad \dots [\log_a a = 1] \\
 &= \frac{1}{\log_a(abc)} + \frac{1}{\log_b(abc)} + \frac{1}{\log_c(abc)} \\
 &\quad \dots [\log_a m + \log_a n = \log_a mn] \\
 &= \frac{\log a}{\log(abc)} + \frac{\log b}{\log(abc)} + \frac{\log c}{\log(abc)} \\
 &\quad \dots \left[\log_y x = \frac{\log x}{\log y} \right] \\
 &= \frac{\log a + \log b + \log c}{\log abc} \\
 &= \frac{\log abc}{\log abc} \\
 &= 1 \\
 &= \text{R.H.S.}
 \end{aligned}$$



Maharashtra Board Solutions

Class 11 Arts & Science Maths

(Part 2)

- Chapter 1- Complex Numbers
- Chapter 2- Sequences and Series
- Chapter 3- Permutations and Combination
- Chapter 4- Methods of Induction and Binomial Theorem
- Chapter 5- Sets and Relations
- Chapter 6- Functions
- Chapter 7- Limits
- Chapter 8- Continuity
- Chapter 9- Differentiation

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-2-chapter-6-functions/>

About About Maharashtra State Board (MSBSHSE)

The Maharashtra State Board of Secondary and Higher Secondary Education or MSBSHSE (Marathi: महाराष्ट्र राज्य माध्यमिक आणि उच्च माध्यमिक शिक्षण मंडळ), is an **autonomous and statutory body established in 1965**. The board was amended in the year 1977 under the provisions of the Maharashtra Act No. 41 of 1965.

The Maharashtra State Board of Secondary & Higher Secondary Education (MSBSHSE), Pune is an independent body of the Maharashtra Government. There are more than 1.4 million students that appear in the examination every year. The Maha State Board conducts the board examination twice a year. This board conducts the examination for SSC and HSC.

The Maharashtra government established the Maharashtra State Bureau of Textbook Production and Curriculum Research, also commonly referred to as Ebalbharati, in 1967 to take up the responsibility of providing quality textbooks to students from all classes studying under the Maharashtra State Board. MSBHSE prepares and updates the curriculum to provide holistic development for students. It is designed to tackle the difficulty in understanding the concepts with simple language with simple illustrations. Every year around 10 lakh students are enrolled in schools that are affiliated with the Maharashtra State Board.

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-2-chapter-6-functions/>

FAQs

Where do I get the Maharashtra State Board Books PDF For free download?

You can download the Maharashtra State Board Books from the eBalbharti official website, i.e. cart.ebalbharati.in or from this article.

Add image

How to Download Maharashtra State Board Books?

Students can get the Maharashtra Books for primary, secondary, and senior secondary classes from here. You can view or download the Maharashtra State Board Books from this page or from the official website for free of cost. Students can follow the detailed steps below to visit the official website and download the e-books for all subjects or a specific subject in different mediums.

Step 1: Visit the official website ebalbharati.in

Step 2: On the top of the screen, select "Download PDF textbooks"

Step 3: From the "Classes" section, select your class.

Step 4: From "Medium", select the medium suitable to you.

Step 5: All Maharashtra board books for class 11th will now be displayed on the right side.

Step 6: Click on the "Download" option to download the PDF book.

Who developed the Maharashtra State board books?

As of now, the MSCERT and Balbharti are responsible for the syllabus and textbooks of Classes 1 to 8, while Classes 9 and 10 are under the Maharashtra State Board of Secondary and Higher Secondary Education (MSBSHSE).

How many state boards are there in Maharashtra?

The Maharashtra State Board of Secondary & Higher Secondary Education, conducts the HSC and SSC Examinations in the state of Maharashtra through its nine Divisional Boards located at Pune, Mumbai, Aurangabad, Nasik, Kolhapur, Amravati, Latur, Nagpur and Ratnagiri.

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-2-chapter-6-functions/>

About IndCareer

IndCareer.com is a leading developer of online career guidance resources for the Indian marketplace. Established in 2007, IndCareer.com is currently used by over thousands of institutions across India, including schools, employment agencies, libraries, colleges and universities.

IndCareer.com is designed to assist you in making the right career decision - a decision that meets your unique interests and personality.

For any clarifications or questions you can write to **info@indcareer.com**

Postal Address

IndCareer.com
52, Shilpa Nagar,
Somalwada
Nagpur - 440015
Maharashtra, India

WhatsApp: +91 9561 204 888

Website: <https://www.indcareer.com>

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-6-functions/>