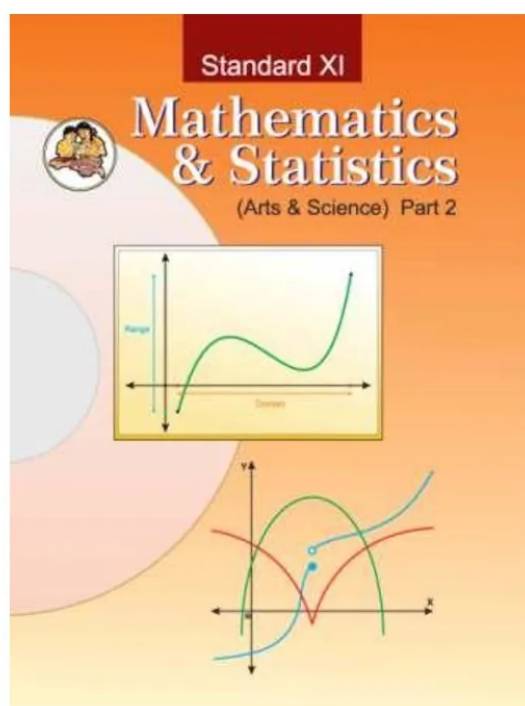


# Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 4- Methods of Induction and Binomial Theorem

## Class 11 - Chapter 4 Methods of Induction and Binomial Theorem



For any clarifications or questions you can write to [info@indcareer.com](mailto:info@indcareer.com)

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# Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 4- Methods of Induction and Binomial Theorem

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## Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 4- Methods of Induction and Binomial Theorem

Maharashtra Board 11th Maths Chapter 4, Class 11 Maths Chapter 4 solutions

### Ex 4.1

Prove by the method of induction, for all  $n \in \mathbb{N}$ .

#### Question 1.

$$2 + 4 + 6 + \dots + 2n = n(n + 1)$$

**Solution:**

Let  $P(n) = 2 + 4 + 6 + \dots + 2n = n(n + 1)$ , for all  $n \in \mathbb{N}$ .

Step I:

Put  $n = 1$

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$$\text{L.H.S.} = 2$$

$$\text{R.H.S.} = 1(1 + 1) = 2$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$\therefore P(n)$  is true for  $n = 1$ .

Step II:

Let us assume that  $P(n)$  is true for  $n = k$ .

$$\therefore 2 + 4 + 6 + \dots + 2k = k(k + 1) \dots\dots(i)$$

Step III:

We have to prove that  $P(n)$  is true for  $n = k + 1$ ,

i.e., to prove that

$$2 + 4 + 6 + \dots + 2(k + 1) = (k + 1)(k + 2)$$

$$\text{L.H.S.} = 2 + 4 + 6 + \dots + 2(k + 1)$$

$$= 2 + 4 + 6 + \dots + 2k + 2(k + 1)$$

$$= k(k + 1) + 2(k + 1) \dots\dots[\text{From (i)}]$$

$$= (k + 1)(k + 2)$$

$$= \text{R.H.S.}$$

$\therefore P(n)$  is true for  $n = k + 1$ .

Step IV:

From all the steps above, by the principle of mathematical induction,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$$\therefore 2 + 4 + 6 + \dots + 2n = n(n + 1) \text{ for all } n \in \mathbb{N}.$$

**Question 2.**

$$3 + 7 + 11 + \dots \text{ to } n \text{ terms} = n(2n + 1)$$

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**Solution:**

Let  $P(n) = 3 + 7 + 11 + \dots$  to  $n$  terms  $= n(2n + 1)$ , for all  $n \in \mathbb{N}$ .

But 3, 7, 11, .... are in A.P.

$$\therefore a = 3 \text{ and } d = 4$$

Let  $t_n$  be the  $n$ th term.

$$\therefore t_n = a + (n - 1)d = 3 + (n - 1)4 = 4n - 1$$

$$\therefore P(n) = 3 + 7 + 11 + \dots + (4n - 1) = n(2n + 1)$$

Step I:

Put  $n = 1$

$$\text{L.H.S.} = 3$$

$$\text{R.H.S.} = 1[2(1) + 1] = 3$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$\therefore P(n)$  is true for  $n = 1$ .

Step II:

Let us assume that  $P(n)$  is true for  $n = k$ .

$$\therefore 3 + 7 + 11 + \dots + (4k - 1) = k(2k + 1) \dots (i)$$

Step III:

We have to prove that  $P(n)$  is true for  $n = k + 1$ ,

i.e., to prove that

$$3 + 7 + 11 + \dots + [4(k + 1) - 1] = (k + 1)(2k + 3)$$

$$\text{L.H.S.} = 3 + 7 + 11 + \dots + [4(k + 1) - 1]$$

$$= 3 + 7 + 11 + \dots + (4k - 1) + [4(k + 1) - 1]$$

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$$= k(2k + 1) + (4k + 4 - 1) \dots [\text{From (i)}]$$

$$= 2k^2 + k + 4k + 3$$

$$= 2k^2 + 2k + 3k + 3$$

$$= 2k(k + 1) + 3(k + 1)$$

$$= (k + 1)(2k + 3)$$

$$= \text{R.H.S.}$$

$\therefore P(n)$  is true for  $n = k + 1$ .

Step IV:

From all the steps above, by the principle of mathematical induction,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$\therefore 3 + 7 + 11 + \dots$  to  $n$  terms  $= n(2n + 1)$  for all  $n \in \mathbb{N}$ .

Question 3.

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Solution:

$$\text{Let } P(n) = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} \text{ for all } n \in \mathbb{N}.$$

Step I:

Put  $n = 1$

$$\text{L.H.S.} = 1^2 = 1$$

$$\text{RHS} = \frac{1(1+1)[2(1)+1]}{6} = \frac{6}{6} = 1$$

$\therefore \text{L.H.S.} = \text{R.H.S.}$

$\therefore P(n)$  is true for  $n = 1$ .

Step II:

Let us assume that  $P(n)$  is true for  $n = k$ .

$$\therefore 1^2 + 2^2 + 3^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6} \dots\dots(i)$$

Step III:

We have to prove that  $P(n)$  is true for  $n = k + 1$ ,

i.e., to prove that

$$\begin{aligned} &1^2 + 2^2 + 3^2 + \dots + (k+1)^2 \\ &= \frac{(k+1)[(k+1)+1][2(k+1)+1]}{6} \\ &= \frac{(k+1)(k+2)(2k+3)}{6} \end{aligned}$$

$$\begin{aligned}\text{L.H.S.} &= 1^2 + 2^2 + 3^2 + \dots + (k+1)^2 \\ &= 1^2 + 2^2 + 3^2 + \dots + k^2 + (k+1)^2 \\ &= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \quad \dots [\text{From (i)}] \\ &= (k+1) \left[ \frac{k(2k+1)}{6} + (k+1) \right] \\ &= (k+1) \left( \frac{2k^2 + k + 6k + 6}{6} \right) \\ &= \frac{(k+1)(2k^2 + 7k + 6)}{6} \\ &= \frac{(k+1)(k+2)(2k+3)}{6} \\ &= \text{R.H.S.}\end{aligned}$$

$\therefore P(n)$  is true for  $n = k + 1$ .

Step IV:

From all the steps above, by the principle of mathematical induction,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$$\therefore 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} \text{ for all } n \in \mathbb{N}.$$

Question 4.

$$1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n}{3} (2n-1)(2n+1)$$

Solution:

$$\text{Let } P(n) = 1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n}{3} (2n-1)(2n+1), \text{ for all } n \in \mathbb{N}.$$

Step I:

Put  $n = 1$

$$\text{L.H.S.} = 1^2 = 1$$

$$\text{R.H.S.} = \frac{1}{3} [2(1) - 1][2(1) + 1] = 1$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$\therefore P(n)$  is true for  $n = 1$ .

Step II:

Let us assume that  $P(n)$  is true for  $n = k$ .

$$\therefore 1^2 + 3^2 + 5^2 + \dots + (2k-1)^2 = \frac{k}{3} (2k-1)(2k+1) \dots\dots(i)$$

Step III:

We have to prove that  $P(n)$  is true for  $n = k + 1$ ,

i.e., to prove that

$$1^2 + 3^2 + 5^2 + \dots + [2(k+1) - 1]^2$$
$$= \frac{(k+1)}{3} [2(k+1) - 1][2(k+1) + 1]$$

$$= \frac{(k+1)(2k+1)(2k+3)}{3}$$

$$\text{L.H.S.} = 1^2 + 3^2 + 5^2 + \dots + [2(k+1) - 1]^2$$
$$= 1^2 + 3^2 + 5^2 + \dots + (2k-1)^2 + (2k+1)^2$$
$$= \frac{k}{3} (2k-1)(2k+1) + (2k+1)^2$$

...[From (i)]

$$= (2k+1) \left[ \frac{k(2k-1)}{3} + (2k+1) \right]$$

$$= (2k+1) \left[ \frac{2k^2 - k + 6k + 3}{3} \right]$$

$$= \frac{(2k+1)}{3} (2k^2 + 2k + 3k + 3)$$

$$= \frac{(2k+1)}{3} [2k(k+1) + 3(k+1)]$$

$$= \frac{(2k+1)(k+1)(2k+3)}{3}$$

$$= \text{R.H.S.}$$

$\therefore P(n)$  is true for  $n = k + 1$ .

Step IV:

From all the steps above, by the principle of mathematical induction,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$$\therefore 1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n}{3} (2n-1)(2n+1) \text{ for all } n \in \mathbb{N}.$$

Question 5.

$$1^3 + 3^3 + 5^3 + \dots \text{ to } n \text{ terms} = n^2 (2n^2 - 1)$$

Solution:

$$\text{Let } P(n) = 1^3 + 3^3 + 5^3 + \dots \text{ to } n \text{ terms} = n^2 (2n^2 - 1), \text{ for all } n \in \mathbb{N}.$$

But 1, 3, 5, are in A.P.

$$\therefore a = 1, d = 2$$

Let  $t_n$  be the  $n$ th term.

$$t_n = a + (n-1)d = 1 + (n-1)2 = 2n-1$$

$$\therefore P(n) = 1^3 + 3^3 + 5^3 + \dots + (2n-1)^3 = n^2 (2n^2 - 1)$$

Step I:

Put  $n = 1$

$$\text{L.H.S.} = 1^3 = 1$$

$$\text{R.H.S.} = 1^2 [2(1)^2 - 1] = 1$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$\therefore P(n)$  is true for  $n = 1$ .

Step II:

Let us assume that  $P(n)$  is true for  $n = k$ .

$$\therefore 1^3 + 3^3 + 5^3 + \dots + (2k-1)^3 = k^2 (2k^2 - 1) \dots (i)$$

Step III:

We have to prove that  $P(n)$  is true for  $n = k + 1$ ,

i.e., to prove that

$$\begin{aligned} & 1^3 + 3^3 + 5^3 + \dots + [2(k+1) - 1]^3 \\ &= (k+1)^2 [2(k+1)^2 - 1] \\ &= (k^2 + 2k + 1)(2k^2 + 4k + 1) \\ \text{L.H.S.} &= 1^3 + 3^3 + 5^3 + \dots + [2(k+1) - 1]^3 \\ &= 1^3 + 3^3 + 5^3 + \dots + (2k-1)^3 + (2k+1)^3 \\ &= k^2(2k^2 - 1) + (2k+1)^3 \quad \dots [\text{From (i)}] \\ &= 2k^4 - k^2 + 8k^3 + 12k^2 + 6k + 1 \\ &= 2k^4 + 8k^3 + 11k^2 + 6k + 1 \\ &= 2k^2(k^2 + 2k + 1) + 4k^3 + 9k^2 + 6k + 1 \\ &= 2k^2(k^2 + 2k + 1) + 4k(k^2 + 2k + 1) \\ &\quad + (k^2 + 2k + 1) \\ &= (k^2 + 2k + 1)(2k^2 + 4k + 1) \\ &= \text{R.H.S.} \end{aligned}$$

$\therefore P(n)$  is true for  $n = k + 1$ .

Step IV:

From all the steps above, by the principle of mathematical induction,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$$\therefore 1^3 + 3^3 + 5^3 + \dots \text{ to } n \text{ terms} = n^2 (2n^2 - 1) \text{ for all } n \in \mathbb{N}.$$

Question 6.

$$1.2 + 2.3 + 3.4 + \dots + n(n+1) = \frac{n}{3} (n+1)(n+2)$$

Solution:

$$\text{Let } P(n) = 1.2 + 2.3 + 3.4 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}, \text{ for all } n \in \mathbb{N}.$$

Step I:

Put  $n = 1$

$$\text{L.H.S.} = 1.2 = 2$$

$$\text{R.H.S.} = \frac{1}{3} (1+1)(1+2) = 2$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$\therefore P(n)$  is true for  $n = 1$ .

Step II:

Let us assume that  $P(n)$  is true for  $n = k$ .

$$\therefore 1.2 + 2.3 + 3.4 + \dots + k(k+1) = \frac{k}{3} (k+1)(k+2) \dots\dots(i)$$

Step III:

We have to prove that  $P(n)$  is true for  $n = k + 1$ ,

i.e., to prove that

$$\begin{aligned} & 1.2 + 2.3 + 3.4 + \dots + (k+1)[(k+1)+1] \\ &= \frac{(k+1)}{3} [(k+1)+1][(k+1)+2] \\ &= \frac{(k+1)}{3} (k+2)(k+3) \\ \text{L.H.S.} &= 1.2 + 2.3 + 3.4 + \dots + (k+1)[(k+1)+1] \\ &= 1.2 + 2.3 + 3.4 + \dots + k(k+1) \\ &\quad + (k+1)(k+2) \\ &= \frac{k}{3} (k+1)(k+2) + (k+1)(k+2) \\ &\quad \dots [\text{From (i)}] \\ &= (k+1)(k+2) \left( \frac{k}{3} + 1 \right) \\ &= \frac{(k+1)(k+2)(k+3)}{3} \\ &= \text{R.H.S.} \end{aligned}$$

$\therefore P(n)$  is true for  $n = k + 1$ .

Step IV:

From all the steps above, by the principle of mathematical induction,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$$\therefore 1.2 + 2.3 + 3.4 + \dots + n(n+1) = \frac{n}{3} (n+1)(n+2), \text{ for all } n \in \mathbb{N}.$$

Question 7.

$$1.3 + 3.5 + 5.7 + \dots \text{ to } n \text{ terms} = \frac{n}{3} (4n^2 + 6n - 1)$$

Solution:

$$\text{Let } P(n) = 1.3 + 3.5 + 5.7 + \dots \text{ to } n \text{ terms} = \frac{n}{3} (4n^2 + 6n - 1), \text{ for all } n \in \mathbb{N}.$$

But first factor in each term, i.e., 1, 3, 5, ... are in A.P. with  $a = 1$  and  $d = 2$ .

$$\therefore \text{nth term} = a + (n-1)d = 1 + (n-1)2 = (2n-1)$$

Also, second factor in each term,

i.e., 3, 5, 7, ... are in A.P. with  $a = 3$  and  $d = 2$ .

$$\therefore \text{nth term} = a + (n-1)d = 3 + (n-1)2 = (2n+1)$$

$$\therefore \text{nth term, } t_n = (2n-1)(2n+1)$$

$$\therefore P(n) \equiv 1.3 + 3.5 + 5.7 + \dots + (2n-1)(2n+1) = \frac{n}{3} (4n^2 + 6n - 1)$$

Step I:

Put  $n = 1$

$$\text{L.H.S.} = 1.3 = 3$$

$$\text{R.H.S.} = \frac{1}{3} [4(1)^2 + 6(1) - 1] = 3$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$\therefore P(n)$  is true for  $n = 1$ .

Step II:

Let us assume that  $P(n)$  is true for  $n = k$ .

$$\therefore 1.3 + 3.5 + 5.7 + \dots + (2k-1)(2k+1) = \frac{k}{3} (4k^2 + 6k - 1) \dots\dots(i)$$

Step III:

We have to prove that  $P(n)$  is true for  $n = k + 1$ ,

i.e., to prove that

$$\begin{aligned} & 1.3 + 3.5 + 5.7 + \dots + [2(k+1)-1][2(k+1)+1] \\ &= \frac{(k+1)}{3} [4(k+1)^2 + 6(k+1) - 1] \\ &= \frac{(k+1)}{3} (4k^2 + 8k + 4 + 6k + 6 - 1) \\ &= \frac{(k+1)}{3} (4k^2 + 14k + 9) \end{aligned}$$

$$\begin{aligned} \text{L.H.S.} &= 1.3 + 3.5 + 5.7 + \dots \\ &\quad + [2(k+1)-1][2(k+1)+1] \\ &= 1.3 + 3.5 + 5.7 + \dots + (2k-1)(2k+1) \\ &\quad + (2k+1)(2k+3) \\ &= \frac{k}{3} (4k^2 + 6k - 1) + (2k+1)(2k+3) \\ &\quad \dots[\text{From (i)}] \\ &= \frac{1}{3} [4k^3 + 6k^2 - k + 3(2k+1)(2k+3)] \\ &= \frac{1}{3} (4k^3 + 6k^2 - k + 12k^2 + 24k + 9) \end{aligned}$$

$$\begin{aligned} &= \frac{1}{3} (4k^3 + 18k^2 + 23k + 9) \\ &= \frac{1}{3} (k+1)(4k^2 + 14k + 9) \\ &= \text{R.H.S.} \end{aligned}$$

$\therefore P(n)$  is true for  $n = k + 1$ .

Step IV:

From all the steps above, by the principle of mathematical induction,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$\therefore 1.3 + 3.5 + 5.7 + \dots$  to  $n$  terms  $= \frac{n}{3} (4n^2 + 6n - 1)$  for all  $n \in \mathbb{N}$ .

Question 8.

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

Solution:

Let  $P(n) \equiv \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$ , for all  $n \in \mathbb{N}$ .

Step I:

Put  $n = 1$

$$\text{L.H.S.} = \frac{1}{1.3} = \frac{1}{3}$$

$$\text{R.H.S.} = \frac{1}{2(1)+1} = \frac{1}{3}$$

$\therefore \text{L.H.S.} = \text{R.H.S.}$

$\therefore P(n)$  is true for  $n = 1$ .

Step II:

Let us assume that  $P(n)$  is true for  $n = k$ .

$$\therefore \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1} \dots (i)$$

Step III:

We have to prove that  $P(n)$  is true for  $n = k + 1$ ,

i.e., to prove that

$$\begin{aligned} & \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{[2(k+1)-1][2(k+1)+1]} \\ &= \frac{k+1}{2(k+1)+1} = \frac{k+1}{2k+3} \\ \text{L.H.S.} &= \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{[2(k+1)-1][2(k+1)+1]} \\ &= \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2k-1)(2k+1)} \\ & \quad + \frac{1}{(2k+1)(2k+3)} \\ &= \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)} \dots [\text{From (i)}] \\ &= \frac{k(2k+3)+1}{(2k+1)(2k+3)} \\ &= \frac{2k^2+3k+1}{(2k+1)(2k+3)} \end{aligned}$$

$$\begin{aligned}
 &= \frac{2k^2 + 2k + k + 1}{(2k+1)(2k+3)} \\
 &= \frac{2k(k+1) + 1(k+1)}{(2k+1)(2k+3)} \\
 &= \frac{(2k+1)(k+1)}{(2k+1)(2k+3)} \\
 &= \frac{k+1}{2k+3} \\
 &= \text{R.H.S.}
 \end{aligned}$$

$\therefore P(n)$  is true for  $n = k + 1$ .

Step IV:

From all the steps above, by the principle of mathematical induction,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$$\therefore \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}, \text{ for all } n \in \mathbb{N}.$$

Question 9.

$$\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots \text{ to } n \text{ terms} = \frac{n}{3(2n+3)}$$

Solution:

$$\text{Let } P(n) \equiv \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots \text{ to } n \text{ terms} = \frac{n}{3(2n+3)}, \text{ for all } n \in \mathbb{N}.$$

But first factor in each term of the denominator,

i.e., 3, 5, 7, ..... are in A.P. with  $a = 3$  and  $d = 2$ .

$$\therefore \text{nth term} = a + (n-1)d = 3 + (n-1)2 = (2n+1)$$

Also, second factor in each term of the denominator,

i.e., 5, 7, 9, ... are in A.P. with  $a = 5$  and  $d = 2$ .

$$\therefore \text{nth term} = a + (n - 1) d = 5 + (n - 1) 2 = (2n + 3)$$

$$\therefore \text{nth term, } t_n = \frac{1}{(2n+1)(2n+3)}$$

$$P(n) \equiv \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$$

Step I:

Put  $n = 1$

$$\text{L.H.S.} = \frac{1}{3.5} = \frac{1}{15}$$

$$\text{R.H.S.} = \frac{1}{3[2(1)+3]} = \frac{1}{3(2+3)} = \frac{1}{15}$$

$\therefore \text{L.H.S.} = \text{R.H.S.}$

$\therefore P(n)$  is true for  $n = 1$ .

Step II:

Let us assume that  $P(n)$  is true for  $n = k$ .

$$\therefore \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots + \frac{1}{(2k+1)(2k+3)} = \frac{k}{3(2k+3)} \dots(i)$$

Step III:

We have to prove that  $P(n)$  is true for  $n = k + 1$ ,

i.e., to prove that

$$\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots + \frac{1}{[2(k+1)+1][2(k+1)+3]}$$

$$= \frac{k+1}{3[2(k+1)+3]} = \frac{k+1}{3(2k+5)}$$

$$\text{L.H.S.} = \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots$$

$$+ \frac{1}{[2(k+1)+1][2(k+1)+3]}$$

$$= \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots + \frac{1}{(2k+1)(2k+3)} + \frac{1}{(2k+3)(2k+5)}$$

$$= \frac{k}{3(2k+3)} + \frac{1}{(2k+3)(2k+5)} \dots [\text{From (i)}]$$

$$= \frac{k(2k+5)+3}{3(2k+3)(2k+5)}$$

$$= \frac{2k^2+5k+3}{3(2k+3)(2k+5)}$$

$$= \frac{2k^2+2k+3k+3}{3(2k+3)(2k+5)}$$

$$= \frac{2k(k+1)+3(k+1)}{3(2k+3)(2k+5)}$$

$$= \frac{(2k+3)(k+1)}{3(2k+3)(2k+5)}$$

$$= \frac{k+1}{3(2k+5)}$$

= R.H.S.

$\therefore P(n)$  is true for  $n = k + 1$ .

Step IV:

From all the steps above, by the principle of mathematical induction,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$\therefore \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots$  to  $n$  terms  $= \frac{n}{3(2n+3)}$ , for all  $n \in \mathbb{N}$ .

Question 10.

$(2^{3n} - 1)$  is divisible by 7.

Solution:

$(2^{3n} - 1)$  is divisible by 7 if and only if  $(2^{3n} - 1)$  is a multiple of 7.

Let  $P(n) \equiv (2^{3n} - 1) = 7m$ , where  $m \in \mathbb{N}$ .

Step I:

Put  $n = 1$

$$\therefore 2^{3n} - 1 = 2^{3(1)} - 1 = 2^3 - 1 = 8 - 1 = 7$$

$\therefore (2^{3n} - 1)$  is a multiple of 7.

$\therefore P(n)$  is true for  $n = 1$ .

Step II:

Let us assume that  $P(n)$  is true for  $n = k$ .

i.e.,  $2^{3k} - 1$  is a multiple of 7.

$$\therefore 2^{3k} - 1 = 7a, \text{ where } a \in \mathbb{N}$$

$$\therefore 2^{3k} = 7a + 1 \dots\dots(i)$$

Step III:

We have to prove that  $P(n)$  is true for  $n = k + 1$ ,

i.e., to prove that

$$2^{3(k+1)} - 1 = 7b, \text{ where } b \in \mathbb{N}.$$

$$\therefore P(k + 1) = 2^{3(k+1)} - 1$$

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$$= 2^{3k+3} - 1$$

$$= 2^{3k} \cdot (2^3) - 1$$

$$= (7a + 1)8 - 1 \dots [\text{From (i)}]$$

$$= 56a + 8 - 1$$

$$= 56a + 7$$

$$= 7(8a + 1)$$

$$7b, \text{ where } b = (8a + 1) \in \mathbb{N}$$

$\therefore P(n)$  is true for  $n = k + 1$ .

Step IV:

From all the steps above, by the principle of mathematical induction,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$\therefore (2^{4n} - 1)$  is divisible by 7, for all  $n \in \mathbb{N}$ .

**Question 11.**

**$(2^{4n} - 1)$  is divisible by 15.**

**Solution:**

$(2^{4n} - 1)$  is divisible by 15 if and only if  $(2^{4n} - 1)$  is a multiple of 15.

Let  $P(n) \equiv (2^{4n} - 1) = 15m$ , where  $m \in \mathbb{N}$ .

Step I:

Put  $n = 1$

$$\therefore 2^{4(1)} - 1 = 16 - 1 = 15$$

$\therefore (2^{4n} - 1)$  is a multiple of 15.

$\therefore P(n)$  is true for  $n = 1$ .

Step II:

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Let us assume that  $P(n)$  is true for  $n = k$ .

$$\therefore 2^{4k} - 1 = 15a, \text{ where } a \in \mathbb{N}$$

$$\therefore 2^{4k} = 15a + 1 \dots\dots(i)$$

Step III:

We have to prove that  $P(n)$  is true for  $n = k + 1$ ,

i.e., to prove that

$$\therefore 2^{4(k+1)} - 1 = 15b, \text{ where } b \in \mathbb{N}$$

$$\therefore P(k + 1) = 2^{4(k+1)} - 1 = 2^{4k+4} - 1$$

$$= 2^{4k} \cdot 2^4 - 1$$

$$= 16 \cdot (2^{4k}) - 1$$

$$= 16(15a + 1) - 1 \dots\dots[\text{From (i)}]$$

$$= 240a + 16 - 1$$

$$= 240a + 15$$

$$= 15(16a + 1)$$

$$= 15b, \text{ where } b = (16a + 1) \in \mathbb{N}$$

$$\therefore P(n) \text{ is true for } n = k + 1.$$

Step IV:

From all the steps above, by the principle of mathematical induction,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$$\therefore (2^{4n} - 1) \text{ is divisible by } 15, \text{ for all } n \in \mathbb{N}.$$

**Question 12.**

**$3^n - 2n - 1$  is divisible by 4.**

**Solution:**

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$(3^n - 2n - 1)$  is divisible by 4 if and only if  $(3^n - 2n - 1)$  is a multiple of 4.

Let  $P(n) \equiv (3^n - 2n - 1) = 4m$ , where  $m \in \mathbb{N}$ .

Step I:

Put  $n = 1$

$$\therefore (3^n - 2n - 1) = 3^{(1)} - 2(1) - 1 = 0 = 4(0)$$

$\therefore (3^n - 2n - 1)$  is a multiple of 4.

$\therefore P(n)$  is true for  $n = 1$ .

Step II:

Let us assume that  $P(n)$  is true for  $n = k$ .

$$\therefore 3^k - 2k - 1 = 4a, \text{ where } a \in \mathbb{N}$$

$$\therefore 3^k = 4a + 2k + 1 \dots (i)$$

Step III:

We have to prove that  $P(n)$  is true for  $n = k + 1$ ,

i.e., to prove that

$$3^{(k+1)} - 2(k+1) - 1 = 4b, \text{ where } b \in \mathbb{N}$$

$$P(k+1) = 3^{k+1} - 2(k+1) - 1$$

$$= 3^k \cdot 3 - 2k - 2 - 1$$

$$= (4a + 2k + 1) \cdot 3 - 2k - 3 \dots \dots \dots [\text{From (i)}]$$

$$= 12a + 6k + 3 - 2k - 3$$

$$= 12a + 4k$$

$$= 4(3a + k)$$

$$= 4b, \text{ where } b = (3a + k) \in \mathbb{N}$$

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$\therefore P(n)$  is true for  $n = k + 1$ .

Step IV:

From all the steps above, by the principle of mathematical induction,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$\therefore 3^n - 2n - 1$  is divisible by 4, for all  $n \in \mathbb{N}$ .

Question 13.

$$5 + 5^2 + 5^3 + \dots + 5^n = \frac{5}{4} (5^n - 1)$$

Solution:

$$\text{Let } P(n) \equiv 5 + 5^2 + 5^3 + \dots + 5^n = \frac{5}{4} (5^n - 1), \text{ for all } n \in \mathbb{N}.$$

Step I:

Put  $n = 1$

$$\text{L.H.S.} = 5$$

$$\text{R.H.S.} = \frac{5}{4} (5^1 - 1) = 5$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$\therefore P(n)$  is true for  $n = 1$ .

Step II:

Let us assume that  $P(n)$  is true for  $n = k$ .

$$\therefore 5 + 5^2 + 5^3 + \dots + 5^k = \frac{5}{4} (5^k - 1) \dots\dots(i)$$

Step III:

We have to prove that  $P(n)$  is true for  $n = k + 1$ ,

i.e., to prove that

$$5 + 5^2 + 5^3 + \dots + 5^{k+1} = \frac{5}{4} (5^{k+1} - 1)$$

$$\begin{aligned} \text{L.H.S.} &= 5 + 5^2 + 5^3 + \dots + 5^{k+1} \\ &= 5 + 5^2 + 5^3 + \dots + 5^k + 5^{k+1} \\ &= \frac{5}{4} (5^k - 1) + 5^{k+1} \quad \dots[\text{From (i)}] \end{aligned}$$

$$\begin{aligned}
 &= \frac{5 \cdot 5^k - 5 + 4 \cdot 5^{k+1}}{4} \\
 &= \frac{5^{k+1} + 4 \cdot 5^{k+1} - 5}{4} \\
 &= \frac{5 \cdot 5^{k+1} - 5}{4} \\
 &= \frac{5}{4} (5^{k+1} - 1) \\
 &= \text{R.H.S.}
 \end{aligned}$$

$\therefore P(n)$  is true for  $n = k + 1$ .

Step IV:

From all the steps above, by the principle of mathematical induction,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$$\therefore 5 + 5^2 + 5^3 + \dots + 5^n = \frac{5}{4} (5^n - 1), \text{ for all } n \in \mathbb{N}.$$

Question 14.

$$(\cos \theta + i \sin \theta)^n = \cos (n\theta) + i \sin (n\theta)$$

Solution:

Let  $P(n) \equiv (\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$ , for all  $n \in \mathbb{N}$ .

Step I:

Put  $n = 1$

$$\text{L.H.S.} = (\cos \theta + i \sin \theta)^1 = \cos \theta + i \sin \theta$$

$$\text{R.H.S.} = \cos[(1)\theta] + i \sin[(1)\theta] = \cos \theta + i \sin \theta$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$\therefore P(n)$  is true for  $n = 1$ .

Step II:

Let us assume that  $P(n)$  is true for  $n = k$ .

$$\therefore (\cos \theta + i \sin \theta)^k = \cos k\theta + i \sin k\theta \dots\dots(i)$$

Step III:

We have to prove that  $P(n)$  is true for  $n = k + 1$ ,

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i.e., to prove that

$$(\cos \theta + i \sin \theta)^{k+1} = \cos (k+1)\theta + i \sin (k+1)\theta$$

$$\text{L.H.S.} = (\cos \theta + i \sin \theta)^{k+1}$$

$$= (\cos \theta + i \sin \theta)^k \cdot (\cos \theta + i \sin \theta)$$

$$= (\cos k\theta + i \sin k\theta) \cdot (\cos \theta + i \sin \theta) \dots\dots[\text{From (i)}]$$

$$= \cos k\theta \cos \theta + i \sin \theta \cos k\theta + i \sin k\theta \cos \theta - \sin k\theta \sin \theta \dots\dots[\because i^2 = -1]$$

$$= (\cos k\theta \cos \theta - \sin k\theta \sin \theta) + i(\sin k\theta \cos \theta + \cos k\theta \sin \theta)$$

$$= \cos(k\theta + \theta) + i \sin(k\theta + \theta)$$

$$= \cos(k+1)\theta + i \sin(k+1)\theta$$

$$= \text{R.H.S.}$$

$\therefore P(n)$  is true for  $n = k + 1$ .

Step IV:

From all the steps above, by the principle of mathematical induction,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$\therefore (\cos \theta + i \sin \theta)^n = \cos(n\theta) + i \sin(n\theta)$ , for all  $n \in \mathbb{N}$ .

### Question 15.

Given that  $t_{n+1} = 5 t_{n+4}$ ,  $t_1 = 4$ , prove by method of induction that  $t_n = 5^n - 1$ .

**Solution:**

Let the statement  $P(n)$  has L.H.S. a recurrence relation  $t_{n+1} = 5 t_{n+4}$ ,  $t_1 = 4$  and R.H.S. a general statement  $t_n = 5^n - 1$ .

Step I:

Put  $n = 1$

$$\text{L.H.S.} = 4$$

$$\text{R.H.S.} = 5^1 - 1 = 4$$

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$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$$\therefore P(n) \text{ is true for } n = 1.$$

Put  $n = 2$

$$\text{L.H.S.} = t_2 = 5t_1 + 4 = 24$$

$$\text{R.H.S.} = t_2 = 5^2 - 1 = 24$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$$\therefore P(n) \text{ is true for } n = 2.$$

Step II:

Let us assume that  $P(n)$  is true for  $n = k$ .

$$\therefore t_{k+1} = 5 t_{k+4} \text{ and } t_k = 5^k - 1$$

Step III:

We have to prove that  $P(n)$  is true for  $n = k + 1$ ,

i.e., to prove that  $t_{k+1} = 5^{k+1} - 1$

Since  $t_{k+1} = 5 t_{k+4}$  and  $t_k = 5^k - 1$  .....[From Step II]

$$t_{k+1} = 5 (5^k - 1) + 4 = 5^{k+1} - 1$$

$$\therefore P(n) \text{ is true for } n = k + 1.$$

Step IV:

From all the steps above, by the principle of mathematical induction,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$$\therefore t_n = 5^n - 1, \text{ for all } n \in \mathbb{N}.$$

Question 16.

Prove by method of induction

$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix} \forall n \in N$$

Solution:

$$\text{Let } P(n) \equiv \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}, \text{ for all } n \in N.$$

**Step I:**

Put  $n = 1$

$$\text{L.H.S.} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^1 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$\text{R.H.S.} = \begin{bmatrix} 1 & 2(1) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$$\therefore P(n) \text{ is true for } n = 1.$$

**Step II:**

Let us assume that  $P(n)$  is true for  $n = k$ .

$$\therefore \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^k = \begin{bmatrix} 1 & 2k \\ 0 & 1 \end{bmatrix} \quad \dots(i)$$

**Step III:**

We have to prove that  $P(n)$  is true for  $n = k + 1$ ,

$$\text{i.e., to prove that } \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^{k+1} = \begin{bmatrix} 1 & 2(k+1) \\ 0 & 1 \end{bmatrix}$$

$$\begin{aligned}\text{L.H.S.} &= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^{k+1} \\ &= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^k \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad \dots [\text{From (i)}] \\ &= \begin{bmatrix} 1 + 2k(0) & 1(2) + 2k(1) \\ 0(1) + 1(0) & 0(2) + 1(1) \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2k + 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2(k+1) \\ 0 & 1 \end{bmatrix} \\ &= \text{R.H.S.}\end{aligned}$$

$\therefore$   $P(n)$  is true for  $n = k + 1$ .

**Step IV:**

From all the steps above, by the principle of mathematical induction,  $P(n)$  is true for all  $n \in \mathbb{N}$ .

$$\therefore \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}, \text{ for all } n \in \mathbb{N}.$$

**Ex 4.2**

**Question 1.**

**Expand:**

(i)  $(\sqrt{3} + \sqrt{2})^4$

**Solution:**

Here,  $a = \sqrt{3}$ ,  $b = \sqrt{2}$  and  $n = 4$ .

Using binomial theorem,

$$\begin{aligned} & (\sqrt{3} + \sqrt{2})^4 \\ &= {}^4C_0(\sqrt{3})^4(\sqrt{2})^0 + {}^4C_1(\sqrt{3})^3(\sqrt{2})^1 \\ &\quad + {}^4C_2(\sqrt{3})^2(\sqrt{2})^2 + {}^4C_3(\sqrt{3})^1(\sqrt{2})^3 \\ &\quad + {}^4C_4(\sqrt{3})^0(\sqrt{2})^4 \end{aligned}$$

$$\text{Now, } {}^4C_0 = {}^4C_4 = 1, {}^4C_1 = {}^4C_3 = 4,$$

$${}^4C_2 = \frac{4 \times 3}{2 \times 1} = 6$$

$$\therefore (\sqrt{3} + \sqrt{2})^4 = 1(9)(1) + 4(3\sqrt{3})(\sqrt{2}) + 6(3)(2) + 4(\sqrt{3})(2\sqrt{2}) + 1(1)(4)$$

$$= 9 + 12\sqrt{6} + 36 + 8\sqrt{6} + 4$$

$$= 49 + 20\sqrt{6}$$

$$\text{(ii) } (\sqrt{5} - \sqrt{2})^5$$

**Solution:**

Here,  $a = \sqrt{5}$ ,  $b = \sqrt{2}$  and  $n = 5$ .

Using binomial theorem,

$$\begin{aligned} & (\sqrt{5} - \sqrt{2})^5 \\ &= {}^5C_0(\sqrt{5})^5(\sqrt{2})^0 - {}^5C_1(\sqrt{5})^4(\sqrt{2})^1 \\ &\quad + {}^5C_2(\sqrt{5})^3(\sqrt{2})^2 - {}^5C_3(\sqrt{5})^2(\sqrt{2})^3 \\ &\quad + {}^5C_4(\sqrt{5})^1(\sqrt{2})^4 - {}^5C_5(\sqrt{5})^0(\sqrt{2})^5 \end{aligned}$$

$$\text{Now, } {}^5C_0 = {}^5C_5 = 1, {}^5C_1 = {}^5C_4 = 5,$$

$${}^5C_2 = {}^5C_3 = \frac{5 \times 4}{2 \times 1} = 10$$

$$\begin{aligned} & (\sqrt{5} - \sqrt{2})^5 \\ &= 1(25\sqrt{5})(1) - 5(25)(\sqrt{2}) + 10(5\sqrt{5})(2) \\ &\quad - 10(5)(2\sqrt{2}) + 5(\sqrt{5})(4) - 1(1)(4\sqrt{2}) \\ &= 25\sqrt{5} - 125\sqrt{2} + 100\sqrt{5} \\ &\quad - 100\sqrt{2} + 20\sqrt{5} - 4\sqrt{2} \\ &= 145\sqrt{5} - 229\sqrt{2} \end{aligned}$$

**Question 2.**

**Expand:**

**(i)  $(2x^2 + 3)^4$**

**Solution:**

Here,  $a = 2x^2$ ,  $b = 3$  and  $n = 4$ .

Using binomial theorem,

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$$(2x^2 + 3)^4 \\ = {}^4C_0(2x^2)^4(3)^0 + {}^4C_1(2x^2)^3(3)^1 + {}^4C_2(2x^2)^2(3)^2 \\ + {}^4C_3(2x^2)^1(3)^3 + {}^4C_4(2x^2)^0(3)^4$$

$$\text{Now, } {}^4C_0 = {}^4C_4 = 1, {}^4C_1 = {}^4C_3 = 4,$$

$${}^4C_2 = \frac{4 \times 3}{2 \times 1} = 6$$

$$(2x^2 + 3)^4 = 1(16x^8)(1) + 4(8x^6)(3) + 6(4x^4)(9) \\ + 4(2x^2)(27) + 1(1)(81) \\ = 16x^8 + 96x^6 + 216x^4 + 216x^2 + 81$$

(ii)  $\left(2x - \frac{1}{x}\right)^6$

Solution:

Here,  $a = 2x$ ,  $b = \frac{1}{x}$  and  $n = 6$ .

Using binomial theorem,

$$\begin{aligned} \left(2x - \frac{1}{x}\right)^6 &= {}^6C_0(2x)^6\left(\frac{1}{x}\right)^0 - {}^6C_1(2x)^5\left(\frac{1}{x}\right)^1 + {}^6C_2(2x)^4\left(\frac{1}{x}\right)^2 \\ &\quad - {}^6C_3(2x)^3\left(\frac{1}{x}\right)^3 + {}^6C_4(2x)^2\left(\frac{1}{x}\right)^4 - {}^6C_5(2x)^1\left(\frac{1}{x}\right)^5 \\ &\quad + {}^6C_6(2x)^0\left(\frac{1}{x}\right)^6 \end{aligned}$$

Now,  ${}^6C_0 = {}^6C_6 = 1$ ,  ${}^6C_1 = {}^6C_5 = 6$ ,

${}^6C_2 = {}^6C_4 = \frac{6 \times 5}{2 \times 1} = 15$ ,  ${}^6C_3 = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20$

$$\begin{aligned} \left(2x - \frac{1}{x}\right)^6 &= 1(64x^6)(1) - 6(32x^4) + 15(16x^2) - 20(8) \\ &\quad + 15\left(\frac{4}{x^2}\right) - 6\left(\frac{2}{x^4}\right) + 1(1)\left(\frac{1}{x^6}\right) \\ &= 64x^6 - 192x^4 + 240x^2 - 160 + \frac{60}{x^2} - \frac{12}{x^4} + \frac{1}{x^6} \end{aligned}$$

**Question 3.**

Find the value of

(i)  $(\sqrt{3} + 1)^4 - (\sqrt{3} - 1)^4$

Solution:

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$$\begin{aligned}
 & (\sqrt{3}+1)^4 \\
 &= {}^4C_0(\sqrt{3})^4(1)^0 + {}^4C_1(\sqrt{3})^3(1)^1 + {}^4C_2(\sqrt{3})^2(1)^2 \\
 &\quad + {}^4C_3(\sqrt{3})^1(1)^3 + {}^4C_4(\sqrt{3})^0(1)^4
 \end{aligned}$$

$$\text{Now, } {}^4C_0 = {}^4C_4 = 1, {}^4C_1 = {}^4C_3 = 4,$$

$${}^4C_2 = \frac{4 \times 3}{2 \times 1} = 6$$

$$\begin{aligned}
 \therefore (\sqrt{3}+1)^4 &= 1(9)(1) + 4(3\sqrt{3})(1) + 6(3)(1) \\
 &\quad + 4(\sqrt{3})(1) + 1(1)(1)
 \end{aligned}$$

$$\therefore (\sqrt{3}+1)^4 = 9 + 12\sqrt{3} + 18 + 4\sqrt{3} + 1 \dots(i)$$

$$\begin{aligned}
 \text{Also, } & (\sqrt{3}-1)^4 \\
 &= {}^4C_0(\sqrt{3})^4(1)^0 - {}^4C_1(\sqrt{3})^3(1)^1 + {}^4C_2(\sqrt{3})^2(1)^2 \\
 &\quad - {}^4C_3(\sqrt{3})^1(1)^3 + {}^4C_4(\sqrt{3})^0(1)^4 \\
 &= 1(9)(1) - 4(3\sqrt{3})(1) + 6(3)(1) \\
 &\quad - 4(\sqrt{3})(1) + 1(1)(1)
 \end{aligned}$$

$$(\sqrt{3}-1)^4 = 9 - 12\sqrt{3} + 18 - 4\sqrt{3} + 1 \dots(ii)$$

Subtracting (ii) from (i), we get

$$\begin{aligned}
 & (\sqrt{3}+1)^4 - (\sqrt{3}-1)^4 \\
 &= (9 + 12\sqrt{3} + 18 + 4\sqrt{3} + 1) \\
 &\quad - (9 - 12\sqrt{3} + 18 - 4\sqrt{3} + 1) \\
 &= 24\sqrt{3} + 8\sqrt{3} \\
 &= 32\sqrt{3}
 \end{aligned}$$

$$(ii) (2 + \sqrt{5})^5 + (2 - \sqrt{5})^5$$

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**Solution:**

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$$\begin{aligned}
 & (2 + \sqrt{5})^5 \\
 &= {}^5C_0(2)^5 (\sqrt{5})^0 + {}^5C_1(2)^4 (\sqrt{5})^1 + {}^5C_2(2)^3 (\sqrt{5})^2 \\
 &\quad + {}^5C_3(2)^2 (\sqrt{5})^3 + {}^5C_4(2)^1 (\sqrt{5})^4 + {}^5C_5(2)^0 (\sqrt{5})^5 \\
 &\text{Now, } {}^5C_0 = {}^5C_5 = 1, {}^5C_1 = {}^5C_4 = 5, \\
 &{}^5C_2 = {}^5C_3 = \frac{5 \times 4}{2 \times 1} = 10
 \end{aligned}$$

$$\begin{aligned}
 \therefore (2 + \sqrt{5})^5 &= 1(32)(1) + 5(16)(\sqrt{5}) + 10(8)(5) \\
 &\quad + 10(4)(5\sqrt{5}) + 5(2)(25) + 1(1)(25\sqrt{5}) \\
 \therefore (2 + \sqrt{5})^5 &= 32 + 80\sqrt{5} + 400 + 200\sqrt{5} + 250 + 25\sqrt{5} \\
 &\quad \dots(i)
 \end{aligned}$$

$$\begin{aligned}
 \text{Also, } (2 - \sqrt{5})^5 &= {}^5C_0(2)^5 (\sqrt{5})^0 - {}^5C_1(2)^4 (\sqrt{5})^1 \\
 &\quad + {}^5C_2(2)^3 (\sqrt{5})^2 - {}^5C_3(2)^2 (\sqrt{5})^3 + {}^5C_4(2)^1 (\sqrt{5})^4 \\
 &\quad - {}^5C_5(2)^0 (\sqrt{5})^5 \\
 &= 1(32)(1) - 5(16)(\sqrt{5}) + 10(8)(5) \\
 &\quad - 10(4)(5\sqrt{5}) + 5(2)(25) - 1(1)(25\sqrt{5}) \\
 \therefore (2 - \sqrt{5})^5 &= 32 - 80\sqrt{5} + 400 - 200\sqrt{5} + 250 - 25\sqrt{5} \\
 &\quad \dots(ii)
 \end{aligned}$$

Adding (i) and (ii), we get

$$\therefore (2 + \sqrt{5})^5 + (2 - \sqrt{5})^5 = (32 + 80\sqrt{5} + 400 + 200\sqrt{5} + 250 + 25\sqrt{5}) + (32 - 80\sqrt{5} + 400 - 200\sqrt{5} + 250 - 25\sqrt{5})$$

$$= 64 + 800 + 500$$

$$= 1364$$

**Question 4.**

**Prove that:**

$$(i) (\sqrt{3} + \sqrt{2})^6 + (\sqrt{3} - \sqrt{2})^6 = 970$$

**Solution:**

$$\begin{aligned} & (\sqrt{3} + \sqrt{2})^6 \\ &= {}^6C_0(\sqrt{3})^6(\sqrt{2})^0 + {}^6C_1(\sqrt{3})^5(\sqrt{2})^1 \\ &\quad + {}^6C_2(\sqrt{3})^4(\sqrt{2})^2 + {}^6C_3(\sqrt{3})^3(\sqrt{2})^3 \\ &\quad + {}^6C_4(\sqrt{3})^2(\sqrt{2})^4 + {}^6C_5(\sqrt{3})^1(\sqrt{2})^5 \\ &\quad + {}^6C_6(\sqrt{3})^0(\sqrt{2})^6 \end{aligned}$$

$$\begin{aligned} \text{Now, } {}^6C_0 &= {}^6C_6 = 1, {}^6C_1 = {}^6C_5 = 6, \\ {}^6C_2 &= {}^6C_4 = \frac{6 \times 5}{2 \times 1} = 15, {}^6C_3 = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20 \end{aligned}$$

$$\begin{aligned} \therefore (\sqrt{3} + \sqrt{2})^6 &= 1(27)(1) + 6(9\sqrt{3})(\sqrt{2}) + 15(9)(2) \\ &\quad + 20(3\sqrt{3})(2\sqrt{2}) + 15(3)(4) + 6(\sqrt{3})(4\sqrt{2}) \\ &\quad + 1(1)(8) \end{aligned}$$

$$\begin{aligned} \therefore (\sqrt{3} + \sqrt{2})^6 &= 27 + 54\sqrt{6} + 270 + 120\sqrt{6} + 180 + 24\sqrt{6} + 8 \\ &\quad \dots(i) \end{aligned}$$

$$\begin{aligned} \text{Also, } (\sqrt{3} - \sqrt{2})^6 &= {}^6C_0(\sqrt{3})^6(\sqrt{2})^0 - {}^6C_1(\sqrt{3})^5(\sqrt{2})^1 \\ &\quad + {}^6C_2(\sqrt{3})^4(\sqrt{2})^2 - {}^6C_3(\sqrt{3})^3(\sqrt{2})^3 \\ &\quad + {}^6C_4(\sqrt{3})^2(\sqrt{2})^4 - {}^6C_5(\sqrt{3})^1(\sqrt{2})^5 \\ &\quad + {}^6C_6(\sqrt{3})^0(\sqrt{2})^6 \\ &= 1(27)(1) - 6(9\sqrt{3})(\sqrt{2}) + 15(9)(2) \\ &\quad - 20(3\sqrt{3})(2\sqrt{2}) + 15(3)(4) - 6(\sqrt{3})(4\sqrt{2}) \\ &\quad + 1(1)(8) \\ &= (\sqrt{3} - \sqrt{2})^6 \\ &= 27 - 54\sqrt{6} + 270 - 120\sqrt{6} + 180 - 24\sqrt{6} + 8 \\ &\quad \dots(\text{ii}) \end{aligned}$$

Adding (i) and (ii), we get

$$(\sqrt{3} + \sqrt{2})^6 + (\sqrt{3} - \sqrt{2})^6$$

$$\begin{aligned} \text{Also, } (\sqrt{3} - \sqrt{2})^6 &= {}^6C_0(\sqrt{3})^6(\sqrt{2})^0 - {}^6C_1(\sqrt{3})^5(\sqrt{2})^1 \\ &\quad + {}^6C_2(\sqrt{3})^4(\sqrt{2})^2 - {}^6C_3(\sqrt{3})^3(\sqrt{2})^3 \\ &\quad + {}^6C_4(\sqrt{3})^2(\sqrt{2})^4 - {}^6C_5(\sqrt{3})^1(\sqrt{2})^5 \\ &\quad + {}^6C_6(\sqrt{3})^0(\sqrt{2})^6 \\ &= 1(27)(1) - 6(9\sqrt{3})(\sqrt{2}) + 15(9)(2) \\ &\quad - 20(3\sqrt{3})(2\sqrt{2}) + 15(3)(4) - 6(\sqrt{3})(4\sqrt{2}) \\ &\quad + 1(1)(8) \\ &= (\sqrt{3} - \sqrt{2})^6 \\ &= 27 - 54\sqrt{6} + 270 - 120\sqrt{6} + 180 - 24\sqrt{6} + 8 \\ &\quad \dots(ii) \end{aligned}$$

Adding (i) and (ii), we get

$$(\sqrt{3} + \sqrt{2})^6 + (\sqrt{3} - \sqrt{2})^6$$

$$(ii) (\sqrt{5} + 1)^5 - (\sqrt{5} - 1)^5 = 352$$

**Solution:**

$$\begin{aligned}
 & (\sqrt{5}+1)^5 \\
 &= {}^5C_0(\sqrt{5})^5(1)^0 + {}^5C_1(\sqrt{5})^4(1)^1 \\
 &\quad + {}^5C_2(\sqrt{5})^3(1)^2 + {}^5C_3(\sqrt{5})^2(1)^3 \\
 &\quad + {}^5C_4(\sqrt{5})^1(1)^4 + {}^5C_5(\sqrt{5})^0(1)^5
 \end{aligned}$$

$$\begin{aligned}
 \text{Now, } {}^5C_0 &= {}^5C_5 = 1, {}^5C_1 = {}^5C_4 = 5, \\
 {}^5C_2 &= {}^5C_3 = 10
 \end{aligned}$$

$$\begin{aligned}
 \therefore (\sqrt{5}+1)^5 &= 1(25\sqrt{5})(1) + 5(25)(1) + 10(5\sqrt{5})(1) \\
 &\quad + 10(5)(1) + 5\sqrt{5}(1) + 1(1)(1) \\
 \therefore (\sqrt{5}+1)^5 &= 25\sqrt{5} + 125 + 50\sqrt{5} + 50 + 5\sqrt{5} + 1 \\
 &\quad \dots(i)
 \end{aligned}$$

$$\begin{aligned}
 \text{Also, } (\sqrt{5}-1)^5 &= {}^5C_0(\sqrt{5})^5(1)^0 - {}^5C_1(\sqrt{5})^4(1)^1 + {}^5C_2(\sqrt{5})^3(1)^2 \\
 &\quad - {}^5C_3(\sqrt{5})^2(1)^3 + {}^5C_4(\sqrt{5})^1(1)^4 - {}^5C_5(\sqrt{5})^0(1)^5 \\
 &= 1(25\sqrt{5})(1) - 5(25)(1) + 10(5\sqrt{5})(1) \\
 &\quad - 10(5)(1) + 5\sqrt{5}(1) - 1(1)(1) \\
 (\sqrt{5}-1)^5 &= 25\sqrt{5} - 125 + 50\sqrt{5} - 50 + 5\sqrt{5} - 1 \\
 &\quad \dots(ii)
 \end{aligned}$$

Subtracting (ii) from (i), we get

$$\begin{aligned}
 & (\sqrt{5}+1)^5 - (\sqrt{5}-1)^5 \\
 &= (25\sqrt{5} + 125 + 50\sqrt{5} + 50 + 5\sqrt{5} + 1) \\
 &\quad - (25\sqrt{5} - 125 + 50\sqrt{5} - 50 + 5\sqrt{5} - 1) \\
 &= 250 + 100 + 2 \\
 &= 352
 \end{aligned}$$

**Question 5.**

Using binomial theorem, find the value of

(i)  $(102)^4$

**Solution:**

$$\begin{aligned}(102)^4 &= (100 + 2)^4 \\&= {}^4C_0(100)^4(2)^0 + {}^4C_1(100)^3(2)^1 \\&\quad + {}^4C_2(100)^2(2)^2 + {}^4C_3(100)^1(2)^3 + {}^4C_4(100)^0(2)^4 \\ \text{Now, } {}^4C_0 &= {}^4C_4 = 1, {}^4C_1 = {}^4C_3 = 4, {}^4C_2 = 6 \\ \therefore (102)^4 &= 1(100000000)(1) + 4(1000000)(2) \\&\quad + 6(10000)(4) + 4(100)(8) + 1(1)(16) \\&= 100000000 + 8000000 + 240000 + 3200 + 16 \\&= 108243216\end{aligned}$$

(ii)  $(1.1)^5$

**Solution:**

$$\begin{aligned}(1.1)^5 &= (1 + 0.1)^5 \\&= {}^5C_0(1)^5(0.1)^0 + {}^5C_1(1)^4(0.1)^1 + {}^5C_2(1)^3(0.1)^2 \\&\quad + {}^5C_3(1)^2(0.1)^3 + {}^5C_4(1)^1(0.1)^4 + {}^5C_5(1)^0(0.1)^5 \\ \text{Now, } {}^5C_0 &= {}^5C_5 = 1, {}^5C_1 = {}^5C_4 = 5, \\&\quad {}^5C_2 = {}^5C_3 = 10 \\ \therefore (1.1)^5 &= 1(1)(1) + 5(1)(0.1) + 10(1)(0.01) \\&\quad + 10(1)(0.001) + 5(1)(0.0001) + 1(1)(0.00001) \\&= 1 + 0.5 + 0.1 + 0.01 + 0.0005 + 0.00001 \\&= 1.61051\end{aligned}$$

**Question 6.**

Using binomial theorem, find the value of

(i)  $(9.9)^3$

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**Solution:**

$$\begin{aligned}(9.9)^3 &= (10 - 0.1)^3 \\&= {}^3C_0(10)^3(0.1)^0 - {}^3C_1(10)^2(0.1)^1 \\&\quad + {}^3C_2(10)^1(0.1)^2 - {}^3C_3(10)^0(0.1)^3 \\ \text{Now, } {}^3C_0 &= {}^3C_3 = 1, {}^3C_1 = {}^3C_2 = 3 \\ \therefore (9.9)^3 &= 1(1000)(1) - 3(100)(0.1) + 3(10)(0.01) \\&\quad - 1(1)(0.001) \\&= 1000 - 30 + 0.3 - 0.001 \\&= 1000.3 - 30.001 \\&= 970.299\end{aligned}$$

**(ii)  $(0.9)^4$**

**Solution:**

$$\begin{aligned}(0.9)^4 &= (1 - 0.1)^4 \\&= {}^4C_0(1)^4(0.1)^0 - {}^4C_1(1)^3(0.1)^1 + {}^4C_2(1)^2(0.1)^2 \\&\quad - {}^4C_3(1)^1(0.1)^3 + {}^4C_4(1)^0(0.1)^4 \\ \text{Now, } {}^4C_0 &= {}^4C_4 = 1, {}^4C_1 = {}^4C_3 = 4, {}^4C_2 = 6 \\ \therefore (0.9)^4 &= 1(1)(1) - 4(1)(0.1) + 6(1)(0.01) \\&\quad - 4(1)(0.001) + 1(1)(0.0001) \\&= 1 - 0.4 + 0.06 - 0.004 + 0.0001 \\&= 1.0601 - 0.404 \\&= 0.6561\end{aligned}$$

**Question 7.**

**Without expanding, find the value of**

$$(i) (x + 1)^4 - 4(x + 1)^3(x - 1) + 6(x + 1)^2(x - 1)^2 - 4(x + 1)(x - 1)^3 + (x - 1)^4$$

**Solution:**

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Let  $x + 1 = a$  and  $x - 1 = b$

We notice that 1, 4, 6, 4, 1 are the values of  ${}^4C_0$ ,  ${}^4C_1$ ,  ${}^4C_2$ ,  ${}^4C_3$ ,  ${}^4C_4$  respectively.

∴ The given expression becomes

$$\begin{aligned} & {}^4C_0 a^4 b^0 - {}^4C_1 a^3 b + {}^4C_2 a^2 b^2 - {}^4C_3 a b^3 + {}^4C_4 a^0 b^4 \\ &= (a - b)^4 \\ &= [x + 1 - (x - 1)]^4 = (x + 1 - x + 1)^4 \\ &= 2^4 \\ &= 16 \end{aligned}$$

$$(ii) (2x - 1)^4 + 4(2x - 1)^3 (3 - 2x) + 6(2x - 1)^2 (3 - 2x)^2 + 4(2x - 1) (3 - 2x)^3 + (3 - 2x)^4$$

**Solution:**

Let  $2x - 1 = a$  and  $3 - 2x = b$

We notice that 1, 4, 6, 4, 1 are the values of  ${}^4C_0$ ,  ${}^4C_1$ ,  ${}^4C_2$ ,  ${}^4C_3$ ,  ${}^4C_4$  respectively.

∴ The given expression becomes

$$\begin{aligned} & {}^4C_0 a^4 b^0 + {}^4C_1 a^3 b + {}^4C_2 a^2 b^2 + {}^4C_3 a b^3 + {}^4C_4 a^0 b^4 \\ &= (a + b)^4 \\ &= (2x - 1 + 3 - 2x)^4 = 2^4 \\ &= 16 \end{aligned}$$

**Question 8.**

Find the value of  $(1.02)^6$ , correct upto four places of decimals.

**Solution:**

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$$\begin{aligned}(1.02)^6 &= (1 + 0.02)^6 \\ &= {}^6C_0(1)^6(0.02)^0 + {}^6C_1(1)^5(0.02)^1 \\ &\quad + {}^6C_2(1)^4(0.02)^2 + {}^6C_3(1)^3(0.02)^3 \\ &\quad + {}^6C_4(1)^2(0.02)^4 + {}^6C_5(1)^1(0.02)^5 \\ &\quad + {}^6C_6(1)^0(0.02)^6\end{aligned}$$

$$\text{Now, } {}^6C_0 = {}^6C_6 = 1, {}^6C_1 = {}^6C_5 = 6,$$

$${}^6C_2 = {}^6C_4 = \frac{6 \times 5}{2 \times 1} = 15, {}^6C_3 = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20$$

$$\begin{aligned}\therefore (1.02)^6 &= 1(1)(1) + 6(1)(0.02) + 15(1)(0.0004) \\ &\quad + 20(1)(0.000008) + \dots \\ &= 1 + 0.12 + 0.0060 + 0.000160 + \dots \\ &= 1.12616 \\ &= 1.1262, \text{ correct upto 4 decimal places.}\end{aligned}$$

#### Question 9.

Find the value of  $(1.01)^5$ , correct upto three places of decimals.

**Solution:**

$$\begin{aligned}(1.01)^5 &= (1 + 0.01)^5 \\ &= {}^5C_0(1)^5(0.01)^0 + {}^5C_1(1)^4(0.01)^1 \\ &\quad + {}^5C_2(1)^3(0.01)^2 + {}^5C_3(1)^2(0.01)^3 \\ &\quad + {}^5C_4(1)^1(0.01)^4 + {}^5C_5(1)^0(0.01)^5\end{aligned}$$

$$\text{Now, } {}^5C_0 = {}^5C_5 = 1, {}^5C_1 = {}^5C_4 = 5,$$

$${}^5C_2 = {}^5C_3 = \frac{5 \times 4}{2 \times 1} = 10$$

$$\begin{aligned}\therefore (1.01)^5 &= 1(1)(1) + 5(1)(0.01) + 10(1)(0.0001) \\ &\quad + 10(1)(0.000001) + \dots \\ &= 1 + 0.05 + 0.001 + 0.00001 + \dots \\ &= 1.05101 \\ &= 1.051, \text{ correct upto 3 decimal places.}\end{aligned}$$

#### Question 10.

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Find the value of  $(0.9)^6$ , correct upto four places of decimals.

**Solution:**

$$\begin{aligned}(0.9)^6 &= (1 - 0.1)^6 \\ &= {}^6C_0(1)^6(0.1)^0 - {}^6C_1(1)^5(0.1)^1 + {}^6C_2(1)^4(0.1)^2 \\ &\quad - {}^6C_3(1)^3(0.1)^3 + {}^6C_4(1)^2(0.1)^4 \\ &\quad - {}^6C_5(1)^1(0.1)^5 + {}^6C_6(1)^0(0.1)^6\end{aligned}$$

$$\text{Now, } {}^6C_0 = {}^6C_6 = 1, {}^6C_1 = {}^6C_5 = 6,$$

$${}^6C_2 = {}^6C_4 = \frac{6 \times 5}{2 \times 1} = 15, {}^6C_3 = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20$$

$$\begin{aligned}\therefore (0.9)^6 &= 1(1)(1) - 6(1)(0.1) + 15(1)(0.01) \\ &\quad - 20(1)(0.001) + 15(1)(0.0001) - 6(1)(0.00001) \\ &\quad + 1(1)(0.000001) \\ &= 1 - 0.6 + 0.15 - 0.02 + 0.0015 \\ &\quad - 0.00006 + 0.000001 \\ &= 0.531441 \\ &= 0.5314, \text{ correct upto 4 decimal places.}\end{aligned}$$

**Ex 4.3**

## Question 1.

In the following expansions, find the indicated term.

(i)  $\left(2x^2 + \frac{3}{2x}\right)^8$ , 3rd term

Solution:

Here,  $a = 2x^2$ ,  $b = \frac{3}{2x}$ ,  $n = 8$ .

For 3<sup>rd</sup> term,  $r = 2$

We have  $t_{r+1} = {}^nC_r a^{n-r} b^r$

$$\begin{aligned}\therefore t_3 &= {}^8C_2 (2x^2)^{8-2} \left(\frac{3}{2x}\right)^2 = \frac{8!}{2!6!} \times (2x^2)^6 \times \left(\frac{3}{2x}\right)^2 \\ &= \frac{8 \times 7}{2 \times 1} \times 64x^{12} \times \frac{9}{4x^2} \\ &= 28 \times 64x^{12} \times \frac{9}{4x^2} \\ &= 4032 x^{10}\end{aligned}$$

$\therefore$  3<sup>rd</sup> term in the expansion of  $\left(2x^2 + \frac{3}{2x}\right)^8$  is  $4032 x^{10}$ .

(ii)  $\left(x^2 - \frac{4}{x^3}\right)^{11}$ , 5th term

Solution:

Here,  $a = x^2$ ,  $b = \frac{-4}{x^3}$ ,  $n = 11$ .

For 5<sup>th</sup> term,  $r = 4$

We have  $t_{r+1} = {}^nC_r a^{n-r} b^r$

$$\begin{aligned}\therefore t_5 &= {}^{11}C_4 (x^2)^{11-4} \left(\frac{-4}{x^3}\right)^4 = \frac{11!}{4!7!} \times (x^2)^7 \times \left(\frac{-4}{x^3}\right)^4 \\ &= \frac{11 \times 10 \times 9 \times 8}{4 \times 3 \times 2 \times 1} \times x^{14} \times \frac{256}{x^{12}} \\ &= 330 \times 256 \times x^2 \\ &= 84480 x^2\end{aligned}$$

$\therefore$  5<sup>th</sup> term in the expansion of  $\left(x^2 - \frac{4}{x^3}\right)^{11}$  is  $84480 x^2$ .

(iii)  $\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$ , 7th term

Solution:

Here,  $a = \frac{4x}{5}$ ,  $b = \frac{-5}{2x}$ ,  $n = 9$ .

For 7<sup>th</sup> term,  $r = 6$

We have  $t_{r+1} = {}^nC_r a^{n-r} b^r$

$$\begin{aligned}\therefore t_7 &= {}^9C_6 \left(\frac{4x}{5}\right)^{9-6} \left(\frac{-5}{2x}\right)^6 = \frac{9!}{6!3!} \times \left(\frac{4x}{5}\right)^3 \times \left(\frac{-5}{2x}\right)^6 \\ &= \frac{9 \times 8 \times 7}{3 \times 2 \times 1} \times \frac{4^3 \cdot x^3}{5^3} \times \frac{5^6}{2^6 \cdot x^6} \\ &= 84 \times 64 \times \frac{5^3}{64 \cdot x^3}\end{aligned}$$

$$= \frac{84 \times 125}{x^3}$$

$$= \frac{10500}{x^3}$$

$\therefore$  7<sup>th</sup> term in the expansion of  $\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$  is  $\frac{10500}{x^3}$ .

(iv) In  $\left(\frac{1}{3} + a^2\right)^{12}$ , 9th term

Solution:

Here,  $a = \frac{1}{3}$ ,  $b = a^2$ ,  $n = 12$ .

For 9<sup>th</sup> term,  $r = 8$

We have  $t_{r+1} = {}^nC_r a^{n-r} b^r$

$$\therefore t_9 = {}^{12}C_8 \left(\frac{1}{3}\right)^{12-8} (a^2)^8 = \frac{12!}{8!4!} \times \left(\frac{1}{3}\right)^4 \times a^{16}$$

$$= \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1} \times \frac{1}{81} \times a^{16}$$

$$= \frac{55}{9} a^{16}$$

$\therefore$  9<sup>th</sup> term in the expansion of  $\left(\frac{1}{3} + a^2\right)^{12}$  is  $\frac{55}{9} a^{16}$ .

(v) In  $\left(3a + \frac{4}{a}\right)^{13}$ , 10th term

Solution:

Here,  $a = 3a$ ,  $b = \frac{4}{a}$ ,  $n = 13$ .

For 10<sup>th</sup> term,  $r = 9$

We have  $t_{r+1} = {}^nC_r a^{n-r} b^r$

$$\begin{aligned}\therefore t_{10} &= {}^{13}C_9 (3a)^{13-9} \left(\frac{4}{a}\right)^9 = \frac{13!}{9!4!} \times (3a)^4 \times \left(\frac{4}{a}\right)^9 \\ &= \frac{13 \times 12 \times 11 \times 10}{4 \times 3 \times 2 \times 1} \times 81a^4 \times \frac{4^9}{a^9} \\ &= \frac{143 \times 5 \times 81 \times 256 \times 256 \times 4}{a^5} \\ &= \frac{15182069760}{a^5} \\ \therefore 10^{\text{th}} \text{ term in the expansion of } \left(3a + \frac{4}{a}\right)^{13} \text{ is } \\ &\frac{15182069760}{a^5}.\end{aligned}$$

Question 2.

In the following expansions, find the indicated coefficients.

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(i)  $x^3$  in  $\left(x^2 + \frac{3\sqrt{2}}{x}\right)^9$

Solution:

Here,  $a = x^2$ ,  $b = \frac{3\sqrt{2}}{x}$ ,  $n = 9$ .

We have  $t_{r+1} = {}^nC_r a^{n-r} b^r$

$$\begin{aligned} &= {}^9C_r (x^2)^{9-r} \left(\frac{3\sqrt{2}}{x}\right)^r \\ &= {}^9C_r x^{18-2r} (3\sqrt{2})^r \cdot x^{-r} \\ &= {}^9C_r (3\sqrt{2})^r x^{18-3r} \end{aligned}$$

To get the coefficient of  $x^3$ , we must have  
 $x^{18-3r} = x^3$

$$\therefore 18 - 3r = 3$$

$$\therefore 15 = 3r$$

$$\therefore r = 5$$

$$\begin{aligned} \therefore \text{Coefficient of } x^3 &= {}^9C_5 (3\sqrt{2})^5 \\ &= \frac{9!}{5!4!} (3\sqrt{2})^5 \\ &= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \times 243 \times 4\sqrt{2} \\ &= 122472\sqrt{2} \end{aligned}$$

(ii)  $x^8$  in  $\left(2x^5 - \frac{5}{x^3}\right)^8$

Solution:

Here,  $a = 2x^5$ ,  $b = \frac{-5}{x^3}$ ,  $n = 8$ .

$$\begin{aligned}\text{We have } t_{r+1} &= {}^nC_r a^{n-r} b^r \\ &= {}^8C_r (2x^5)^{8-r} \left(\frac{-5}{x^3}\right)^r \\ &= {}^8C_r (2)^{8-r} x^{40-5r} (-5)^r x^{-3r} \\ &= {}^8C_r (2)^{8-r} (-5)^r x^{40-8r}\end{aligned}$$

To get the coefficient of  $x^8$ , we must have

$$x^{40-8r} = x^8$$

$$\therefore 40 - 8r = 8$$

$$\therefore 8r = 32$$

$$\therefore r = 4$$

$$\begin{aligned}\therefore \text{Coefficient of } x^8 &= {}^8C_4 (2)^4 (-5)^4 \\ &= \frac{8!}{4!4!} (2)^4 (-5)^4 \\ &= \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} \times 16 \times 625 \\ &= 700000\end{aligned}$$

(iii)  $x^9$  in  $\left(\frac{1}{x} + x^2\right)^{18}$

Solution:

Here,  $a = \frac{1}{x}$ ,  $b = x^2$ ,  $n = 18$ .

$$\begin{aligned}\text{We have } t_{r+1} &= {}^nC_r a^{n-r} b^r \\ &= {}^{18}C_r \left(\frac{1}{x}\right)^{18-r} (x^2)^r \\ &= {}^{18}C_r (x^{-1})^{18-r} x^{2r} \\ &= {}^{18}C_r x^{r-18} \cdot x^{2r} \\ &= {}^{18}C_r x^{3r-18}\end{aligned}$$

To get the coefficient of  $x^9$ , we must have  
 $x^{3r-18} = x^9$

$$\therefore 3r - 18 = 9$$

$$\therefore 3r = 27$$

$$\therefore r = 9$$

$$\therefore \text{Coefficient of } x^9$$

$$= {}^{18}C_9$$

$$= \frac{18!}{9!9!}$$

$$= \frac{18 \times 17 \times 16 \times 15 \times 14 \times 13 \times 12 \times 11 \times 10}{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$= 48620$$

$$\text{(iv) } x^{-3} \text{ in } \left(x - \frac{1}{2x}\right)^5$$

Solution:

$$\text{Here, } a = x, b = \frac{-1}{2x}, n = 5.$$

$$\text{We have } t_{r+1} = {}^nC_r a^{n-r} b^r$$

$$\begin{aligned}
 &= {}^5C_r (x)^{5-r} \left(\frac{-1}{2x}\right)^r \\
 &= {}^5C_r x^{5-r} \left(\frac{-1}{2}\right)^r x^{-r} \\
 &= {}^5C_r \left(\frac{-1}{2}\right)^r x^{5-2r}
 \end{aligned}$$

To get the coefficient of  $x^{-3}$ , we must have  
 $x^{5-2r} = x^{-3}$

$$\therefore 5 - 2r = -3$$

$$\therefore 2r = 8$$

$$\therefore r = 4$$

$$\begin{aligned}
 \therefore \text{Coefficient of } x^{-3} &= {}^5C_4 \left(\frac{-1}{2}\right)^4 \\
 &= \frac{5!}{4!!} \times \frac{1}{16} = 5 \times \frac{1}{16} \\
 &= \frac{5}{16}
 \end{aligned}$$

$$(v) x^{-20} \text{ in } \left(x^3 - \frac{1}{2x^2}\right)^{15}$$

Solution:

$$\text{Here, } a = x^3, b = \frac{-1}{2x^2}, n = 15.$$

$$\text{We have } t_{r+1} = {}^nC_r a^{n-r} b^r$$

$$\begin{aligned} &= {}^{15}C_r (x^3)^{15-r} \left(\frac{-1}{2x^2}\right)^r \\ &= {}^{15}C_r x^{45-3r} \left(\frac{-1}{2}\right)^r x^{-2r} \\ &= {}^{15}C_r \left(\frac{-1}{2}\right)^r x^{45-5r} \end{aligned}$$

To get the coefficient of  $x^{-20}$ , we must have  
 $x^{45-5r} = x^{-20}$

$$\therefore 45 - 5r = -20$$

$$\therefore 5r = 65$$

$$\therefore r = 13$$

$$\begin{aligned} \therefore \text{Coefficient of } x^{-20} &= {}^{15}C_{13} \left(\frac{-1}{2}\right)^{13} \\ &= \frac{15!}{13! 2!} \left(\frac{-1}{2}\right)^{13} \\ &= \frac{15 \times 14}{2 \times 1} \times \left(\frac{-1}{8192}\right) \\ &= \frac{-105}{8192} \end{aligned}$$

Question 3.

Find the constant term (term independent of x) in the expansion of

(i)  $\left(2x + \frac{1}{3x^2}\right)^9$

Solution:

Here,  $a = 2x$ ,  $b = \frac{1}{3x^2}$ ,  $n = 9$ .

$$\begin{aligned}\text{We have } t_{r+1} &= {}^nC_r a^{n-r} b^r \\ &= {}^9C_r (2x)^{9-r} \left(\frac{1}{3x^2}\right)^r \\ &= {}^9C_r (2)^{9-r} x^{9-r} \left(\frac{1}{3}\right)^r x^{-2r} \\ &= {}^9C_r (2)^{9-r} \left(\frac{1}{3}\right)^r x^{9-3r}\end{aligned}$$

To get the term independent of  $x$ , we must have

$$x^{9-3r} = x^0$$

$$\therefore 9 - 3r = 0$$

$$\therefore r = 3$$

$$\begin{aligned}\therefore \text{The term independent of } x &= {}^9C_3 (2)^6 \left(\frac{1}{3}\right)^3 \\ &= \frac{9!}{3!6!} (2)^6 \left(\frac{1}{3}\right)^3 \\ &= \frac{9 \times 8 \times 7}{3 \times 2 \times 1} \times \frac{64}{27} \\ &= \frac{1792}{9}\end{aligned}$$



(ii)  $\left(x - \frac{2}{x^2}\right)^{15}$

Solution:

Here,  $a = x$ ,  $b = \frac{-2}{x^2}$ ,  $n = 15$ .

$$\begin{aligned}\text{We have } t_{r+1} &= {}^nC_r a^{n-r} b^r \\ &= {}^{15}C_r (x)^{15-r} \left(\frac{-2}{x^2}\right)^r \\ &= {}^{15}C_r x^{15-r} (-2)^r x^{-2r} \\ &= {}^{15}C_r (-2)^r x^{15-3r}\end{aligned}$$

To get the term independent of  $x$ , we must have

$$x^{15-3r} = x^0$$

$$\therefore 15 - 3r = 0$$

$$\therefore r = 5$$

$\therefore$  The term independent of  $x$

$$= {}^{15}C_5 (-2)^5$$

$$= \frac{15!}{5!10!} (-2)^5$$

$$= \frac{15 \times 14 \times 13 \times 12 \times 11}{5 \times 4 \times 3 \times 2 \times 1} \times (-32)$$

$$= -96096$$

(iii)  $\left(\sqrt{x} - \frac{3}{x^2}\right)^{10}$

Solution:

(iii)  $\left(\sqrt{x} - \frac{3}{x^2}\right)^{10}$

Solution:

Here,  $a = \sqrt{x}$ ,  $b = \frac{-3}{x^2}$ ,  $n = 10$ .

$$\begin{aligned}\text{We have } t_{r+1} &= {}^nC_r a^{n-r} b^r \\ &= {}^{10}C_r (\sqrt{x})^{10-r} \left(\frac{-3}{x^2}\right)^r \\ &= {}^{10}C_r (x)^{\frac{10-r}{2}} (-3)^r x^{-2r} \\ &= {}^{10}C_r (-3)^r x^{\frac{10-r}{2}} x^{-2r} \\ &= {}^{10}C_r (-3)^r x^{\frac{10-5r}{2}}\end{aligned}$$

To get the term independent of  $x$ , we must have

$$x^{\frac{10-5r}{2}} = x^0$$

$$\therefore \frac{10-5r}{2} = 0$$

$$\therefore 10 - 5r = 0$$

$$\therefore r = 2$$

$$\begin{aligned}\therefore \text{The term independent of } x &= {}^{10}C_2 (-3)^2 \\ &= \frac{10!}{2!8!} (-3)^2 \\ &= \frac{10 \times 9}{2 \times 1} \times 9 \\ &= 405\end{aligned}$$

(iv)  $\left(x^2 - \frac{1}{x}\right)^9$

Solution:

Here,  $a = x^2$ ,  $b = \frac{-1}{x}$ ,  $n = 9$ .

We have  $t_{r+1} = {}^nC_r a^{n-r} b^r$

$$\begin{aligned} &= {}^9C_r (x^2)^{9-r} \left(\frac{-1}{x}\right)^r \\ &= {}^9C_r x^{18-2r} (-1)^r x^{-r} \\ &= {}^9C_r (-1)^r x^{18-3r} \end{aligned}$$

To get the term independent of  $x$ , we must have  $x^{18-3r} = x^0$

$\therefore 18 - 3r = 0$

$\therefore r = 6$

$\therefore$  The term independent of  $x = {}^9C_6 (-1)^6$

$$\begin{aligned} &= \frac{9!}{6!3!} \times 1 \\ &= \frac{9 \times 8 \times 7}{3 \times 2 \times 1} \\ &= 84 \end{aligned}$$

(v)  $\left(2x^2 - \frac{5}{x}\right)^9$

Solution:

Here,  $a = 2x^2$ ,  $b = \frac{-5}{x}$ ,  $n = 9$ .

We have  $t_{r+1} = {}^nC_r a^{n-r} b^r$

$$\begin{aligned} &= {}^9C_r (2x^2)^{9-r} \left(\frac{-5}{x}\right)^r \\ &= {}^9C_r 2^{9-r} x^{18-2r} (-5)^r x^{-r} \\ &= {}^9C_r 2^{9-r} (-5)^r x^{18-3r} \end{aligned}$$

To get the term independent of  $x$ , we must have

$$x^{18-3r} = x^0$$

$$\therefore 18 - 3r = 0$$

$$\therefore r = 6$$

$\therefore$  The term independent of  $x$

$$= {}^9C_6 2^3 (-5)^6$$

$$= \frac{9!}{6!3!} (2)^3 (-5)^6$$

$$= \frac{9 \times 8 \times 7}{3 \times 2 \times 1} \times 8 \times 15625$$

$$= 10500000$$

Question 4.

Find the middle terms in the expansion of

$$(i) \left(\frac{x}{y} + \frac{y}{x}\right)^{12}$$

Solution:

$$\text{Here, } a = \frac{x}{y}, b = \frac{y}{x}, n = 12.$$

Now,  $n$  is even.

$$\therefore \frac{n+2}{2} = \frac{12+2}{2} = 7$$

$\therefore$  Middle term is  $t_7$ , for which  $r = 6$ .

We have  $t_{r+1} = {}^nC_r a^{n-r} b^r$

$$\begin{aligned}\therefore t_7 &= {}^{12}C_6 \left(\frac{x}{y}\right)^6 \cdot \left(\frac{y}{x}\right)^6 = \frac{12!}{6!6!} \\ &= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = 924\end{aligned}$$

$\therefore$  Middle term is 924.

(ii)  $\left(x^2 + \frac{1}{x}\right)^7$

Solution:

Here,  $a = x^2$ ,  $b = \frac{1}{x}$ ,  $n = 7$ .

Now,  $n$  is odd.

$$\therefore \frac{n+1}{2} = \frac{7+1}{2} = 4, \quad \frac{n+3}{2} = \frac{7+3}{2} = 5$$

$\therefore$  Middle terms are  $t_4$  and  $t_5$ , for which  $r = 3$  and  $r = 4$  respectively.

We have  $t_{r+1} = {}^nC_r a^{n-r} b^r$

$$\therefore t_4 = {}^7C_3 (x^2)^4 \left(\frac{1}{x}\right)^3 = \frac{7!}{3!4!} (x^8) \times \frac{1}{x^3}$$

$$\begin{aligned} &= \frac{7 \times 6 \times 5}{3 \times 2 \times 1} (x^8) \times \frac{1}{x^3} \\ &= 35x^5 \end{aligned}$$

$$\begin{aligned} \text{and } t_5 &= {}^7C_4 (x^2)^3 \left(\frac{1}{x}\right)^4 = \frac{7!}{4!3!} (x^6) \times \frac{1}{x^4} \\ &= \frac{7 \times 6 \times 5}{3 \times 2 \times 1} (x^6) \times \frac{1}{x^4} \\ &= 35x^2 \end{aligned}$$

$\therefore$  Middle terms are  $35x^5$  and  $35x^2$ .

(iii)  $\left(x^2 - \frac{2}{x}\right)^8$

Solution:

Here,  $a = x^2$ ,  $b = \frac{-2}{x}$ ,  $n = 8$ .

Now,  $n$  is even.

$$\therefore \frac{n+2}{2} = \frac{8+2}{2} = 5$$

$\therefore$  Middle term is  $t_5$ , for which  $r = 4$

We have  $t_{r+1} = {}^nC_r a^{n-r} b^r$

$$\begin{aligned} \therefore t_5 &= {}^8C_4 (x^2)^4 \left(\frac{-2}{x}\right)^4 \\ &= \frac{8!}{4!4!} (x^8) \left(\frac{16}{x^4}\right) \end{aligned}$$

$$\begin{aligned} &= \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} \times \frac{x^8}{x^4} \times 16 \\ &= 1120x^4 \end{aligned}$$

$\therefore$  Middle term is  $1120x^4$ .

(iv)  $\left(\frac{x}{a} - \frac{a}{x}\right)^{10}$

Solution:

Here,  $a = \frac{x}{a}$ ,  $b = \frac{-a}{x}$ ,  $n = 10$ .

Now,  $n$  is even.

$$\therefore \frac{n+2}{2} = \frac{10+2}{2} = 6$$

$\therefore$  Middle term is  $t_6$ , for which  $r = 5$ .

We have  $t_{r+1} = {}^nC_r a^{n-r} b^r$

$$\begin{aligned} \therefore t_6 &= {}^{10}C_5 \cdot \left(\frac{x}{a}\right)^5 \cdot \left(\frac{-a}{x}\right)^5 \\ &= {}^{10}C_5 \times -1 \\ &= \frac{-10!}{5!5!} \\ &= -\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} \\ &= -252 \end{aligned}$$

$\therefore$  Middle term is  $-252$ .

$$(v) \left( x^4 - \frac{1}{x^3} \right)^{11}$$

Solution:

$$\text{Here, } a = x^4, b = \frac{-1}{x^3}, n = 11.$$

Now,  $n$  is odd.

$$\therefore \frac{n+1}{2} = \frac{11+1}{2} = 6, \frac{n+3}{2} = \frac{11+3}{2} = 7$$

$\therefore$  Middle terms are  $t_6$  and  $t_7$ , for which  $r = 5$  and  $r = 6$  respectively.

$$\text{We have } t_{r+1} = {}^nC_r a^{n-r} b^r$$

$$\begin{aligned} \therefore t_6 &= {}^{11}C_5 (x^4)^6 \left( \frac{-1}{x^3} \right)^5 \\ &= \frac{11!}{5!6!} (x^{24}) \times \left( \frac{-1}{x^{15}} \right) \\ &= \frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2 \times 1} (-x^9) \\ &= -462x^9 \end{aligned}$$

$$\begin{aligned} \text{and } t_7 &= {}^{11}C_6 (x^4)^5 \left( \frac{-1}{x^3} \right)^6 \\ &= \frac{11!}{6!5!} (x^{20}) \times \left( \frac{1}{x^{18}} \right) \\ &= \frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2 \times 1} (x^2) \times \frac{1}{x^{18}} \\ &= 462x^2 \end{aligned}$$

$\therefore$  Middle terms are  $-462x^9$  and  $462x^2$ .



## Question 5.

In the expansion of  $(k + x)^8$ , the coefficient of  $x^5$  is 10 times the coefficient of  $x^6$ . Find the value of  $k$ .

Solution:

$$\begin{aligned}\text{Coefficient of } x^5 \text{ in } (k + x)^8 &= {}^8C_5 k^{8-5} \\ &= {}^8C_5 k^3\end{aligned}$$

$$\text{Coefficient of } x^6 \text{ in } (k + x)^8 = {}^8C_6 k^2$$

The coefficient of  $x^5$  is 10 times the coefficient of  $x^6$ .

$$\therefore {}^8C_5 k^3 = 10 ({}^8C_6 k^2)$$

$$\therefore \frac{8 \times 7 \times 6}{3 \times 2 \times 1} k = 10 \left( \frac{8 \times 7}{2 \times 1} \right)$$

$$\therefore k = 5$$

## Question 6.

Find the term containing  $x^6$  in the expansion of  $(2 - x)(3x + 1)^9$ .

Solution:

$$(2 - x)(3x + 1)^9 = 2(3x + 1)^9 - x(3x + 1)^9$$

Consider  $(3x + 1)^9$

Here,  $a = 3x$ ,  $b = 1$ ,  $n = 9$

$$\begin{aligned}\text{We have } t_{r+1} &= {}^nC_r a^{n-r} \cdot b^r \\ &= {}^9C_r (3x)^{9-r} \cdot (1)^r \\ &= {}^9C_r 3^{9-r} x^{9-r}\end{aligned}$$

To get the coefficient of  $x^6$  in  $2(3x + 1)^9$ , we must have

$$\begin{aligned}x^{9-r} &= x^6 \\ \therefore 9-r &= 6 \\ \therefore r &= 3 \quad \dots(i) \\ \text{Also, to get the coefficient of } x^6 \text{ in } x(3x+1)^9, \\ \text{we must have} \\ x \cdot x^{9-r} &= x^6 \\ \therefore x^{9-r} &= x^5 \\ \therefore 9-r &= 5 \\ \therefore r &= 4 \quad \dots(ii) \\ \therefore \text{The term containing } x^6 \text{ in the expansion of} \\ 2(3x+1)^9 - x(3x+1)^9 \\ &= 2 {}^9C_r 3^{9-r} - {}^9C_r 3^{9-r} \\ &= 2 {}^9C_3 3^{9-3} - {}^9C_4 3^{9-4} \quad \dots[\text{From (i) and (ii)}] \\ &= 2 \times 84 \times (3^6) - 126 \times 3^5 \\ &= 2 \times 3^5 (3 \times 84 - 63) \\ &= 2 \times 243 (252 - 63) \\ &= 486 \times 189 = 91854\end{aligned}$$

Question 7.

The coefficient of  $x^2$  in the expansion of  $(1+2x)^m$  is 112. Find  $m$ .

Solution:

The coefficient of  $x^2$  in  $(1+2x)^m = {}^mC_2 (2^2)$

Given that the coefficient of  $x^2 = 112$

$$\therefore {}^mC_2 (4) = 112$$

$$\therefore {}^mC_2 = 28$$

$$\therefore \frac{m!}{2!(m-2)!} = 28$$

$$\therefore \frac{m(m-1)(m-2)!}{2 \times (m-2)!} = 28$$

$$\therefore m(m-1) = 56$$

$$\therefore m^2 - m - 56 = 0$$

$$\therefore (m-8)(m+7) = 0$$

As  $m$  cannot be negative.

$$\therefore m = 8$$

#### Ex 4.4

##### Question 1.

State, by writing the first four terms, the expansion of the following, where  $|x| < 1$ .

(i)  $(1+x)^{-4}$

Solution:

$$\begin{aligned} (1+x)^{-4} &= 1 + (-4)x + \frac{(-4)(-4-1)}{2!}x^2 \\ &\quad + \frac{(-4)(-4-1)(-4-2)}{3!}x^3 + \dots \\ &= 1 + (-4)x + \frac{(-4)(-5)}{2}x^2 + \frac{(-4)(-5)(-6)}{6}x^3 + \dots \\ &= 1 - 4x + 10x^2 - 20x^3 + \dots \end{aligned}$$

(ii)  $(1-x)^{1/3}$

Solution:

$$\begin{aligned}
 (1-x)^{\frac{1}{3}} &= 1 - \frac{1}{3}x + \frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)}{2!}x^2 \\
 &\quad - \frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3!}x^3 + \dots \\
 &= 1 - \frac{x}{3} + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2}x^2 \\
 &\quad - \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{6}x^3 + \dots \\
 &= 1 - \frac{x}{3} - \frac{1}{9}x^2 - \frac{5}{81}x^3 - \dots
 \end{aligned}$$

(iii)  $(1 - x^2)^{-3}$

**Solution:**

$$\begin{aligned}
 (1 - x^2)^{-3} &= 1 + (-3)(-x^2) + \frac{(-3)(-3-1)}{2!}(-x^2)^2 \\
 &\quad + \frac{(-3)(-3-1)(-3-2)}{3!}(-x^2)^3 + \dots \\
 &= 1 + 3x^2 + \frac{(-3)(-4)}{2}x^4 - \frac{(-3)(-4)(-5)}{6}x^6 + \dots \\
 &= 1 + 3x^2 + 6x^4 + 10x^6 + \dots
 \end{aligned}$$

(iv)  $(1 + x)^{-1/5}$

**Solution:**

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-4-methods-of-induction-and-binomial-theorem/>

$$\begin{aligned}
 & (1+x)^{-1/5} \\
 &= 1 + \left(\frac{-1}{5}\right)x + \frac{\left(\frac{-1}{5}\right)\left(\frac{-1}{5}-1\right)}{2!}x^2 \\
 &\quad + \frac{\left(\frac{-1}{5}\right)\left(\frac{-1}{5}-1\right)\left(\frac{-1}{5}-2\right)}{3!}x^3 + \dots \\
 &= 1 - \frac{x}{5} + \frac{\left(\frac{-1}{5}\right)\left(\frac{-6}{5}\right)}{2}x^2 + \frac{\left(\frac{-1}{5}\right)\left(\frac{-6}{5}\right)\left(\frac{-11}{5}\right)}{6}x^3 + \dots \\
 &= 1 - \frac{x}{5} + \frac{3x^2}{25} - \frac{11x^3}{125} + \dots
 \end{aligned}$$

(v)  $(1+x^2)^{-1}$

**Solution:**

$$\begin{aligned}
 & (1+x^2)^{-1} \\
 &= 1 + (-1)x^2 + \frac{(-1)(-1-1)}{2!}(x^2)^2 \\
 &\quad + \frac{(-1)(-1-1)(-1-2)}{3!}(x^2)^3 + \dots \\
 &= 1 - x^2 + \frac{(-1)(-2)}{2}x^4 + \frac{(-1)(-2)(-3)}{6}x^6 + \dots \\
 &= 1 - x^2 + x^4 - x^6 + \dots
 \end{aligned}$$

**Question 2.**

State by writing first four terms, the expansion of the following, where  $|b| < |a|$ .

(i)  $(a-b)^{-3}$

**Solution:**

$$(a-b)^{-3} = [a(1-ba)]^{-3}$$

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-4-methods-of-induction-and-binomial-theorem/>

$$\begin{aligned}
 &= a^{-3} \left( 1 - \frac{b}{a} \right)^{-3} \\
 &= a^{-3} \left[ 1 + (-3) \left( -\frac{b}{a} \right) + \frac{(-3)(-3-1)}{2!} \left( -\frac{b}{a} \right)^2 \right. \\
 &\quad \left. + \frac{(-3)(-3-1)(-3-2)}{3!} \left( -\frac{b}{a} \right)^3 + \dots \right] \\
 &= a^{-3} \left[ 1 + \frac{3b}{a} + \frac{(-3)(-4)}{2} \cdot \frac{b^2}{a^2} - \frac{(-3)(-4)(-5)}{6} \cdot \frac{b^3}{a^3} + \dots \right] \\
 &= a^{-3} \left[ 1 + \frac{3b}{a} + \frac{6b^2}{a^2} + \frac{10b^3}{a^3} + \dots \right]
 \end{aligned}$$

(ii)  $(a + b)^{-4}$

**Solution:**

$$\begin{aligned}
 &(a + b)^{-4} \\
 &= \left[ a \left( 1 + \frac{b}{a} \right) \right]^{-4} \\
 &= a^{-4} \left( 1 + \frac{b}{a} \right)^{-4} \\
 &= a^{-4} \left[ 1 + (-4) \frac{b}{a} + \frac{(-4)(-4-1)}{2!} \left( \frac{b}{a} \right)^2 \right. \\
 &\quad \left. + \frac{(-4)(-4-1)(-4-2)}{3!} \left( \frac{b}{a} \right)^3 + \dots \right] \\
 &= a^{-4} \left[ 1 - \frac{4b}{a} + \frac{(-4)(-5)}{2} \cdot \frac{b^2}{a^2} + \frac{(-4)(-5)(-6)}{6} \cdot \frac{b^3}{a^3} + \dots \right] \\
 &= a^{-4} \left[ 1 - \frac{4b}{a} + \frac{10b^2}{a^2} - \frac{20b^3}{a^3} + \dots \right]
 \end{aligned}$$

(iii)  $(a + b)^{1/4}$

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-4-methods-of-induction-and-binomial-theorem/>

**Solution:**

$$\begin{aligned}(a+b)^{1/4} &= \left[ a \left( 1 + \frac{b}{a} \right) \right]^{1/4} \\&= a^{1/4} \left( 1 + \frac{b}{a} \right)^{1/4} \\&= a^{1/4} \left[ 1 + \left( \frac{1}{4} \right) \frac{b}{a} + \frac{\frac{1}{4} \left( \frac{1}{4} - 1 \right)}{2!} \left( \frac{b}{a} \right)^2 + \frac{\frac{1}{4} \left( \frac{1}{4} - 1 \right) \left( \frac{1}{4} - 2 \right)}{3!} \left( \frac{b}{a} \right)^3 + \dots \right] \\&= a^{1/4} \left[ 1 + \frac{b}{4a} + \frac{\frac{1}{4} \left( \frac{-3}{4} \right)}{2} \cdot \frac{b^2}{a^2} + \frac{\frac{1}{4} \left( \frac{-3}{4} \right) \left( \frac{-7}{4} \right)}{6} \cdot \frac{b^3}{a^3} + \dots \right] \\&= a^{1/4} \left[ 1 + \frac{b}{4a} - \frac{3b^2}{32a^2} + \frac{7b^3}{128a^3} - \dots \right]\end{aligned}$$

(iv)  $(a-b)^{-1/4}$

**Solution:**

$$(a-b)^{-1/4} = [a(1-ba)]^{-1/4}$$

$$\begin{aligned}
 &= a^{-\frac{1}{4}} \left( 1 - \frac{b}{a} \right)^{-\frac{1}{4}} \\
 &= a^{-\frac{1}{4}} \left[ 1 + \left( -\frac{1}{4} \right) \left( -\frac{b}{a} \right) + \frac{\left( -\frac{1}{4} \right) \left( -\frac{1}{4} - 1 \right)}{2!} \left( -\frac{b}{a} \right)^2 \right. \\
 &\quad \left. + \frac{\left( -\frac{1}{4} \right) \left( -\frac{1}{4} - 1 \right) \left( -\frac{1}{4} - 2 \right)}{3!} \left( -\frac{b}{a} \right)^3 + \dots \right] \\
 &= a^{-\frac{1}{4}} \left[ 1 + \frac{b}{4a} + \frac{\left( -\frac{1}{4} \right) \left( -\frac{5}{4} \right)}{2} \cdot \frac{b^2}{a^2} \right. \\
 &\quad \left. - \frac{\left( -\frac{1}{4} \right) \left( -\frac{5}{4} \right) \left( -\frac{9}{4} \right)}{6} \cdot \frac{b^3}{a^3} + \dots \right] \\
 &= a^{-\frac{1}{4}} \left[ 1 + \frac{b}{4a} + \frac{5b^2}{32a^2} + \frac{15b^3}{128a^3} + \dots \right]
 \end{aligned}$$

(v)  $(a + b)^{-1/3}$

**Solution:**

$$\begin{aligned}
 (a+b)^{-1/3} &= \left[ a \left( 1 + \frac{b}{a} \right) \right]^{-1/3} \\
 &= a^{-1/3} \left( 1 + \frac{b}{a} \right)^{-1/3} \\
 &= a^{-1/3} \left[ 1 + \left( \frac{-1}{3} \right) \frac{b}{a} + \frac{\left( \frac{-1}{3} \right) \left( \frac{-1}{3} - 1 \right)}{2!} \left( \frac{b}{a} \right)^2 \right. \\
 &\quad \left. + \frac{\left( \frac{-1}{3} \right) \left( \frac{-1}{3} - 1 \right) \left( \frac{-1}{3} - 2 \right)}{3!} \left( \frac{b}{a} \right)^3 + \dots \right] \\
 &= a^{-1/3} \left[ 1 - \frac{b}{3a} + \frac{\left( \frac{-1}{3} \right) \left( \frac{-4}{3} \right)}{2} \cdot \frac{b^2}{a^2} \right. \\
 &\quad \left. + \frac{\left( \frac{-1}{3} \right) \left( \frac{-4}{3} \right) \left( \frac{-7}{3} \right)}{6} \cdot \frac{b^3}{a^3} + \dots \right] \\
 &= a^{-1/3} \left[ 1 - \frac{b}{3a} + \frac{2b^2}{9a^2} - \frac{14b^3}{81a^3} + \dots \right]
 \end{aligned}$$

**Question 3.**

**Simplify the first three terms in the expansion of the following:**

**(i)  $(1 + 2x)^{-4}$**

**Solution:**

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-4-methods-of-induction-and-binomial-theorem/>

$$\begin{aligned}(1 + 2x)^{-4} \\&= 1 + (-4)(2x) + \frac{(-4)(-4-1)}{2!}(2x)^2 + \dots \\&= 1 - 8x + \frac{(-4)(-5)}{2}(4x^2) + \dots \\&= 1 - 8x + 40x^2 + \dots\end{aligned}$$

(ii)  $(1 + 3x)^{-1/2}$

**Solution:**

$$\begin{aligned}(1 + 3x)^{-\frac{1}{2}} \\&= 1 + \left(\frac{-1}{2}\right)(3x) + \frac{\left(\frac{-1}{2}\right)\left(\frac{-1}{2}-1\right)}{2!}(3x)^2 + \dots \\&= 1 - \frac{3x}{2} + \frac{\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)}{2}(9x^2) + \dots \\&= 1 - \frac{3x}{2} + \frac{27}{8}x^2 + \dots\end{aligned}$$

(iii)  $(2 - 3x)^{1/3}$

**Solution:**

$$\begin{aligned}
 & (2-3x)^{\frac{1}{3}} \\
 &= \left[ 2 \left( 1 - \frac{3x}{2} \right) \right]^{\frac{1}{3}} \\
 &= 2^{\frac{1}{3}} \left( 1 - \frac{3x}{2} \right)^{\frac{1}{3}} \\
 &= 2^{\frac{1}{3}} \left[ 1 - \left( \frac{1}{3} \right) \left( \frac{3x}{2} \right) + \frac{\left( \frac{1}{3} \right) \left( \frac{1}{3} - 1 \right)}{2!} \left( \frac{3x}{2} \right)^2 - \dots \right] \\
 &= 2^{\frac{1}{3}} \left[ 1 - \frac{x}{2} + \frac{\frac{1}{3} \left( \frac{-2}{3} \right)}{2} \left( \frac{9x^2}{4} \right) - \dots \right] \\
 &= 2^{\frac{1}{3}} \left[ 1 - \frac{x}{2} - \frac{x^2}{4} - \dots \right]
 \end{aligned}$$

(iv)  $(5 + 4x)^{-1/2}$

**Solution:**

$$\begin{aligned}
 & (5+4x)^{-\frac{1}{2}} \\
 &= \left[ 5 \left( 1 + \frac{4}{5}x \right) \right]^{-\frac{1}{2}} \\
 &= 5^{-\frac{1}{2}} \left( 1 + \frac{4x}{5} \right)^{-\frac{1}{2}} \\
 &= 5^{-\frac{1}{2}} \left[ 1 + \left( \frac{-1}{2} \right) \left( \frac{4x}{5} \right) + \frac{\left( \frac{-1}{2} \right) \left( \frac{-1}{2} - 1 \right)}{2!} \left( \frac{4x}{5} \right)^2 + \dots \right] \\
 &= 5^{-\frac{1}{2}} \left[ 1 - \frac{2x}{5} + \frac{\left( \frac{-1}{2} \right) \left( \frac{-3}{2} \right)}{2} \left( \frac{16x^2}{25} \right) + \dots \right] \\
 &= 5^{-\frac{1}{2}} \left[ 1 - \frac{2x}{5} + \frac{6x^2}{25} + \dots \right]
 \end{aligned}$$

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(v)  $(5 - 3x)^{-1/3}$

**Solution:**

$$\begin{aligned} & (5 - 3x)^{-\frac{1}{3}} \\ &= \left[ 5 \left( 1 - \frac{3x}{5} \right) \right]^{-\frac{1}{3}} \\ &= 5^{-\frac{1}{3}} \left( 1 - \frac{3x}{5} \right)^{-\frac{1}{3}} \\ &= 5^{-\frac{1}{3}} \left[ 1 + \left( \frac{-1}{3} \right) \left( -\frac{3x}{5} \right) + \frac{\left( \frac{-1}{3} \right) \left( \frac{-1}{3} - 1 \right)}{2!} \left( -\frac{3x}{5} \right)^2 + \dots \right] \\ &= 5^{-\frac{1}{3}} \left[ 1 + \frac{x}{5} + \frac{\left( \frac{-1}{3} \right) \left( \frac{-4}{3} \right)}{2} \left( \frac{9x^2}{25} \right) + \dots \right] \\ &= 5^{-\frac{1}{3}} \left[ 1 + \frac{x}{5} + \frac{2x^2}{25} + \dots \right] \end{aligned}$$

**Question 4.**

Use the binomial theorem to evaluate the following upto four places of decimals.

(i)  $\sqrt{99}$

**Solution:**

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$$\begin{aligned}
 \sqrt{99} &= (99)^{\frac{1}{2}} \\
 &= (100-1)^{\frac{1}{2}} \\
 &= \left[ 100 \left( 1 - \frac{1}{100} \right) \right]^{\frac{1}{2}} \\
 &= (100)^{\frac{1}{2}} (1-0.01)^{\frac{1}{2}} \\
 &= 10 \left[ 1 - \frac{1}{2}(0.01) + \frac{\frac{1}{2} \left( \frac{1}{2} - 1 \right)}{2!} (0.01)^2 - \dots \right] \\
 &= 10 \left[ 1 - \frac{1}{2}(0.01) + \frac{\frac{1}{2} \left( \frac{-1}{2} \right)}{2} (0.01)^2 - \dots \right] \\
 &= 10 \left[ 1 - 0.005 - \frac{0.0001}{8} - \dots \right]
 \end{aligned}$$

$$= 10 [1 - 0.005 - 0.0000125 - \dots]$$

$$= 10(0.9949875)$$

$$= 9.949875$$

$$= 9.9499$$

$$(ii) \sqrt{126} \approx \sqrt{3}$$

**Solution:**

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$$\begin{aligned}
 & \sqrt[3]{126} \\
 &= (125+1)^{\frac{1}{3}} \\
 &= \left[ 125 \left( 1 + \frac{1}{125} \right) \right]^{\frac{1}{3}} \\
 &= (125)^{\frac{1}{3}} \left( 1 + \frac{1}{125} \right)^{\frac{1}{3}} \\
 &= 5 \left( 1 + 0.008 \right)^{\frac{1}{3}} \\
 &= 5 \left[ 1 + \frac{1}{3}(0.008) + \frac{\frac{1}{3} \left( \frac{1}{3} - 1 \right)}{2!} (0.008)^2 + \dots \right] \\
 &= 5 \left[ 1 + \frac{1}{3}(0.008) + \frac{\frac{1}{3} \left( \frac{-2}{3} \right)}{2} (0.008)^2 + \dots \right] \\
 &= 5 \left[ 1 + 0.00266 - \frac{0.000064}{9} \right] \\
 &= 5 (1.00266 - 0.000007) \\
 &= 5 (1.002653) \\
 &= 5.013265 \\
 &= 5.0133
 \end{aligned}$$

(iii)  $16.08 \text{-----} \sqrt{4}$

**Solution:**

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$$\begin{aligned} & \sqrt[4]{16.08} \\ &= (16 + 0.08)^{1/4} \\ &= [16(1 + 0.005)]^{1/4} \\ &= 16^{1/4} (1 + 0.005)^{1/4} \\ &= 2 \left[ 1 + \frac{1}{4}(0.005) + \frac{\frac{1}{4} \left( \frac{1}{4} - 1 \right)}{2!} (0.005)^2 + \dots \right] \\ &= 2 \left[ 1 + \frac{1}{4}(0.005) + \frac{\frac{1}{4} \left( \frac{-3}{4} \right)}{2} (0.005)^2 + \dots \right] \\ &= 2 \left[ 1 + 0.00125 - \frac{0.000075}{32} + \dots \right] \\ &= 2 [1 + 0.00125 - 0.0000023 + \dots] \\ &= 2 (1.0012477) \\ &= 2.0024954 \\ &= 2.0025 \end{aligned}$$

(iv)  $(1.02)^{-5}$

**Solution:**

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$$\begin{aligned}(1.02)^{-5} &= (1 + 0.02)^{-5} \\&= 1 + (-5)(0.02) + \frac{(-5)(-5-1)}{2!} (0.02)^2 \\&\quad + \frac{(-5)(-5-1)(-5-2)}{3!} (0.02)^3 + \dots \\&= 1 + (-5)(0.02) + \frac{(-5)(-6)}{2} (0.02)^2 \\&\quad + \frac{(-5)(-6)(-7)}{6} (0.02)^3 + \dots \\&= 1 - 0.1 + 0.006 - 0.00028 + \dots \\&= 0.90572 \\&= 0.9057\end{aligned}$$

(v)  $(0.98)^{-3}$

**Solution:**

$$\begin{aligned}(0.98)^{-3} &= (1 - 0.02)^{-3} \\&= 1 + (-3)(-0.02) + \frac{(-3)(-3-1)}{2!} (-0.02)^2 \\&\quad + \frac{(-3)(-3-1)(-3-2)}{3!} (-0.02)^3 + \dots \\&= 1 + 0.06 + \frac{(-3)(-4)}{2} (0.02)^2 \\&\quad - \frac{(-3)(-4)(-5)}{6} (0.02)^3 + \dots \\&= 1 + 0.06 + 0.0024 + 0.00008 + \dots \\&= 1.06248 \\&= 1.0625\end{aligned}$$

**Ex 4.5**

**Question 1.**

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-4-methods-of-induction-and-binomial-theorem/>

**Show that  $C_0 + C_1 + C_2 + \dots + C_8 = 256$**

**Solution:**

Since  $C_0 + C_1 + C_2 + C_3 + \dots + C_n = 2^n$

Putting  $n = 8$ , we get

$$C_0 + C_1 + C_2 + \dots + C_8 = 2^8$$

$$\therefore C_0 + C_1 + C_2 + \dots + C_8 = 256$$

**Question 2.**

**Show that  $C_0 + C_1 + C_2 + \dots + C_9 = 512$**

**Solution:**

Since  $C_0 + C_1 + C_2 + C_3 + \dots + C_n = 2^n$

Putting  $n = 9$ , we get

$$C_0 + C_1 + C_2 + \dots + C_9 = 2^9$$

$$\therefore C_0 + C_1 + C_2 + \dots + C_9 = 512$$

**Question 3.**

**Show that  $C_1 + C_2 + C_3 + \dots + C_7 = 127$**

**Solution:**

Since  $C_0 + C_1 + C_2 + C_3 + \dots + C_n = 2^n$

Putting  $n = 7$ , we get

$$C_0 + C_1 + C_2 + \dots + C_7 = 2^7$$

$$\therefore C_0 + C_1 + C_2 + \dots + C_7 = 128$$

But,  $C_0 = 1$

$$\therefore 1 + C_1 + C_2 + \dots + C_7 = 128$$

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$$\therefore C_1 + C_2 + \dots + C_7 = 128 - 1 = 127$$

**Question 4.**

**Show that  $C_1 + C_2 + C_3 + \dots + C_6 = 63$**

**Solution:**

$$\text{Since } C_0 + C_1 + C_2 + C_3 + \dots + C_n = 2^n$$

Putting  $n = 6$ , we get

$$C_0 + C_1 + C_2 + \dots + C_6 = 2^6$$

$$\therefore C_0 + C_1 + C_2 + \dots + C_6 = 64$$

$$\text{But, } C_0 = 1$$

$$\therefore 1 + C_1 + C_2 + \dots + C_6 = 64$$

$$\therefore C_1 + C_2 + \dots + C_6 = 64 - 1 = 63$$

**Question 5.**

**Show that  $C_0 + C_2 + C_4 + C_6 + C_8 = C_1 + C_3 + C_5 + C_7 = 128$**

**Solution:**

$$\text{Since } C_0 + C_1 + C_2 + C_3 + \dots + C_n = 2^n$$

Putting  $n = 8$ , we get

$$C_0 + C_1 + C_2 + C_3 + \dots + C_8 = 2^8$$

But, sum of even coefficients = sum of odd coefficients

$$\therefore C_0 + C_2 + C_4 + C_6 + C_8 = C_1 + C_3 + C_5 + C_7$$

$$\text{Let } C_0 + C_2 + C_4 + C_6 + C_8 = C_1 + C_3 + C_5 + C_7 = k$$

$$\text{Now, } C_0 + C_1 + C_2 + C_3 + C_4 + C_5 + C_6 + C_7 + C_8 = 256$$

$$\therefore (C_0 + C_2 + C_4 + C_6 + C_8) + (C_1 + C_3 + C_5 + C_7) = 256$$

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$$\therefore k + k = 256$$

$$\therefore 2k = 256$$

$$\therefore k = 128$$

$$\therefore C_0 + C_2 + C_4 + C_6 + C_8 = C_1 + C_3 + C_5 + C_7 = 128$$

**Question 6.**

Show that  $C_1 + C_2 + C_3 + \dots + C_n = 2^n - 1$

**Solution:**

$$\text{Since } C_0 + C_1 + C_2 + C_3 + \dots + C_n = 2^n$$

$$\text{But, } C_0 = 1$$

$$\therefore 1 + C_1 + C_2 + C_3 + \dots + C_n = 2^n$$

$$\therefore C_1 + C_2 + C_3 + \dots + C_n = 2^n - 1$$

**Question 7.**

Show that  $C_0 + 2C_1 + 3C_2 + 4C_3 + \dots + (n + 1)C_n = (n + 2) 2^{n-1}$

**Solution:**

$$\begin{aligned}
 &\text{Since } C_0 + C_1 + C_2 + C_3 + \dots + C_n = 2^n \\
 &C_0 + 2C_1 + 3C_2 + 4C_3 + \dots + (n+1)C_n \\
 &= (C_0 + C_1 + C_2 + C_3 + \dots + C_n) \\
 &\quad + (C_1 + 2C_2 + 3C_3 + \dots + nC_n) \\
 &= 2^n + (C_1 + 2C_2 + 3C_3 + \dots + nC_n) \\
 &= 2^n + \left[ n + \frac{2n(n-1)}{2!} + \frac{3n(n-1)(n-2)}{3!} + \dots + n(1) \right] \\
 &= 2^n + n \left[ 1 + \frac{(n-1)}{1!} + \frac{(n-1)(n-2)}{2!} + \dots + 1 \right] \\
 &= 2^n + n \left[ {}^{n-1}C_0 + {}^{n-1}C_1 + {}^{n-1}C_2 + \dots + {}^{n-1}C_{n-1} \right] \\
 &= 2^n + n \cdot 2^{n-1} \\
 &= 2^{n-1} \cdot 2 + n \cdot 2^{n-1} \\
 &= (2+n) \cdot 2^{n-1} \\
 &= (n+2) \cdot 2^{n-1}
 \end{aligned}$$



# Maharashtra Board Solutions

## Class 11 Arts & Science Maths

### (Part 2)

- Chapter 1- Complex Numbers
- Chapter 2- Sequences and Series
- Chapter 3- Permutations and Combination
- Chapter 4- Methods of Induction and Binomial Theorem
- Chapter 5- Sets and Relations
- Chapter 6- Functions
- Chapter 7- Limits
- Chapter 8- Continuity
- Chapter 9- Differentiation

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## About About Maharashtra State Board (MSBSHSE)

The Maharashtra State Board of Secondary and Higher Secondary Education or MSBSHSE (Marathi: महाराष्ट्र राज्य माध्यमिक आणि उच्च माध्यमिक शिक्षण मंडळ), is an **autonomous and statutory body established in 1965**. The board was amended in the year 1977 under the provisions of the Maharashtra Act No. 41 of 1965.

The Maharashtra State Board of Secondary & Higher Secondary Education (MSBSHSE), Pune is an independent body of the Maharashtra Government. There are more than 1.4 million students that appear in the examination every year. The Maha State Board conducts the board examination twice a year. This board conducts the examination for SSC and HSC.

The Maharashtra government established the Maharashtra State Bureau of Textbook Production and Curriculum Research, also commonly referred to as Ebalbharati, in 1967 to take up the responsibility of providing quality textbooks to students from all classes studying under the Maharashtra State Board. MSBHSE prepares and updates the curriculum to provide holistic development for students. It is designed to tackle the difficulty in understanding the concepts with simple language with simple illustrations. Every year around 10 lakh students are enrolled in schools that are affiliated with the Maharashtra State Board.

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