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# K-1059

## M.A./M.Sc. (Final) Examination, 2022

## MATHEMATICS

## (Fuzzy Sets and their Applications)

### [ Paper : Seventh ]

***Time Allowed : Three Hours***

***Maximum Marks : 100***

***Minimum Passing Marks : 36***

**Note :** Attempt **any five** questions. All questions carry **equal** marks.

1. (a) Prove that the sum and difference of two convex fuzzy sets are convex.

   (b) Let $\tilde{A}$ be a fuzzy set defined by

$$\tilde{A} = \left\{ \frac{0.5}{x_1}, \frac{0.4}{x_2}, \frac{0.7}{x_3}, \frac{0.8}{x_4}, \frac{1}{x_5} \right\}.$$

Find list of $\alpha$-cuts set and strong $\alpha$-cut sets.

2. (a) Let $\tilde{A}_i \in F(x)$ for all $i \in I$, where $I$ is an index set. Then prove that $\bigcup_{i=1}^{\alpha} A_i \leq {}^{\alpha}\left( \bigcup_{i=1} A_i \right)$.

(b) State and prove second Decomposition theorem.

3. State and prove that first characterization Theorem of Fuzzy complements.

4. Let $A$ and $B$ be two fuzzy number whose membership function are given by

$$A(x) = \begin{cases} (x+2)/2, & -2 < x \leq 0 \\ (2-x)/2, & 0 < x < 2 \\ 0, & \text{othewise} \end{cases} \quad ; \quad B = \begin{cases} (x-2)/2, & 2 < x < 0 \\ (6-x)/2, & 0 < x \leq 6 \\ 0, & \text{otherwise} \end{cases}$$

Find the fuzzy numbers $(A+B), (A-B), (A.B)$ and $(A/B)$.

5. Let

$$P = \begin{bmatrix} .3 & .5 & .8 \\ 0 & .7 & 1 \\ .4 & .6 & .5 \end{bmatrix}, Q = \begin{bmatrix} .9 & .5 & .7 & .7 \\ .3 & .2 & 0 & .9 \\ 1 & 0 & .5 & .5 \end{bmatrix}$$

find the value of $PoQ$.

6. (a) Explain the Belief measure.

(b) Prove that a belief measure Bel on a finite power set $P(X)$ is a probability measure if and only if the associated basic probability assignment function $\mu$ is given by $\mu(\{x\}) = Bel(\{x\})$ and $\mu(A) = 0$ for all subsets of $X$ that are not singletons.

7. (a) Explain Fuzzy implications.

(b) Prove that every possibility measure $\pi$ on $p(x)$ can be uniquely determined by a possibility distribution function $r : X \to [0,1]$

viz the formula $\pi(A) = \max_{x \in A} r(X)$

8. Suppose the fuzzy conditional and qualified preposition is given by $P$: "If $x$ is $A$ then $\psi$ is $B$ is very true" where

$A = \frac{1}{x_1} + \frac{.5}{x_2} + \frac{.7}{x_3}$, $B = \frac{.6}{y_1} + \frac{1}{y_2}$ and $s$ stands for "very true", let $s(a) = a^2, \forall a \in (0,1)$.

9. Explain methods of Defuzzification.

10. Solve the following fuzzy linear programming problem :

Max $z$ = $0.5x_1 + 0.2x_2$

such that $x_1 + x_2 \le B_1$

$2x_1 + x_2 \le B_2 \quad x_1 x_2 \ge 0$

whose $B_1(x) = \begin{cases} 1 & \text{for } x \le 300 \\ \frac{400 - x}{100} & \text{for } 300 < x \le 400 \\ 0 & \text{for } x > 400 \end{cases}$

$B_2(x) = \begin{cases} 1 & \text{for } x \le 400 \\ \frac{500 - x}{100} & \text{for } 400 < x \le 500 \\ 0 & \text{for } x > 500 \end{cases}$

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