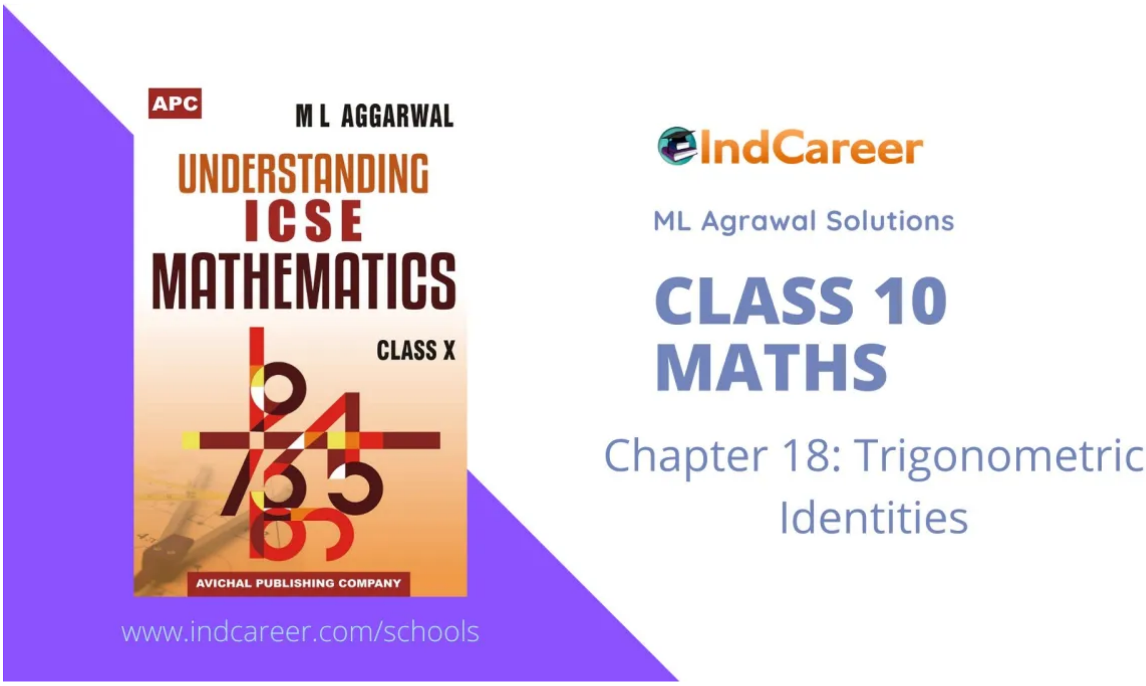
# ML Aggarwal Solutions for Class 10 Maths Chapter 18- Trigonometric Identities





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# ML Aggarwal Solutions for Class 10 Maths Chapter 18- Trigonometric Identities

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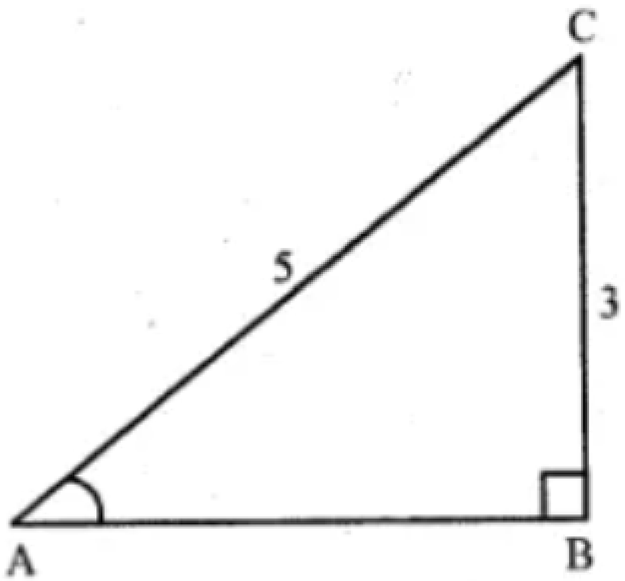
ML Aggarwal 10th Maths Chapter 18, Class 10 Maths Chapter 18 solutions

**Exercise 18**

**1. If A is an acute angle and sin A = 3/5, find all other trigonometric ratios of angle A (using trigonometric identities).**

**Solution:**




Given,

sin A = 3/5 and A is an acute angle

So, in △ABC we have ∠B = 90°

And,

AC = 5 and BC = 3

By Pythagoras theorem,

AB = $\sqrt{(AC^2 - BC^2)}$

$= \sqrt{(5^2 - 3^2)} = \sqrt{(25 - 9)} = \sqrt{16}$

$= 4$

Now,

cos A = AB/AC = 4/5

tan A = BC/AB = 3/4

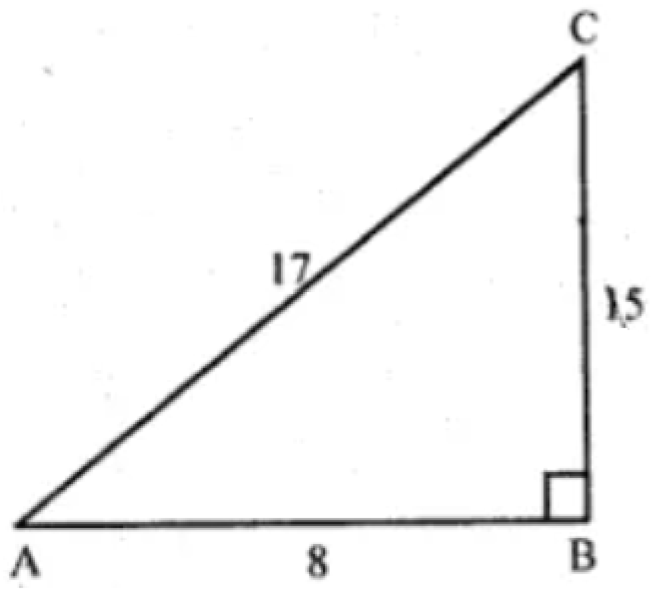
cot A = 1/tan θ = 4/3

sec A = 1/cos θ = 5/4

cosec A = 1/sin θ = 5/3

**2. If A is an acute angle and sec A = 17/8, find all other trigonometric ratios of angle A (using trigonometric identities).**

**Solution:**




Given,

sec A = 17/8 and A is an acute angle

So, in ∆ ABC we have ∠B = 90°

And,

AC = 17 and AB = 8

By Pythagoras theorem,

$BC = \sqrt{(AC^2 - AB^2)}$

$= \sqrt{(17^2 - 8^2)} = \sqrt{(289 - 64)} = \sqrt{225}$

$= 15$

Now,

sin A = BC/AC = 15/17

cos A = 1/sec A = 8/17

tan A = BC/AB = 15/8

cot A = 1/tan A= 8/15

cosec A = 1/sin A = 17/15

**3. Express the ratios cos A, tan A and sec A in terms of sin A.**

**Solution:**

We know that,

$\sin^2 A + \cos^2 A = 1$

So,

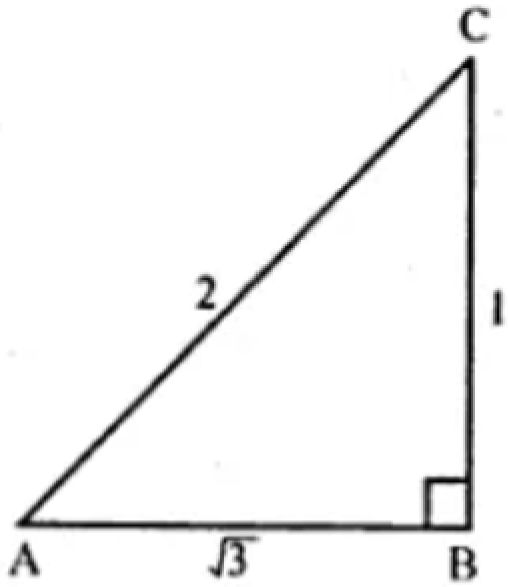
$\cos A = \sqrt{(1 - \sin^2 A)}$

$\tan A = \sin A/\cos A = \sin A/ \sqrt{(1 - \sin^2 A)}$

$\sec A = 1/\cos A = 1/ (\sqrt{1 - \sin^2 A})$

**4. If tan A = 1/√3, find all other trigonometric ratios of angle A.**

**Solution:**




Given, tan A = 1/√3

In right ∆ ABC,

tan A = BC/AB = 1/√3

So,

BC = 1 and AB = √3

By Pythagoras theorem,

AC = $\sqrt{(AB^2 + BC^2)} = \sqrt{[(\sqrt{3})^2 + (1)^2]}$

= √(3 + 1) = √4 = 2

Hence,

sin A = BC/AC = ½

cos A = AB/AC = √3/2

cot A = 1/tan A = √3

sec A = 1/cos A = 2/√3

cosec A = 1/sin A = 2/1 = 2

**5. If 12 cosec θ = 13, find the value of (2 sin θ – 3 cos θ)/ (4 sin θ – 9 cos θ)**

**Solution:**

Given,

12 cosec θ = 13

⇒ cosec θ = 13/12

In right Δ ABC,

∠A = θ

So, cosec θ = AC/BC = 13/12

AC = 13 and BC = 12

By Pythagoras theorem,

AB = $\sqrt{(AC^2 - BC^2)}$

= $\sqrt{[(13)^2 - (12)^2]}$

= √(169 – 144)

= √25

= 5

Now,

sin θ = BC/AC = 12/13

cos θ = AB/AC = 5/13

Hence,

$$\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \frac{2\times\frac{12}{13} - 3\times\frac{5}{13}}{4\times\frac{12}{13} - 9\times\frac{5}{13}}$$

$$= \frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} - \frac{45}{13}} = \frac{\frac{9}{13}}{\frac{3}{13}} = \frac{9}{13} \times \frac{13}{3} = 3$$

**Without using trigonometric tables, evaluate the following (6 to 10):**

**6. (i) $\cos^2 26^\circ + \cos 64^\circ \sin 26^\circ + (\tan 36^\circ / \cot 54^\circ)$**

**(ii) $(\sec 17^\circ / \text{cosec } 73^\circ) + (\tan 68^\circ / \cot 22^\circ) + \cos^2 44^\circ + \cos^2 46^\circ$**

**Solution:**

Given,

(i) $\cos^2 26^\circ + \cos 64^\circ \sin 26^\circ + (\tan 36^\circ / \cot 54^\circ)$

$= \cos^2 26^\circ + \cos (90^\circ – 16^\circ) \sin 26^\circ + [\tan 36^\circ / \cot (90^\circ – 54^\circ)]$

$= [\cos^2 26^\circ + \sin^2 26^\circ] + (\tan 36^\circ / \tan 36^\circ)$

$= 1 + 1 = 2$

(ii) $(\sec 17^\circ / \text{cosec } 73^\circ) + (\tan 68^\circ / \cot 22^\circ) + \cos^2 44^\circ + \cos^2 46^\circ$

$= [\sec 17^\circ / \text{cosec } (90^\circ – 73^\circ)] + [(\tan 90^\circ – 22^\circ) / \cot 22^\circ] + \cos^2 (90^\circ – 44^\circ) + \cos^2 46^\circ$

$= [\sec 17^\circ / \sec 17^\circ] + [\cot 22^\circ / \cot 22^\circ] + [\sin^2 46^\circ + \cos^2 46^\circ]$

$= 1 + 1 + 1$

$= 3$

**7. (i) $(\sin 65^\circ / \cos 25^\circ) + (\cos 32^\circ / \sin 58^\circ) – \sin 28^\circ \sec 62^\circ + \text{cosec}^2 30^\circ$**

**(ii) (sin 29°/ cosec 61°) + 2 cot 8° cot 17° cot 45° cot 73° cot 82° – 3($\sin^2$ 38° + $\sin^2$ 52°).**

**Solution:**

Given,

(i) (sin 65°/ cos 25°) + (cos 32°/sin 58°) – sin 28° sec 62° + $\text{cosec}^2$ 30°

= (sin 65°/ cos (90° – 65°)) + (cos 32°/sin (90° – 32°)) – sin 28° sec (90° – 28°) + $2^2$

= (sin 65°/sin 65°) + (cos 32°/cos 32°) – [sin 28° x cosec 28°] + 4

= 1 + 1 – 1 + 4

= 5

(ii) (sin 29°/ cosec 61°) + 2 cot 8° cot 17° cot 45° cot 73° cot 82° – 3($\sin^2$ 38° + $\sin^2$ 52°).

= (sin 29°/ cosec (90° – 29°)) + [2 cot 8° cot 17° cot 45° cot (90° – 17°) cot (90° – 8°)] – 3($\sin^2$ 38° + $\sin^2$ (90° – 38°))

= (sin 29°/ sin 29°) + [2 cot 8° cot 17° cot 45° tan 17° tan 8°] – 3($\sin^2$ 38° + $\cos^2$ 38°)

= 1 + 2[(cot 8° tan 8°) (cot 17° tan 17°) cot 45°] – 3(1)

= 1 + 2[1 x 1 x 1] – 3

= 1 + 2 – 3

= 0

**8. (i) (sin 35° cos 55° + cos 35° sin 55°)/ ($\text{cosec}^2$ 10° – $\tan^2$ 80°)**

**(ii) $\sin^2$ 34° + $\sin^2$ 56° + 2 tan18° tan 72° – $\cot^2$ 30°**

**Solution:**

Given,

(i) (sin 35° cos 55° + cos 35° sin 55°)/ ($\text{cosec}^2$ 10° – $\tan^2$ 80°)

$$= \frac{\sin 35^\circ \cos(90^\circ - 35^\circ) + \cos 35^\circ \sin(90^\circ - 35^\circ)}{\operatorname{cosec}^2 10^\circ - \tan^2(90^\circ - 10^\circ)}$$

$$= \frac{\sin 35^\circ \sin 35^\circ + \cos 35^\circ \cos 35^\circ}{\operatorname{cosec}^2 10^\circ - \cot^2 10^\circ}$$

$$= \frac{\sin^2 35^\circ + \cos^2 35^\circ}{\operatorname{cosec}^2 10^\circ - \cot^2 10^\circ} = \frac{1}{1} = 1$$

(ii) $\sin^2 34^\circ + \sin^2 56^\circ + 2 \tan 18^\circ \tan 72^\circ - \cot^2 30^\circ$

$= \sin^2 34^\circ + \sin^2 (90^\circ - 34^\circ) + 2 \tan 18^\circ \tan (90^\circ - 18^\circ) - \cot^2 30^\circ$

$= [\sin^2 34^\circ + \cos^2 34^\circ] + 2 \tan 18^\circ \cot 18^\circ - \cot^2 30^\circ$

$= 1 + 2 \times 1 - (\sqrt{3})^2$

$= 1 + 2 - 3$

$= 0$

**9. (i) $(\tan 25^\circ / \operatorname{cosec} 65^\circ)^2 + (\cot 25^\circ / \sec 65^\circ)^2 + 2 \tan 18^\circ \tan 45^\circ \tan 75^\circ$**

**(ii) $(\cos^2 25^\circ + \cos^2 65^\circ) + \operatorname{cosec} \theta \sec (90^\circ - \theta) - \cot \theta \tan (90^\circ - \theta)$**

**Solution:**

Given,

(i) $(\tan 25^\circ / \operatorname{cosec} 65^\circ)^2 + (\cot 25^\circ / \sec 65^\circ)^2 + 2 \tan 18^\circ \tan 45^\circ \tan 75^\circ$

$$= \left(\frac{\tan 25^\circ}{\operatorname{cosec}(90^\circ - 25^\circ)}\right)^2 + \left(\frac{\cot 25^\circ}{\sec(90^\circ - 25^\circ)}\right)^2 + 2 \tan 18^\circ \tan (90^\circ - 18^\circ) \tan 45^\circ$$

$$= \left(\frac{\tan 25^\circ}{\sec 25^\circ}\right)^2 + \left(\frac{\cot 25^\circ}{\operatorname{cosec} 25^\circ}\right)^2 + 2 \tan 18^\circ \cot 18^\circ \tan 45^\circ$$

$$= \left(\frac{\sin 25^\circ \times \cos 25^\circ}{\cos 25^\circ \times 1}\right)^2 + \left(\frac{\cos 25^\circ \times \sin 25^\circ}{\sin 25^\circ \times 1}\right) + 2 \times 1 \times 1$$

$$= \sin^2 25^\circ + \cos^2 25^\circ + 2 \qquad \left\{\begin{array}{l} \because \sin^2\theta + \cos^2\theta = 1 \\ \tan\theta \cot\theta = 1 \end{array}\right\}$$

$$= 1 + 2 = 3$$

(ii) $(\cos^2 25^\circ + \cos^2 65^\circ) + \operatorname{cosec} \theta \sec (90^\circ - \theta) - \cot \theta \tan (90^\circ - \theta)$

$= \cos^2 25^\circ + \cos^2 (90^\circ - 25^\circ) + \operatorname{cosec} \theta \sec (90^\circ - \theta) - \cot \theta. \cot \theta$

$= (\cos^2 25^\circ + \sin^2 25^\circ) + (\operatorname{cosec}^2 \theta - \cot^2 \theta)$

$= 1 + 1 = 2$

**10. (i) $2(\sec^2 35^\circ - \cot^2 55^\circ)$ –**

**(ii)**

**Solution:**

Given,

(i) $2(\sec^2 35° - \cot^2 55°) - \frac{\cos 28^O \operatorname{cosec} 62^O}{\tan 18^O \tan 36^O \tan 30^O \tan 54^O \tan 72^O}$

$$= 2[\sec^2 35° - \cot^2 (90° - 35°)] - \frac{\cos 28° \operatorname{cosec}(90° - 28°)}{\tan 18° \tan(90° - 18°) \tan 36° \tan(90° - 36°) \tan 30°}$$

$$= 2[\sec^2 35° - \tan^2 35°] - \frac{\cos 28° \sec 28°}{\tan 18° \cot 18° \tan 36° \cot 36° \tan 30°}$$

$$\left\{\begin{array}{l} \because \sec^2\theta - \tan^2\theta = 1 \\ \tan\theta \cot\theta = 1 \\ \cos\theta \sec\theta = 1 \end{array}\right\}$$

$$= 2(1) - \frac{1}{1 \times 1 \times \frac{1}{\sqrt{3}}} = 2 - \frac{\sqrt{3}}{1} = 2 - \sqrt{3}$$

(ii) $\frac{\operatorname{cosec}^2(90° - \theta) - \tan^2\theta}{2(\cos^2 48° + \cos^2 42°)} - \frac{2(\tan^2 30° \sec^2 52° \sin^2 38°)}{\operatorname{cosec}^2 70° - \tan^2 20°}$

$$= \frac{\sec^2\theta - \tan^2\theta}{2(\cos^2 48° + \cos^2(90° - 48°))} - \frac{2[\tan^2 30° \sec^2 52° \sin^2(90° - 52°)]}{\operatorname{cosec}^2 70° - \tan^2(90° - 70°)}$$

$$= \frac{1}{2[\cos^2 48° + \sin^2 48°]} - \frac{2\left[\left(\frac{1}{\sqrt{3}}\right)^2 \sec^2 52° \cos^2 52°\right]}{\operatorname{cosec}^2 70° - \cot^2 70}$$

$$\left\{\begin{array}{l} \because \sin^2\theta + \cos^2\theta = 1 \\ \sec^2\theta - \tan^2\theta = 1 \\ \operatorname{cosec}^2\theta - \cot^2\theta = 1 \end{array}\right\}$$

$$= \frac{1}{2 \times 1} - \frac{2\left[\frac{1}{3} \times 1\right]}{1}$$

$$= \frac{1}{2} - \frac{2}{3} = \frac{3-4}{6} = \frac{-1}{6}$$

**11. Prove that following:**

**(i) cos θ sin (90° – θ) + sin θ cos (90° – θ) = 1**

**(ii) tan θ/ tan (90° – θ) + sin (90° – θ)/ cos θ = sec$^2$ θ**

**(iii) (cos (90° – θ) cos θ)/ tan θ + cos$^2$ (90° – θ) = 1**

**(iv) sin (90° – θ) cos (90° – θ) = tan θ/ (1 + tan$^2$ θ)**

**Solution:**

(i) L.H.S. = cos θ sin (90° – θ) + sin θ cos (90° – θ)

= cos θ x cos θ + sin θ x sin θ

= cos$^2$ θ + sin$^2$ θ

= 1 = R.H.S.

(ii) L.H.S = tan θ/ tan (90° – θ) + sin (90° – θ)/ cos θ

= tan θ/ cot θ + cos θ/ cos θ

= tan θ/ (1/tan θ) + 1

= $\tan^2\theta + 1 = \sec^2\theta$ = R.H.S.

(iii) L.H.S. = (cos (90° – θ) cos θ)/ tan θ + $\cos^2 (90° – θ)$

= (sin θ cos θ)/ tan θ + $\sin^2\theta$

= (sin θ cos θ)/ (sin θ/ cos θ) + $\sin^2\theta$

= $\cos^2\theta + \sin^2\theta$

= 1 = R.H.S.

(iv) $\sin(90° - \theta)\cos(90° - \theta) = \dfrac{\tan\theta}{1+\tan^2\theta}$

$$\text{L.H.S.} = \sin(90° - \theta)\cos(90° - \theta) \qquad \left\{\begin{array}{l}\because \sin(90° - \theta) - \cos\theta \\ \sin^2\theta + \cos^2\theta = 1\end{array}\right\}$$

$$= \cos\theta\sin\theta$$

$$\text{R.H.S.} = \frac{\tan\theta}{1+\tan^2\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}} = \frac{\sin\theta}{\cos\theta} \times \cos^2\theta = \sin\theta\cos\theta$$

$\therefore$ L.H.S. = R.H.S.

**Prove that following (12 to 30) identities, where the angles involved are acute angles for which the trigonometric ratios as defined:**

**12. (i) (sec A + tan A) (1 – sin A) = cos A**

**(ii) (1 + $\tan^2$ A) (1 – sin A) (1 + sin A) = 1.**

**Solution:**

Given,

(i) L.H.S = (sec A + tan A) (1 – sin A)

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{(1 + \sin A) \times (1 - \sin A)}{\cos A} = \frac{1 - \sin^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\cos A} = \cos A = \text{R.H.S.}$$

$$(1 - \sin^2 A = \cos^2 A)$$

(ii) L.H.S. = (1 + $\tan^2$ A) (1 – sin A) (1 + sin A)

$$= \left(1 + \frac{\sin^2 A}{\cos^2 A}\right)(1 - \sin^2 A)$$

$$= \frac{\cos^2 A + \sin^2 A}{\cos^2 A} \times \cos^2 A \quad \left\{\begin{array}{l} \because 1 - \sin^2 A = \cos^2 A \\ \sin^2 A + \cos^2 A = 1 \end{array}\right\}$$

$$= \frac{1}{\cos^2 A} \times \cos^2 A = 1 = \text{R.H.S.}$$

**13. (i) tan A + cot A = sec A cosec A**

**(ii) (1 – cos A) (1 + sec A) = tan A sin A.**

**Solution:**

(i) L.H.S. = tan A + cot A

= sin A/cos A + cos A/sin A

= ($\sin^2$ A + $\cos^2$ A)/ (sin A cos A)

= 1/ (sin A cos A)

= sec A cosec A

= R.H.S

(ii) L.H.S. = (1 – cos A) (1 + sec A)

$$= (1 - \cos A)\left(1 + \frac{1}{\cos A}\right)$$

$$= (1 - \cos A)\frac{(\cos A + 1)}{\cos A}$$

$$= \frac{(1 - \cos A)(1 + \cos A)}{\cos A} = \frac{1 - \cos^2 A}{\cos A} = \frac{\sin^2 A}{\cos A}$$

$$= \frac{\sin^2 A}{\cos A} = \sin A \times \frac{\sin A}{\cos A}$$

$$\{1 - \cos^2 A = \sin^2 A\}$$

$$= \tan A \sin A = \text{R.H.S.}$$

**14. (i) 1/ (1 + cos A) + 1/ (1 – cos A) = 2 cosec$^2$ A**

**(ii) 1/(sec A + tan A) + 1/(sec A – tan A) = 2 sec A**

**Solution:**

(i) L.H.S. = 1/ (1 + cos A) + 1/ (1 – cos A)

$$= \frac{1 - \cos A + 1 + \cos A}{(1 + \cos A)(1 - \cos A)}$$

$$= \frac{2}{1 - \cos^2 A} = \frac{2}{\sin^2 A}$$

$$(\because 1 - \cos^2 A = \sin^2 A)$$

$$= 2 \operatorname{cosec}^2 A = \text{R.H.S.}$$

(ii)

$$\text{L.H.S.} = \frac{1}{\sec A + \tan A} + \frac{1}{\sec A - \tan A}$$

$$= \frac{\sec A - \tan A + \sec A + \tan A}{(\sec A + \tan A)(\sec A - \tan A)}$$

$$= \frac{2 \sec A}{\sec^2 A - \tan^2 A} = \frac{2 \sec A}{1}$$

$$(\because \sec^2 A - \tan^2 A = 1)$$

$$= 2 \sec A = \text{R.H.S.}$$

**15. (i) sin A/ (1 + cos A) = (1 – cos A)/ sin A**

**(ii) (1 – tan$^2$ A)/ (cot$^2$ A – 1) = tan$^2$ A**

**(iii) sin A/ (1 + cos A) = cosec A – cot A**

**Solution:**

(i) L.H.S. = sin A/ (1 + cos A)

On multiplying and dividing by (1 – cos A), we have

$$= \frac{\sin A(1-\cos A)}{1-\cos^2 A} = \frac{\sin A(1-\cos A)}{\sin^2 A}$$

$$(\because 1-\cos^2 A = \sin^2 A)$$

$$= \frac{1-\cos A}{\sin A} = \text{R.H.S.}$$

(ii) $$\text{L.H.S.} = \frac{1-\tan^2 A}{\cot^2 A - 1} = \frac{1-\frac{\sin^2 A}{\cos^2 A}}{\frac{\cos^2 A}{\sin^2 A}-1}$$

$$= \frac{\frac{\cos^2 A-\sin^2 A}{\cos^2 A}}{\frac{\cos^2 A-\sin^2 A}{\sin^2 A}} = \frac{\cos^2 A-\sin^2 A}{\cos^2 A} \times \frac{\sin^2 A}{\cos^2 A-\sin^2 A}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A = \text{R.H.S.}$$

(iii) $$\text{R.H.S.} = \operatorname{cosec} A - \cot A$$

$$= \frac{1}{\sin A} - \frac{\cos A}{\sin A} = \frac{1-\cos A}{\sin A}$$

$$= \frac{(1-\cos A)(1+\cos A)}{\sin A(1+\cos A)} \quad \{\text{Multiplying and dividing by } 1+\cos A\}$$

$$= \frac{1-\cos^2 A}{\sin A(1+\cos A)} = \frac{\sin^2 A}{\sin A(1+\cos A)} \quad \{\because 1-\cos^2 A = \sin^2 A\}$$

$$= \frac{\sin A}{1+\cos A} = \text{L.H.S.}$$

**16. (i) (sec A – 1)/(sec A + 1) = (1 – cos A)/(1 + cos A)**

**(ii) $\tan^2 \theta$/ $(\sec \theta - 1)^2$ = (1 + cos θ)/ (1 – cos θ)**

**(iii) $(1 + \tan A)^2 + (1 - \tan A)^2 = 2 \sec^2 A$**

**(iv) $\sec^2 A + \operatorname{cosec}^2 A = \sec^2 A . \operatorname{cosec}^2 A$**

**Solution:**

$$= \frac{\frac{1-\cos A}{\cos A}}{\frac{1+\cos A}{\cos A}} = \frac{1-\cos A}{\cos A} \times \frac{\cos A}{1+\cos A}$$

$$= \frac{1-\cos A}{1+\cos A} = \text{R.H.S.}$$

$$(ii)\ \text{LHS} = \frac{\tan^2\theta}{(\sec\theta - 1)^2} = \frac{\tan^2\theta}{\sec^2\theta + 1 - 2\sec\theta}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta} + 1 - \frac{2}{\cos\theta}} = \frac{\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{1 + \cos^2\theta - 2\cos\theta}$$

$$= \frac{\sin^2\theta}{(1-\cos\theta)^2} = \frac{(1-\cos^2\theta)}{(1-\cos\theta)^2} = \frac{(1+\cos\theta)(1-\cos\theta)}{(1-\cos\theta)^2}$$

$$= \frac{1+\cos\theta}{1-\cos\theta} = \text{R.H.S. Proved.}$$

(iii) L.H.S. = $(1 + \tan A)^2 + (1 - \tan A)^2$

$= 1 + 2 \tan A + \tan^2 A + 1 - 2 \tan A + \tan^2 A$

$= 2 + 2 \tan^2 A$

$= 2(1 + \tan^2 A)$ [As $1 + \tan^2 A = \sec^2 A$]

$= 2 \sec^2 A$

= R.H.S.

(iv) L.H.S = $\sec^2 A + \text{cosec}^2 A$

$= 1/\cos^2 A + 1/\sin^2 A$

$= (\sin^2 A + \cos^2 A)/ (\sin^2 A \cos^2 A)$

$= 1/ (\sin^2 A \cos^2 A)$

$= \sec^2 A \,\text{cosec}^2 A$ = R.H.S

**17. (i) (1 + sin A)/ cos A + cos A/ (1 + sin A) = 2 sec A**

**(ii) tan A/ (sec A – 1) + tan A/ (sec A + 1) = 2cosec A**

**Solution:**

(i) L.H.S. = (1 + sin A)/ cos A + cos A/ (1 + sin A)

$$= \frac{(1+\sin A)(1+\sin A)+\cos^2 A}{\cos A\,(1+\sin A)}$$

$$= \frac{1+\sin A+\sin A+\sin^2 A+\cos^2 A}{\cos A\,(1+\sin A)}$$

$$= \frac{1+2\sin A+1}{\cos A\,(1+\sin A)} = \frac{2+2\sin A}{\cos A\,(1+\sin A)}$$

$$= \frac{2(1+\sin A)}{\cos A\,(1+\sin A)} = \frac{2}{\cos A} = 2\sec A$$

$= \text{R.H.S.}$

(*ii*) $$\frac{\tan A}{\sec A-1}+\frac{\tan A}{\sec A+1} = 2\operatorname{cosec} A$$

$$\text{L.H.S.} = \frac{\tan A}{\sec A-1}+\frac{\tan A}{\sec A+1}$$

$$= \tan A\left(\frac{1}{\sec A-1}+\frac{1}{\sec A+1}\right)$$

$$= \tan A\left(\frac{\sec A+1+\sec A-1}{(\sec A-1)(\sec A+1)}\right)$$

$$= \frac{\tan A\times 2\sec A}{\sec^2 A-1} = \frac{2\sec A\tan A}{\tan^2 A} = \frac{2\sec A}{\tan A}$$



$$= \frac{2\times 1\times\cos A}{\cos A\times\sin A}$$



$$= \frac{2}{\sin A} = 2\operatorname{cosec} A = \text{R.H.S.}$$

**18. (i) cosec A/ (cosec A – 1) + cosec A/ (cosec A + 1) = 2 sec $^2$ A**

**(ii) cot A – tan A = (2cos $^2$ A – 1)/ (sin A – cos A)**

**(iii) (cot A – 1)/ (2 – sec $^2$ A) = cot A/ (1 + tan A)**

**Solution:**

(i) L.H.S. =

$$\frac{cosecA}{cosecA-1}+\frac{cosecA}{cosecA+1}$$

$$= \operatorname{cosec} A\left[\frac{1}{\operatorname{cosec} A - 1} + \frac{1}{\operatorname{cosec} A + 1}\right]$$

$$= \operatorname{cosec} A\left[\frac{\operatorname{cosec} A + 1 + \operatorname{cosec} A - 1}{(\operatorname{cosec} A - 1)(\operatorname{cosec} A + 1)}\right]$$

$$= \frac{\operatorname{cosec} A \times 2\operatorname{cosec} A}{\operatorname{cosec}^2 A - 1} = \frac{2\operatorname{cosec}^2 A}{\cot^2 A}$$

$$= \frac{2 \times \sin^2 A}{\sin^2 A \times \cos^2 A} = \frac{2}{\cos^2 A}$$

$$= 2\sec^2 A = \text{R.H.S.}$$

(*ii*) L.H.S. = $\cot A - \tan A$

$$= \frac{\cos A}{\sin A} - \frac{\sin A}{\cos A} = \frac{\cos^2 A - \sin^2 A}{\sin A \cos A}$$

$$= \frac{\cos^2 A - (1 - \cos^2 A)}{\sin A \cos A}$$

$$= \frac{\cos^2 A - 1 + \cos^2 A}{\sin A \cos A}$$

$$= \frac{2\cos^2 A - 1}{\sin A \cos A} = \text{R.H.S.}$$

(*iii*) $$\text{L.H.S.} = \frac{\cot A - 1}{2 - \sec^2 A} = \frac{\frac{\cos A}{\sin A} - 1}{2 - \frac{1}{\cos^2 A}}$$





$$= \frac{\frac{\cos A - \sin A}{\sin A}}{\frac{2\cos^2 A - 1}{\ldots}}$$

$$\text{R.H.S.} = \frac{\cot A}{1 + \tan A} = \frac{\frac{\cos A}{\sin A}}{1 + \frac{\sin A}{\cos A}}$$

$$= \frac{\frac{\cos A}{\sin A}}{\frac{\cos A + \sin A}{\cos A}}$$

$$= \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A + \sin A}$$

$$= \frac{\cos^2 A}{\sin A(\cos A + \sin A)}$$

$\therefore$ L.H.S. = R.H.S.

**19. (i) $\tan^2 \theta - \sin^2 \theta = \tan^2 \theta \sin^2 \theta$**

**(ii) $\cos \theta / (1 - \tan \theta) - \sin^2 \theta / (\cos \theta - \sin \theta) = \cos \theta + \sin \theta$**

**Solution:**

(i) L.H.S = $\tan^2 \theta - \sin^2 \theta$

$$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$$

$$= \frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta} = \sin^2\theta \times \frac{\sin^2\theta}{\cos^2\theta}$$

$$\sin^2\theta \times \tan^2\theta = \tan^2\theta\sin^2\theta = \text{R.H.S.}$$

(*ii*) $$\frac{\cos\theta}{1-\tan\theta} - \frac{\sin^2\theta}{\cos\theta - \sin\theta} = \cos\theta + \sin\theta$$

$$\text{L.H.S.} = \frac{\cos\theta}{1-\tan\theta} - \frac{\sin^2\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}} - \frac{\sin^2\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\cos\theta}{\frac{\cos\theta-\sin\theta}{\cos\theta}} - \frac{\sin^2\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta-\sin\theta} - \frac{\sin^2\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta}$$



$$= \frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{\cos\theta-\sin\theta}$$



$$= \cos\theta + \sin\theta = \text{R.H.S.}$$

$$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$$

$$= \frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta} = \sin^2\theta \times \frac{\sin^2\theta}{\cos^2\theta}$$

$$\sin^2\theta \times \tan^2\theta = \tan^2\theta\sin^2\theta = \text{R.H.S.}$$

(*ii*) $\dfrac{\cos\theta}{1-\tan\theta} - \dfrac{\sin^2\theta}{\cos\theta-\sin\theta} = \cos\theta + \sin\theta$

$$\text{L.H.S.} = \frac{\cos\theta}{1-\tan\theta} - \frac{\sin^2\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\cos\theta}{1-\dfrac{\sin\theta}{\cos\theta}} - \frac{\sin^2\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\cos\theta}{\dfrac{\cos\theta-\sin\theta}{\cos\theta}} - \frac{\sin^2\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta-\sin\theta} - \frac{\sin^2\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta}$$



$$= \frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{\cos\theta-\sin\theta}$$



$$= \cos\theta + \sin\theta = \text{R.H.S.}$$

**20. (i) $\text{cosec}^4 \theta - \text{cosec}^2 \theta = \cot^4 \theta + \cot^2 \theta$**

**(ii) $2 \sec^2 \theta - \sec^4 \theta - 2 \text{cosec}^2 \theta + \text{cosec}^4 \theta = \cot^4 \theta - \tan^4 \theta$.**

**Solution:**

(i) L.H.S. = $\text{cosec}^4 \theta - \text{cosec}^2 \theta$

$= \text{cosec}^2 \theta (\text{cosec}^2 \theta - 1)$

$= \text{cosec}^2 \theta \cot^2 \theta$ $[\text{cosec}^2 \theta - 1 = \cot^2 \theta]$

$= (\cot^2 \theta + 1) \cot^2 \theta$

$= \cot^4 \theta + \cot^2 \theta$

= R.H.S.

(ii) L.H.S. = $2 \sec^2 \theta - \sec^4 \theta - 2 \text{cosec}^2 \theta + \text{cosec}^4 \theta$

$= 2 (\tan^2 \theta + 1) - (\tan^2 \theta + 1)^2 - 2 (1 + \cot^2 \theta) + (1 + \cot^2 \theta)^2$

$$\left\{\begin{array}{l} \because \sec^2 \theta = \tan^2 \theta + 1 \\ \quad \text{cosec}^2 \theta = 1 + \cot^2 \theta \end{array}\right\}$$

$= 2 \tan^2 \theta + 2 - (\tan^4 \theta + 2 \tan^2 \theta + 1) - 2 - 2 \cot^2 \theta + (1 + 2 \cot^2 \theta + \cot^4 \theta)$

$= 2 \tan^2 \theta + 2 - \tan^4 \theta - 2 \tan^2 \theta - 1 - 2 - 2 \cot^2 \theta + 1 + 2 \cot^2 \theta + \cot^4 \theta$

$= \cot^4 \theta - \tan^4 \theta$ = R.H.S.

**21. (i)  = $\cot \theta$**

**(ii) $(\tan^3 \theta - 1)/ (\tan \theta - 1) = \sec^2 \theta + \tan \theta$**

**Solution:**

(i) L.H.S. $= \dfrac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)} = \dfrac{\cos\theta+\cos^2\theta}{\sin\theta(1+\cos\theta)}$ $\{\because 1-\sin^2\theta=\cos^2\theta\}$

$= \dfrac{\cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)} = \dfrac{\cos\theta}{\sin\theta} = \cot\theta$

= R.H.S.

(*ii*) $\dfrac{\tan^3\theta-1}{\tan\theta-1} = \sec^2\theta+\tan\theta$

L.H.S. $= \dfrac{(\tan\theta-1)}{\tan\theta-1}(\tan^2\theta+\tan\theta+1)$ $\{\because a^3-b^3=(a-b)(a^2+ab+b^2)\}$

$= \tan^2\theta+\tan\theta+1 = \tan^2\theta+1+\tan\theta$

$= \sec^2\theta+\tan\theta$ $\{\because \sec^2\theta=\tan^2\theta+1\}$

= R.H.S.

(i) L.H.S. $= \dfrac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)} = \dfrac{\cos\theta+\cos^2\theta}{\sin\theta(1+\cos\theta)}$ $\{\because 1-\sin^2\theta=\cos^2\theta\}$

$= \dfrac{\cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)} = \dfrac{\cos\theta}{\sin\theta} = \cot\theta$

= R.H.S.

(*ii*) $\dfrac{\tan^3\theta-1}{\tan\theta-1} = \sec^2\theta+\tan\theta$

L.H.S. $= \dfrac{(\tan\theta-1)}{\tan\theta-1}(\tan^2\theta+\tan\theta+1)$ $\{\because a^3-b^3=(a-b)(a^2+ab+b^2)\}$

$= \tan^2\theta+\tan\theta+1 = \tan^2\theta+1+\tan\theta$

$= \sec^2\theta+\tan\theta$ $\{\because \sec^2\theta=\tan^2\theta+1\}$

= R.H.S.

**22. (i) (1 + cosec A)/ cosec A = cos$^2$ A/ (1 – sin A)**

**(ii)**

**Solution:**

(i) $\frac{1+cosecA}{cosecA} = \frac{cos^2 A}{1-sin A}$

L.H.S $= \frac{1+cosecA}{cosecA}$

$$\text{L.H.S.} = \frac{1+\operatorname{cosec} A}{\operatorname{cosec} A} = \frac{1+\frac{1}{\sin A}}{\frac{1}{\sin A}} = \frac{\sin A+1}{\sin A} \times \frac{\sin A}{1}$$

$= \sin A + 1$

$$\text{R.H.S.} = \frac{\cos^2 A}{1-\sin A} = \frac{1-\sin^2 A}{1-\sin A}$$

$$= \frac{(1+\sin A)(1-\sin A)}{1-\sin A} = 1 + \sin A = \sin A + 1$$

$\therefore$ L.H.S. = R.H.S.

(ii) $\sqrt{\frac{1-\cos A}{1+\cos A}} = \operatorname{cosec} A - \cot A$

$$\text{L.H.S.} = \sqrt{\frac{1-\cos A}{1+\cos A}}$$

Rationalising the denominator

$$= \sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{1-\cos^2 A}} = \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}$$

$$= \frac{1-\cos A}{\sin A} = \frac{1}{\sin A} - \frac{\cos A}{\sin A}$$

$= \operatorname{cosec} A - \cot A =$ R.H.S.

(i) $\frac{1+cosecA}{cosecA} = \frac{cos^2 A}{1-sin A}$

L.H.S $= \frac{1+cosecA}{cosecA}$

$$\text{L.H.S.} = \frac{1+\operatorname{cosec} A}{\operatorname{cosec} A} = \frac{1+\frac{1}{\sin A}}{\frac{1}{\sin A}} = \frac{\sin A+1}{\sin A} \times \frac{\sin A}{1}$$

$= \sin A + 1$

$$\text{R.H.S.} = \frac{\cos^2 A}{1-\sin A} = \frac{1-\sin^2 A}{1-\sin A}$$

$$= \frac{(1+\sin A)(1-\sin A)}{1-\sin A} = 1 + \sin A = \sin A + 1$$

$\therefore$ L.H.S. = R.H.S.

(ii) $\sqrt{\frac{1-\cos A}{1+\cos A}} = \operatorname{cosec} A - \cot A$

$$\text{L.H.S.} = \sqrt{\frac{1-\cos A}{1+\cos A}}$$

Rationalising the denominator

$$= \sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{1-\cos^2 A}} = \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}$$

$$= \frac{1-\cos A}{\sin A} = \frac{1}{\sin A} - \frac{\cos A}{\sin A}$$

$= \operatorname{cosec} A - \cot A =$ R.H.S.

**23. (i)  = tan A + sec A**

**(ii)  = cosec A – cot A**

**Solution:**

(i) $\sqrt{\frac{1+sinA}{1-sinA}} = tanA + secA$

L.H.S $= \sqrt{\frac{1+sinA}{1-sinA}}$

L.H.S. $= \sqrt{\frac{1+\sin A}{1-\sin A}}$

Rationalising the denominator,

$$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}} = \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$= \sec A + \tan A = \tan A + \sec A =$ R.H.S.

(*ii*) $\sqrt{\frac{1-\cos A}{1+\cos A}} = \operatorname{cosec} A - \cot A$

L.H.S. $= \sqrt{\frac{1-\cos A}{1+\cos A}}$

Rationalising the denominator,

$$= \sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}}$$





$$= \sqrt{\frac{(1-\cos A)^2}{1-\cos^2 A}} = \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}} = \frac{1-\cos A}{\sin A}$$

(i) $\sqrt{\frac{1+sinA}{1-sinA}} = tanA + secA$

L.H.S $= \sqrt{\frac{1+sinA}{1-sinA}}$

L.H.S. $= \sqrt{\frac{1+\sin A}{1-\sin A}}$

Rationalising the denominator,

$$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}} = \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$= \sec A + \tan A = \tan A + \sec A =$ R.H.S.

(*ii*) $\sqrt{\frac{1-\cos A}{1+\cos A}} = \operatorname{cosec} A - \cot A$

L.H.S. $= \sqrt{\frac{1-\cos A}{1+\cos A}}$

Rationalising the denominator,

$$= \sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}}$$





$$= \sqrt{\frac{(1-\cos A)^2}{1-\cos^2 A}} = \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}} = \frac{1-\cos A}{\sin A}$$

**24. (i) = 2 cosec A**

**(ii) cos A cot A/ (1 – sin A) = 1 + cosec A**

**Solution:**

(i) $\sqrt{\frac{secA-1}{secA+1}}+\sqrt{\frac{secA+1}{secA-1}}=2cosecA$

L.H.S $=\sqrt{\frac{secA-1}{secA+1}}+\sqrt{\frac{secA+1}{secA-1}}$

$$\text{L.H.S.} = \sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}}$$

$$= \frac{\sqrt{\sec A - 1}}{\sqrt{\sec A + 1}} + \frac{\sqrt{\sec A + 1}}{\sqrt{\sec A - 1}}$$

$$= \frac{\sec A - 1 + \sec A + 1}{\sqrt{(\sec A + 1)(\sec A - 1)}} = \frac{2\sec A}{\sqrt{\sec^2 A - 1}}$$

$\{\because \sec^2 A - 1 = \tan^2 A\}$

$$= \frac{2\sec A}{\sqrt{\tan^2 A}} = \frac{2\sec A}{\tan A}$$

$$= \frac{2 \times \cos A}{\cos A \times \sin A} = \frac{2}{\sin A}$$

$= 2 \text{ cosec } A = \text{R.H.S.}$

(ii) $\frac{\cos A \cot A}{1 - \sin A} = 1 + \text{cosec } A$

$$\text{L.H.S.} = \frac{\cos A \cot A}{1 - \sin A} = \frac{\cos A \cos A}{\sin A(1 - \sin A)} \quad \left\{\cos A = \frac{\cos A}{\sin A}\right\}$$

$$= \frac{\cos^2 A}{\sin A(1 - \sin A)} = \frac{1 - \sin^2 A}{\sin A(1 - \sin A)}$$

$\{\because \cos^2 A = 1 - \sin^2 A\}$

$$= \frac{(1 + \sin A)(1 - \sin A)}{\sin A(1 - \sin A)} = \frac{1 + \sin A}{\sin A}$$



$$= \frac{1}{\sin A} + \frac{\sin A}{\sin A} = \text{cosec } A + 1$$

(i) $\sqrt{\frac{secA-1}{secA+1}}+\sqrt{\frac{secA+1}{secA-1}}=2cosecA$

L.H.S $=\sqrt{\frac{secA-1}{secA+1}}+\sqrt{\frac{secA+1}{secA-1}}$

$$\text{L.H.S.} = \sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}}$$

$$= \frac{\sqrt{\sec A - 1}}{\sqrt{\sec A + 1}} + \frac{\sqrt{\sec A + 1}}{\sqrt{\sec A - 1}}$$

$$= \frac{\sec A - 1 + \sec A + 1}{\sqrt{(\sec A + 1)(\sec A - 1)}} = \frac{2\sec A}{\sqrt{\sec^2 A - 1}}$$

$\{\because \sec^2 A - 1 = \tan^2 A\}$

$$= \frac{2\sec A}{\sqrt{\tan^2 A}} = \frac{2\sec A}{\tan A}$$

$$= \frac{2 \times \cos A}{\cos A \times \sin A} = \frac{2}{\sin A}$$

$= 2 \text{ cosec } A = \text{R.H.S.}$

(*ii*) $\frac{\cos A \cot A}{1 - \sin A} = 1 + \text{cosec } A$

$$\text{L.H.S.} = \frac{\cos A \cot A}{1 - \sin A} = \frac{\cos A \cos A}{\sin A(1 - \sin A)} \quad \left\{\cos A = \frac{\cos A}{\sin A}\right\}$$

$$= \frac{\cos^2 A}{\sin A(1 - \sin A)} = \frac{1 - \sin^2 A}{\sin A(1 - \sin A)}$$

$\{\because \cos^2 A = 1 - \sin^2 A\}$

$$\frac{(1 + \sin A)(1 - \sin A)}{\sin A(1 - \sin A)} = \frac{1 + \sin A}{\sin A}$$



$$= \frac{1}{\sin A} + \frac{\sin A}{\sin A} = \text{cosec } A + 1$$

**25. (i) (1 + tan A)/ sin A + (1 + cot A)/ cos A = 2 (sec A + cosec A)**

**(ii) sec⁴ A – tan⁴ A = 1 + 2 tan² A**

**Solution:**

(i) $\frac{1+tan A}{sin A} + \frac{1+cot A}{cos A} = 2(secA + cosecA)$

L.H.S $= \frac{1+tan A}{sin A} + \frac{1+cot A}{cos A}$

$$= \frac{1+\frac{\sin A}{\cos A}}{\sin A} + \frac{1+\frac{\cos A}{\sin A}}{\cos A}$$

$$= \frac{\cos A + \sin A}{\cos A \times \sin A} + \frac{\sin A + \cos A}{\cos A \times \sin A}$$

$$= 2\left[\frac{\cos A + \sin A}{\cos A \sin A}\right]$$

$$= 2\left[\frac{\cos A}{\cos A \sin A} + \frac{\sin A}{\cos A \sin A}\right]$$

$$= 2\left[\frac{1}{\sin A} + \frac{1}{\cos A}\right]$$

$= 2 (\text{cosec } A + \sec A)$

$= 2 (\sec A + \text{cosec } A) = \text{R.H.S.}$

(ii) $\sec^4 A - \tan^4 A = 1 + 2 \tan^2 A$

L.H.S. $= \sec^4 A - \tan^4 A$

$= (\sec^2 A - \tan^2 A)(\sec^2 A + \tan^2 A)$

$= (1 + \tan^4 A - \tan^4 A)(1 + \tan^4 A + \tan^4 A)$ [As $\sec^2 A = \tan^4 A + 1$]

$= 1(1 + 2\tan^2 A)$

$= 1 + 2\tan^2 A$ = R.H.S.

**26. (i) $\operatorname{cosec}^6 A - \cot^6 A = 3\cot^2 A \operatorname{cosec}^2 A + 1$**

**(ii) $\sec^6 A - \tan^6 A = 1 + 3\tan^2 A + 3\tan^4 A$**

**Solution:**

(i) $\operatorname{cosec}^6 A - \cot^6 A = 3\cot^2 A \operatorname{cosec}^2 A + 1$

$$\begin{aligned}
\text{L.H.S} &= \operatorname{cosec}^6 A - \cot^6 A \\
&= (\operatorname{cosec}^2 A)^3 - (\cot^2 A)^3 \\
&= (\operatorname{cosec}^2 \theta - \cot^2 A)^3 + 3\operatorname{cosec}^2 A \cot^2 A (\operatorname{cosec}^2 A - \cot^2 A) \\
&= (1)^3 + 3\operatorname{cosec}^2 A \cot^2 A \times 1 \\
&= 1 + 3\cot^2 A \operatorname{cosec}^2 A \\
&= 3\cot^2 A \operatorname{cosec}^2 A + 1 = \text{R.H.S.}
\end{aligned}$$

(*ii*) $\sec^6 A - \tan^6 A = 1 + 3\tan^2 A + 3\tan^4 A$

$$\begin{aligned}
\text{L.H.S.} &= \sec^6 A - \tan^6 A \\
&= (\sec^2 A)^3 - (\tan^2 A)^3 \\
&= (\sec^2 A - \tan^2 A)^3 + 3\sec^2 A \tan^2 A (\sec^2 A - \tan^2 A) \\
&= (1)^3 + 3\sec^2 A \tan^2 A \times 1 \\
&= 1 + 3\sec^2 A \tan^2 A \\
&= 1 + 3[(1 + \tan^2 A)(\tan^2 A)] \\
&= 1 + 3[\tan^2 A + \tan^4 A] \\
&= 1 + 3\tan^2 A + 3\tan^4 A = \text{R.H.S.}
\end{aligned}$$

**27.**

(i) $\frac{cot\theta - cosec\theta - 1}{cot\theta - cosec\theta + 1} = \frac{1 + cos\theta}{sin\theta}$

(ii) $\frac{sin\theta}{cot\theta + cosec\theta} = 2 + \frac{sin\theta}{cot\theta - cosec\theta}$

**Solution:**

(i)

$$L.H.S = \frac{cot\theta - cosec\theta - 1}{cot\theta - cosec\theta + 1}$$

$$= \frac{\dfrac{\cos\theta}{\sin\theta} + \dfrac{1}{\sin\theta} - 1}{\dfrac{\cos\theta}{\sin\theta} - \dfrac{1}{\sin\theta} + 1}$$

$$= \frac{\cos\theta + 1 - \sin\theta}{\sin\theta} \times \frac{\sin\theta}{\cos\theta - 1 + \sin\theta}$$

$$= \frac{\cos\theta + 1 - \sin\theta}{\cos\theta - 1 + \sin\theta}$$

$$= \frac{\cos\theta + (1 - \sin\theta)}{\cos\theta - (1 - \sin\theta)}$$

$$= \frac{[\cos\theta + (1 - \sin\theta)][\cos\theta + (1 - \sin\theta)]}{[\cos\theta - (1 - \sin\theta)][\cos\theta + (1 - \sin\theta)]}$$

$$= \frac{[\cos\theta + (1 - \sin\theta)]^2}{\cos^2\theta - (1 - \sin\theta)^2}$$

$$= \frac{(\cos\theta + 1 - \sin\theta)^2}{\cos^2\theta - (1 + \sin^2\theta - 2\sin\theta)}$$

$$= \frac{\cos^2\theta + \sin^2\theta + 1 + 2\cos\theta - 2\sin\theta - 2\sin\theta\cos\theta}{\cos^2\theta - 1 - \sin^2\theta + 2\sin\theta}$$

$$= \frac{1 + 1 + 2\cos\theta - 2\sin\theta - 2\sin\theta\cos\theta}{1 - \sin^2\theta - 1 - \sin^2\theta + 2\sin\theta}$$

$$= \frac{2 + 2\cos\theta - 2\sin\theta - 2\sin\theta\cos\theta}{2\sin\theta - 2\sin^2\theta}$$

$$= \frac{2(1 + \cos\theta) - 2\sin\theta(1 + \cos\theta)}{2\sin\theta(1 - \sin\theta)}$$



$$= \frac{(1 + \cos\theta)2(1 - \sin\theta)}{}$$

(ii) $$\frac{\sin\theta}{\cot\theta+\operatorname{cosec}\theta}=2+\frac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta}$$

$$\text{L.H.S.}=\frac{\sin\theta}{\frac{\cos\theta}{\sin\theta}+\sin\theta}=\frac{\sin\theta}{\frac{\cos\theta+1}{\sin\theta}}$$

$$=\frac{\sin^2\theta}{1+\cos\theta}$$

$$=\frac{1-\cos^2\theta}{1+\cos\theta}=\frac{(1+\cos\theta)(1-\cos\theta)}{1+\cos\theta}$$

$$=1-\cos\theta$$

$$\text{R.H.S.}=2+\frac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta}$$

$$=2+\frac{\sin\theta}{\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}}$$

$$=2+\frac{\sin^2\theta}{\cos\theta-1}$$

$$=\frac{2\cos\theta-2+\sin^2\theta}{\cos\theta-1}$$

$$=\frac{2\cos\theta-2+(1-\cos^2\theta)}{\cos\theta-1}$$

$$=\frac{2(\cos\theta-1)+(1+\cos\theta)(1-\cos\theta)}{\cos\theta-1}$$

= (cos θ – 1) (2 – 1 – cos θ)/ (cos θ – 1)

= 1 – cos θ

Hence, L.H.S = R.H.S.

**28. (i) (sin θ + cos θ) (sec θ + cosec θ) = 2 + sec θ cosec θ**

**(ii) (cosec A – sin A) (sec A – cos A) sec $^2$A = tan A**

**Solution:**

(i) (sin θ + cos θ) (sec θ + cosec θ) = 2 + sec θ cosec θ

L.H.S. = (sin θ + cos θ) (sec θ + cosec θ)

$$= (\sin\theta + \cos\theta)\left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)$$

$$= \frac{(\sin\theta + \cos\theta)(\sin\theta + \cos\theta)}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta + \sin\theta\cos\theta + \sin\theta\cos\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1 + 2\sin\theta\cos\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} + \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \operatorname{cosec}\theta\sec\theta + 2$$

$$= 2 + \sec\theta\operatorname{cosec}\theta.$$

= R.H.S.

(ii)

L.H.S = (cosec A – sin A) (sec A – cos A) $\sec^2 A$

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\frac{1}{\cos^2 A}$$

$$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)\frac{1}{\cos^2 A}$$

$$= \frac{\cos^2 A}{\sin A}\cdot\frac{\sin^2 A}{\cos A}\cdot\frac{1}{\cos^2 A} = \frac{\sin A}{\cos A} = \tan A$$

= R.H.S.

**29.**

(i) $\frac{sin^3 A+cos^3 A}{sin A+cos A} + \frac{sin^3 A-cos^3 A}{sin A-cos A} = 2$

(ii) $\frac{tan^2 A}{1+tan^2 A} + \frac{cot^2 A}{1+cot^2 A} = 1$

**Solution:**

(i) $\frac{sin^3A+cos^3A}{sinA+cosA}+\frac{sin^3A-cos^3A}{sinA-cosA}=2$

L.H.S $=\frac{sin^3A+cos^3A}{sinA+cosA}+\frac{sin^3A-cos^3A}{sinA-cosA}$

$$=\frac{(\sin A+\cos A)(\sin^2 A-\sin A\cos A+\cos^2 A)}{(\sin A+\cos A)}+\frac{(\sin A-\cos A)(\sin^2 A+\sin A\cos A+\cos^2 A)}{(\sin A-\cos A)}$$

$= (1-\sin A\cos A)+(1+\sin A\cos A)$

$[\because \sin^2 A+\cos^2 A=1\}$

$= 1-\sin A\cos A+1+\sin A\cos A=2=$ R.H.S.

(*ii*) $\frac{\tan^2 A}{1+\tan^2 A}+\frac{\cot^2 A}{1+\cot^2 A}=1$

$$\text{L.H.S.}=\frac{\tan^2 A}{1+\tan^2 A}+\frac{\frac{1}{\tan^2 A}}{1+\frac{1}{\tan^2 A}}$$

$$=\frac{\tan^2 A}{1+\tan^2 A}+\frac{\frac{1}{\tan^2 A}}{\frac{\tan^2 A+1}{\tan^2 A}}$$

$$=\frac{\tan^2 A}{1+\tan^2 A}+\frac{\tan^2 A}{\tan^2 A(\tan^2 A+1)}$$

$$=\frac{\tan^2 A}{1+\tan^2 A}+\frac{1}{1+\tan^2 A}$$

$$=\frac{1+\tan^2 A}{1+\tan^2 A}=1=\text{R.H.S.}$$

(i) $\frac{sin^3 A+cos^3 A}{sin A+cos A} + \frac{sin^3 A-cos^3 A}{sin A-cos A} = 2$

L.H.S $= \frac{sin^3 A+cos^3 A}{sin A+cos A} + \frac{sin^3 A-cos^3 A}{sin A-cos A}$

$= \frac{(\sin A + \cos A)(\sin^2 A - \sin A \cos A + \cos^2 A)}{(\sin A + \cos A)} + \frac{(\sin A - \cos A)(\sin^2 A + \sin A \cos A + \cos^2 A)}{(\sin A - \cos A)}$

$= (1 - \sin A \cos A) + (1 + \sin A \cos A)$

$[\because \sin^2 A + \cos^2 A = 1\}$

$= 1 - \sin A \cos A + 1 + \sin A \cos A = 2 = \text{R.H.S.}$

(ii) $\frac{\tan^2 A}{1+\tan^2 A} + \frac{\cot^2 A}{1+\cot^2 A} = 1$

$$\text{L.H.S.} = \frac{\tan^2 A}{1+\tan^2 A} + \frac{\frac{1}{\tan^2 A}}{1+\frac{1}{\tan^2 A}}$$

$$= \frac{\tan^2 A}{1+\tan^2 A} + \frac{\frac{1}{\tan^2 A}}{\frac{\tan^2 A+1}{\tan^2 A}}$$

$$= \frac{\tan^2 A}{1+\tan^2 A} + \frac{\tan^2 A}{\tan^2 A(\tan^2 A+1)}$$

$$= \frac{\tan^2 A}{1+\tan^2 A} + \frac{1}{1+\tan^2 A}$$

$$= \frac{1+\tan^2 A}{1+\tan^2 A} = 1 = \text{R.H.S.}$$

**30. (i) 1/ (sec A + tan A) – 1/cos A = 1/cos A – 1/(sec A – tan A)**

**(ii) (sin A + sec A)$^2$ + (cos A + cosec A)$^2$ = (1 + sec A cosec A)$^2$**

**(iii) (tan A + sin A)/ (tan A – sin A) = (sec A + 1)/ (sec A – 1)**

**Solution:**

(i) $\frac{1}{secA+tanA} - \frac{1}{cosA} = \frac{1}{cosA} - \frac{1}{secA-tanA}$

L.H.S $= \frac{1}{secA+tanA} - \frac{1}{cosA}$

$$= \frac{1}{\frac{1}{\cos A} + \frac{\sin A}{\cos A}} - \frac{1}{\cos A}$$

$$= \frac{\cos A}{1+\sin A} - \frac{1}{\cos A}$$

$$= \frac{\cos^2 A - 1 - \sin A}{\cos A(1+\sin A)} = \frac{-\sin^2 - \sin A}{\cos A(1+\sin A)}$$

$$= \frac{-\sin A(1+\sin A)}{\cos A(1+\sin A)} = -\tan A$$

R.H.S. $= \frac{1}{\cos A} - \frac{1}{\sec A - \tan A}$

$$= \frac{1}{\cos A} - \frac{1}{\frac{1}{\cos A} - \frac{\sin A}{\cos A}}$$

$$= \frac{1}{\cos A} - \frac{\cos A}{1 - \sin A}$$

$$= \frac{1 - \sin A - \cos^2 A}{\cos A(1-\sin A)} = \frac{\sin^2 A - \sin A}{\cos A(1-\sin A)}$$

$$= \frac{-\sin A + \sin^2 A}{\cos A(1-\sin A)} = \frac{-\sin A(1-\sin A)}{\cos A(1-\sin A)}$$

$$= \frac{-\sin A}{\cos A} = -\tan A$$

$\therefore$ L.H.S. = R.H.S.

$$= \frac{1}{\cos A} - \frac{1}{\dfrac{1}{\cos A} - \dfrac{\sin A}{\cos A}}$$

$$= \frac{1}{\cos A} - \frac{\cos A}{1 - \sin A}$$

$$= \frac{1 - \sin A - \cos^2 A}{\cos A\,(1 - \sin A)} = \frac{\sin^2 A - \sin A}{\cos A\,(1 - \sin A)}$$

$$= \frac{-\sin A + \sin^2 A}{\cos A\,(1 - \sin A)} = \frac{-\sin A\,(1 - \sin A)}{\cos A\,(1 - \sin A)}$$

$$= \frac{-\sin A}{\cos A} = -\tan A$$

$\therefore$ L.H.S. = R.H.S.

(*ii*) $(\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2 = (1 + \sec A \operatorname{cosec} A)^2$

L.H.S. $= (\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2$

$$= \sin^2 A + \sec^2 A + 2\sin A \sec A + \cos^2 A + \operatorname{cosec}^2 A + 2\cos A \operatorname{cosec} A$$

$$= (\sin^2 A + \cos^2 A) + (\sec^2 A + \operatorname{cosec}^2 A) + 2\sin A \times \frac{1}{\cos A} + 2 \times \cos A \times \frac{1}{\sin A}$$

$$= 1 + \left[\frac{1}{\cos^2 A} + \frac{1}{\sin^2 A}\right] + \frac{2\sin^2 A + 2\cos^2 A}{\sin A \cos A}$$

$$= 1 + \left[\frac{\sin^2 A + \cos^2 A}{\cos^2 A \sin^2 A}\right] + \frac{2[\sin^2 A + \cos^2 A]}{\sin A \cos A}$$

$$= 1 + \frac{1}{\cos^2 A \sin^2 A} + \frac{2}{\sin A \cos A} \qquad [\because \sin^2\theta + \cos^2\theta + 1]$$

$$= \left(1 + \frac{1}{\cos A \sin A}\right)^2 \qquad [\because (a + b)^2 = a^2 + (b)^2 + 2ab]$$

$$= (1 + \operatorname{cosec} A \sec A)^2$$

= R.H.S.

(*iii*) $\dfrac{\tan A + \sin A}{\tan A - \sin A} = \dfrac{\sec A + 1}{\sec A - 1}$

$$\text{L.H.S.} = \frac{\tan A + \sin A}{\tan A - \sin A} = \frac{\dfrac{\sin A}{\cos A} + \sin A}{\dfrac{\sin A}{\cos A} - \sin A} = \frac{\dfrac{\sin A + \sin A \cos A}{\cos A}}{\dfrac{\sin A - \sin A \cos A}{\cos A}}$$

$$= \frac{\sin A\,(1 + \cos A)}{\sin A\,(1 - \cos A)} = \frac{1 + \cos A}{1 - \cos A}$$

**On dividing each term by cos A, we have**

$$= \frac{\dfrac{1}{\cos A} + 1}{\dfrac{1}{\cos A} - 1} = \frac{\sec A + 1}{\sec A - 1} = \text{R.H.S.}$$

**31. If $\sin\theta + \cos\theta = \sqrt{2}\sin(90° - \theta)$, show that $\cot\theta = \sqrt{2} + 1$**

**Solution:**

Given, $\sin\theta + \cos\theta = \sqrt{2}\sin(90° - \theta)$

$\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

On dividing by $\sin\theta$, we have

$1 + \cot\theta = \sqrt{2}\cot\theta$

$1 = \sqrt{2}\cot\theta - \cot\theta$

$(\sqrt{2} - 1)\cot\theta = 1$

$\cot\theta = 1/(\sqrt{2} - 1)$

$$= \frac{1\times(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} \quad \text{(Rationalising the denominator)}$$

$$= \frac{(\sqrt{2}+1)}{(\sqrt{2})^2-(1)^2} = \frac{\sqrt{2}+1}{2-1} = \frac{\sqrt{2}+1}{1}$$

$$= \sqrt{2} + 1 = \text{R.H.S.}$$

Hence, $\cot\theta = \sqrt{2} + 1$

**32. If $7\sin^2\theta + 3\cos^2\theta = 4$, $0° \le \theta \le 90°$, then find the value of $\theta$.**

**Solution:**

Given,

$7\sin^2\theta + 3\cos^2\theta = 4$, $0° \le \theta \le 90°$

$3\sin^2\theta + 3\cos^2\theta + 4\sin^2\theta = 4$

$3(\sin^2\theta + 3\cos^2\theta) + 4\sin^2\theta = 4$

$3(1) + 4\sin^2\theta = 4$

$4\sin^2\theta = 4 - 3$

$\sin^2 \theta = ¼$

Taking square-root on both sides, we get

$\sin \theta = ½$

Thus, $\theta = 30^\circ$

**33. If sec θ + tan θ = m and sec θ – tan θ = n, prove that mn = 1.**

**Solution:**

Given,

$\sec \theta + \tan \theta = m$

$\sec \theta - \tan \theta = n$

Now,

$mn = (\sec \theta + \tan \theta)(\sec \theta - \tan \theta)$

$= \sec^2 \theta - \tan^2 \theta = 1$

Thus, mn = 1

**34. If x – a sec θ + b tan θ and y = a tan θ + b sec θ, prove that $x^2 - y^2 = a^2 - b^2$.**

**Solution:**

Given,

x =

$a \sec \theta + b \tan \theta$,

$y = a \tan \theta + b \sec \theta$

Now,

$x^2 - y^2 = (a \sec \theta + b \tan \theta)^2 - (a \tan \theta + b \sec \theta)^2$

$$= (a^2 \sec^2 \theta + b^2 \tan^2 \theta + 2ab \sec \theta \tan \theta) - (a^2 \tan^2 \theta + b^2 \sec^2 \theta + 2ab \sec \theta \tan \theta)$$
$$= a^2 \sec^2 \theta + b^2 \tan^2 \theta + 2ab \sec \theta \tan \theta - a^2 \tan^2 \theta - b^2 \sec^2 \theta - 2ab \sec \theta \tan \theta$$
$$= a^2 (\sec^2 \theta - \tan^2 \theta) - b^2 (\sec^2 \theta - \tan^2 \theta)$$
$$= a^2 \times 1 - b^2 \times 1 \qquad \{\sec^2 \theta - \tan^2 \theta = 1\}$$
$$= a^2 - b^2$$

– Hence proved.

**35. If x = h + a cos θ and y = k + a sin θ, prove that $(x – h)^2 + (y – k)^2 = a^2$.**

**Solution:**

Given,

x = h + a cos θ

y = k + a sin θ

Now,

x – h = a cos θ

y – k = a sin θ

On squaring and adding we get

$(x – h)^2 + (y – k)^2 = a^2 \cos^2 θ + a^2 \sin^2 θ$

$= a^2 (\sin^2 θ + \cos^2 θ)$

$= a^2 (1)$ [Since, $\sin^2 θ + \cos^2 θ = 1$]

– Hence proved

Chapter Test

**1. (i) If θ is an acute angle and cosec θ = √5, find the value of cot θ – cos θ.**

**(ii) If θ is an acute angle and tan θ = 8/15, find the value of sec θ + cosec θ.**

**Solution:**

Given, θ is an acute angle and cosec θ = √5

So,

sin θ = 1/√5

And, cos θ = $\sqrt{(1 – \sin^2 θ)}$

cos θ = $\sqrt{(1 – (1/\sqrt{5})^2)}$

= √(1 – (1/5))

= √(4/5)

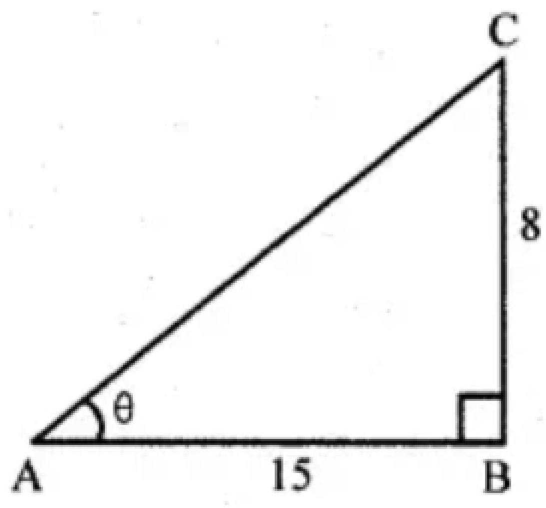
cos θ = 2/√5

Now,

cot θ – cos θ = (cos θ/sin θ) – cos θ

$$= \sqrt{1-\left(\frac{1}{\sqrt{5}}\right)^2} = \sqrt{1-\frac{1}{5}} = \sqrt{\frac{5-1}{5}}$$

$$= \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$




(ii) Given, θ is an acute angle and tan θ = 8/15

In fig. we have

tan θ = BC/AB = 8/15

So, BC = 8 and AB = 15

By Pythagoras theorem, we have

AC = $\sqrt{(AB^2 + BC^2)} = \sqrt{(5^2 + 8^2)} = \sqrt{(25 + 64)} = \sqrt{289}$

⇒ AC = 17

Now,

sec θ = AC/AB = 17/15

cosec θ = AC/BC = 17/8

So,

sec θ + cosec θ = 17/15 + 17/8

= (136 + 255)/ 120

= 391/120

=

$$3\frac{31}{120}$$

**2. Evaluate the following:**

**(i) 2 x () – tan 45° + tan 13° tan 23° tan 30° tan 67° tan 77°**

**(ii)  + sin$^2$ 63° + cos 63° sin 27°**

**Solution:**

(i) $2 \times \left(\frac{cos^2 20^O + cos^2 70^O}{sin^2 25^O + sin^2 65^O}\right)$ – tan 45° + tan 13° tan 23° tan 30° tan 67° tan 77°

$= 2\left[\frac{\cos^2(90° - 70°) + \cos^2 70°}{\sin^2 25° + \sin^2(90° - 25°)}\right] - \tan 45° + \tan 13° \tan 77° \tan 23° \tan 67° \tan 30°$

$= 2\left[\frac{\sin^2 70° + \cos^2 70}{\sin^2 25° + \cos^2 25°}\right] - 1 + \tan 13° \tan(90° - 13°) \tan 23° \tan(90° - 23°) \times \frac{1}{\sqrt{3}}$

$= 2\left(\frac{1}{1}\right) - 1 + \tan 13 \cot 13° \tan 23° \cot 23° \times \frac{1}{\sqrt{3}}$

$= 2 - 1 + 1 \times 1 \times \frac{1}{\sqrt{3}}$

$= 2 - 1 + \frac{1}{\sqrt{3}} = 1 + \frac{1}{\sqrt{3}} = \frac{\sqrt{3}+1}{\sqrt{3}}$

$= \frac{(\sqrt{3}+1)\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{3+\sqrt{3}}{3}$

(ii) $\frac{\sec 29°}{\operatorname{cosec} 61°} + 2 \cot 8° \cot 17° \cot 45° \cot 73° \cot 82° - 3(\sin^2 38° + \sin^2 52°)$

$= \frac{\sec 29°}{\operatorname{cosec}(90° - 29°)} + 2 \cot 8° \times \cot(90° - 8°) \times \cot 17° \times \cot(90° - 17°) \cot 45° - 3[\sin^2 38° + \sin^2(90° - 38°)]$

$= \frac{\sec 29°}{\sec 29°} + 2 \cot 8° \tan 8° \times \cot 17° \tan 17° \times 1 - 3(\sin^2 38° + \cos^2 38°)$

$= 1 + 2 \times 1 \times 1 \times 1 - 3 \times 1 = 1 + 2 - 3 = 0$

(iii) $\frac{\sin^2 22° + \sin^2 68°}{\cos^2 22° + \cos^2 68°} + \sin^2 63° + \cos 63° \sin 27°$

$= \frac{\sin^2 22° + \sin^2(90° - 22°)}{\cos^2 22° + \cos^2(90° - 22°)} + \sin^2 63° + \cos 63° \sin(90° - 63°)$

$= \frac{\sin^2 22° + \cos^2 22°}{\cos^2 22° + \sin^2 22°} + \sin^2 63° + \cos 63° \times \cos 63°$

$= \frac{1}{1} + (\sin^2 63° + \cos^2 63°) = 1 + 1 = 2$

**3. If 4/3 ($\sec^2 59°$ – $\cot^2 31°$) – 2/3 sin 90° + 3 $\tan^2 56°$ $\tan^2 34°$ = x/2, then find the value of x.**

**Solution:**

Given,

4/3 ($\sec^2$ 59° – $\cot^2$ 31°) – 2/3 sin 90° + 3 $\tan^2$ 56° $\tan^2$ 34° = x/2

$$\Rightarrow \frac{4}{3}[\sec^2 59° - \cot^2 (90° - 59°)] - \frac{2}{3} \sin 90° + 3 \tan^2 56° \tan^2 (90° - 56°) = \frac{x}{3}$$

$$\frac{4}{3}[\sec^2 59° - \tan^2 59°] - \frac{2}{3} \times 1 + 3 \tan^2 56° \cot^2 56° = \frac{x}{3}$$

$$\frac{4}{3} \times 1 - \frac{2}{3} + 3 \times 1 = \frac{x}{3}$$

$$\frac{4}{3} - \frac{2}{3} + 3 = \frac{x}{3}$$

$$\frac{4-2+9}{3} = \frac{x}{3}$$

$$\frac{11}{3} = \frac{x}{3} \Rightarrow x = \frac{11 \times 3}{3} = 11$$

$\therefore x = 11$

**4. (i) cos A/ (1 – sin A) + cos A/ (1 + sin A) = 2 sec A**

**(ii) cos A/ (cosec A + 1) + cos A/ (cosec A – 1) = 2 tan A**

**Solution:**

(i) $\frac{\cos A}{1-\sin A} + \frac{\cos A}{1+\sin A} = 2\sec A$

L.H.S = $\frac{\cos A}{1-\sin A} + \frac{\cos A}{1+\sin A}$

$$= \cos A\left[\frac{1}{1-\sin A} + \frac{1}{1+\sin A}\right]$$

$$= \cos A\left[\frac{1+\sin A + 1 - \sin A}{(1-\sin A)(1+\sin A)}\right]$$

$$= \cos A\left[\frac{2}{1-\sin^2 A}\right] = \frac{2\cos A}{\cos^2 A}$$

$$= \frac{2}{\cos A} = 2\sec A = \text{R.H.S.}$$

(ii) $\frac{\cos A}{\operatorname{cosec} A + 1} + \frac{\cos A}{\operatorname{cosec} A - 1} = 2\tan A$

L.H.S.= $\frac{\cos A}{\operatorname{cosec} A + 1} + \frac{\cos A}{\operatorname{cosec} A - 1}$

$$= \cos A\left[\frac{1}{\operatorname{cosec} A + 1} + \frac{1}{\operatorname{cosec} A - 1}\right]$$

$$= \cos A\left[\frac{\operatorname{cosec} A - 1 + \operatorname{cosec} A + 1}{(\operatorname{cosec} A + 1)(\operatorname{cosec} A - 1)}\right]$$

$$= \frac{\cos A\,[2\operatorname{cosec} A]}{\operatorname{cosec}^2 A - 1} = \frac{2\cos A}{\sin A\,(\cot^2 A)}$$





$$= \frac{2\cot A}{\cot^2 A} = \frac{2}{\cot A} = 2\tan A = \text{R.H.S.}$$

(i) $\frac{\cos A}{1-\sin A}+\frac{\cos A}{1+\sin A}=2\sec A$

L.H.S = $\frac{\cos A}{1-\sin A}+\frac{\cos A}{1+\sin A}$

$$=\cos A\left[\frac{1}{1-\sin A}+\frac{1}{1+\sin A}\right]$$

$$=\cos A\left[\frac{1+\sin A+1-\sin A}{(1-\sin A)(1+\sin A)}\right]$$

$$=\cos A\left[\frac{2}{1-\sin^2 A}\right]=\frac{2\cos A}{\cos^2 A}$$

$$=\frac{2}{\cos A}=2\sec A=\text{R.H.S.}$$

(ii) $\frac{\cos A}{\operatorname{cosec} A+1}+\frac{\cos A}{\operatorname{cosec} A-1}=2\tan A$

L.H.S.= $\frac{\cos A}{\operatorname{cosec} A+1}+\frac{\cos A}{\operatorname{cosec} A-1}$

$$=\cos A\left[\frac{1}{\operatorname{cosec} A+1}+\frac{1}{\operatorname{cosec} A-1}\right]$$

$$=\cos A\left[\frac{\operatorname{cosec} A-1+\operatorname{cosec} A+1}{(\operatorname{cosec} A+1)(\operatorname{cosec} A-1)}\right]$$

$$=\frac{\cos A\,[2\operatorname{cosec} A]}{\operatorname{cosec}^2 A-1}=\frac{2\cos A}{\sin A\,(\cot^2 A)}$$





$$=\frac{2\cot A}{\cot^2 A}=\frac{2}{\cot A}=2\tan A=\text{R.H.S.}$$

**5. (i)**

**(ii) (cosec θ – sin θ) (sec θ – cos θ) (tan θ + cot θ) = 1.**

**Solution:**

(i) $\frac{(cos\theta - sin\theta)(1+tan\theta)}{2cos^2\theta - 1} = sec\theta$

L.H.S = $\frac{(cos\theta - sin\theta)(1+tan\theta)}{2cos^2\theta - 1}$

$$= \frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{\cos\theta\,(2\cos^2\theta - 1)}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos\theta\,(2\cos^2\theta - 1)} = \frac{\cos^2\theta - (1 - \cos^2\theta)}{\cos\theta\,(2\cos^2\theta - 1)}$$

$$= \frac{\cos^2\theta - 1 + \cos^2\theta}{\cos\theta\,(2\cos^2\theta - 1)} = \frac{2\cos^2\theta - 1}{\cos\theta\,(2\cos^2\theta - 1)}$$

$$= \frac{1}{\cos\theta} = \sec\theta = \text{R.H.S.}$$

(*ii*) $(\text{cosec}\,\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1.$

L.H.S. $= (\text{cosec}\,\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \left(\frac{1 - \sin^2\theta}{\sin\theta}\right)\left(\frac{1 - \cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)$$

$$= \frac{(\cos^2\theta \times \sin^2\theta)}{\sin\theta\cos\theta} \times \frac{1}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\cos^2\theta}{\sin^2\theta\cos^2\theta} = 1 = \text{R.H.S.}$$

(i) $\frac{(cos\theta - sin\theta)(1+tan\theta)}{2cos^2\theta - 1} = sec\theta$

L.H.S = $\frac{(cos\theta - sin\theta)(1+tan\theta)}{2cos^2\theta - 1}$

$$= \frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{\cos\theta\,(2\cos^2\theta - 1)}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos\theta\,(2\cos^2\theta - 1)} = \frac{\cos^2\theta - (1 - \cos^2\theta)}{\cos\theta\,(2\cos^2\theta - 1)}$$

$$= \frac{\cos^2\theta - 1 + \cos^2\theta}{\cos\theta\,(2\cos^2\theta - 1)} = \frac{2\cos^2\theta - 1}{\cos\theta\,(2\cos^2\theta - 1)}$$

$$= \frac{1}{\cos\theta} = \sec\theta = \text{R.H.S.}$$

(*ii*) $(\text{cosec}\,\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1.$

L.H.S. $= (\text{cosec}\,\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \left(\frac{1 - \sin^2\theta}{\sin\theta}\right)\left(\frac{1 - \cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)$$

$$= \frac{(\cos^2\theta \times \sin^2\theta)}{\sin\theta\cos\theta} \times \frac{1}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\cos^2\theta}{\sin^2\theta\cos^2\theta} = 1 = \text{R.H.S.}$$

**6. (i) $\sin^2\theta + \cos^4\theta = \cos^2\theta + \sin^4\theta$**

**(ii)**

**Solution:**

Given,

(i) $\sin^2 \theta + \cos^4 \theta = \cos^2 \theta + \sin^4 \theta$

L.H.S. = $\sin^2 \theta + \cos^4 \theta$

$= (1 - \cos^2 \theta) + \cos^4 \theta$

$= \cos^4 \theta - \cos^2 \theta + 1$

$= \cos^2 \theta (\cos^2 \theta - 1) + 1$

$= \cos^2 \theta (- \sin^2 \theta) + 1$

$= 1 - \sin^2 \theta \cos^2 \theta$

Now,

R.H.S. = $\cos^2 \theta + \sin^4 \theta$

$= (1 - \sin^2 \theta) + \sin^4 \theta$

$= \sin^4 \theta - \sin^2 \theta + 1$

$= \sin^2 \theta (\sin^2 \theta - 1) + 1$

$= \sin^2 \theta (-\cos^2 \theta) + 1$

$= 1 - \sin^2 \theta \cos^2 \theta$

Hence, L.H.S. = R.H.S.

(ii) $\dfrac{\cot\theta}{\operatorname{cosec}\theta+1}+\dfrac{\operatorname{cosec}\theta+1}{\cot\theta}=2\sec\theta$

$$\text{L.H.S.}=\frac{\cot\theta}{\operatorname{cosec}\theta+1}+\frac{\operatorname{cosec}\theta+1}{\cot\theta}$$

$$=\frac{\dfrac{\cos\theta}{\sin\theta}}{\dfrac{1}{\sin\theta}+1}+\frac{\dfrac{1}{\sin\theta}+1}{\dfrac{\cos\theta}{\sin\theta}}$$

$$=\frac{\dfrac{\cos\theta}{\sin\theta}}{\dfrac{1+\sin\theta}{\sin\theta}}+\frac{\dfrac{1+\sin\theta}{\sin\theta}}{\dfrac{\cos\theta}{\sin\theta}}$$

$$=\frac{\cos\theta}{\sin\theta}\times\frac{\sin\theta}{1+\sin\theta}+\frac{1+\sin\theta}{\sin\theta}\times\frac{\sin\theta}{\cos\theta}$$

$$=\frac{\cos\theta}{1+\sin\theta}+\frac{1+\sin\theta}{\cos\theta}$$

$$=\frac{\cos^2\theta+1+\sin^2\theta+2\sin\theta}{(1+\sin\theta)\cos\theta}$$

$$=\frac{1+1+2\sin\theta}{(1+\sin\theta)\cos\theta}=\frac{2+2\sin\theta}{(1+\sin\theta)\cos\theta}$$

$$=\frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}=\frac{2}{\cos\theta}$$

$$=2\sec\theta=\text{R.H.S.}$$

(ii) $\dfrac{\cot\theta}{\operatorname{cosec}\theta+1}+\dfrac{\operatorname{cosec}\theta+1}{\cot\theta}=2\sec\theta$

$$\text{L.H.S.}=\frac{\cot\theta}{\operatorname{cosec}\theta+1}+\frac{\operatorname{cosec}\theta+1}{\cot\theta}$$

$$=\frac{\dfrac{\cos\theta}{\sin\theta}}{\dfrac{1}{\sin\theta}+1}+\frac{\dfrac{1}{\sin\theta}+1}{\dfrac{\cos\theta}{\sin\theta}}$$

$$=\frac{\dfrac{\cos\theta}{\sin\theta}}{\dfrac{1+\sin\theta}{\sin\theta}}+\frac{\dfrac{1+\sin\theta}{\sin\theta}}{\dfrac{\cos\theta}{\sin\theta}}$$

$$=\frac{\cos\theta}{\sin\theta}\times\frac{\sin\theta}{1+\sin\theta}+\frac{1+\sin\theta}{\sin\theta}\times\frac{\sin\theta}{\cos\theta}$$

$$=\frac{\cos\theta}{1+\sin\theta}+\frac{1+\sin\theta}{\cos\theta}$$

$$=\frac{\cos^2\theta+1+\sin^2\theta+2\sin\theta}{(1+\sin\theta)\cos\theta}$$

$$=\frac{1+1+2\sin\theta}{(1+\sin\theta)\cos\theta}=\frac{2+2\sin\theta}{(1+\sin\theta)\cos\theta}$$

$$=\frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}=\frac{2}{\cos\theta}$$

$$=2\sec\theta=\text{R.H.S.}$$

**7. (i) $\sec^4 A (1 - \sin^4 A) - 2 \tan^2 A = 1$**

**(ii) = sec A + cosec A**

**Solution:**

(i) $\sec^4 A (1 - \sin^4 A) - 2 \tan^2 A = 1$

L.H.S. = $\sec^4 A (1 - \sin^4 A) - 2 \tan^2 A$

$$= \frac{1}{\cos^4 A}(1 + \sin^2 A)(1 - \sin^2 A) - 2\tan^2 A$$

$[\because a^2 - b^2 = (a + b)(a - b)]$

$$= \frac{(1+\sin^2 A)\cos^2 A}{\cos^4 A} - 2\frac{\sin^2 A}{\cos^2 A}$$

$$= \frac{1+\sin^2 A}{\cos^2 A} - \frac{2\sin^2 A}{\cos^2 A}$$

$$= \frac{1+\sin^2 A - 2\sin^2 A}{\cos^2 A}$$

$$= \frac{1-\sin^2 A}{\cos^2 A} = \frac{\cos^2 A}{\cos^2 A} = 1$$

$(\because 1 - \sin^2 A = \cos^2 A)$

= R.H.S.

(ii) $\frac{1}{\sin A + \cos A + 1} + \frac{1}{\sin A + \cos A - 1} = \sec A + \operatorname{cosec} A$

$$\text{L.H.S.} = \frac{1}{\sin A + \cos A + 1} + \frac{1}{\sin A + \cos A - 1}$$

$$= \frac{\sin A + \cos A - 1 + \sin A + \cos A + 1}{(\sin A + \cos A + 1)(\sin A + \cos A - 1)}$$

$$= \frac{2(\sin A + \cos A)}{(\sin A + \cos A)^2 - (1)^2}$$

$$= \frac{2(\sin A + \cos A)}{\sin^2 A + \cos^2 A + 2\sin A\cos A - 1}$$

$$= \frac{2(\sin A + \cos A)}{1 + 2\sin A\cos A - 1}$$

$$= \frac{2(\sin A + \cos A)}{2\sin A\cos A}$$

$$= \frac{\sin A + \cos A}{\sin A\cos A} = \frac{\sin A}{\sin A\cos A} + \frac{\cos A}{\sin A\cos A}$$

$$= \frac{1}{\cos A} + \frac{1}{\sin A}$$

$= \sec A + \operatorname{cosec} A =$ R.H.S.

$$= \frac{1}{\cos^4 A}(1 + \sin^2 A)(1 - \sin^2 A) - 2\tan^2 A$$

$$[\because a^2 - b^2 = (a+b)(a-b)]$$

$$= \frac{(1+\sin^2 A)\cos^2 A}{\cos^4 A} - 2\frac{\sin^2 A}{\cos^2 A}$$

$$= \frac{1+\sin^2 A}{\cos^2 A} - \frac{2\sin^2 A}{\cos^2 A}$$

$$= \frac{1+\sin^2 A - 2\sin^2 A}{\cos^2 A}$$

$$= \frac{1-\sin^2 A}{\cos^2 A} = \frac{\cos^2 A}{\cos^2 A} = 1$$

$$(\because 1 - \sin^2 A = \cos^2 A)$$

= R.H.S.

(*ii*) $\frac{1}{\sin A + \cos A + 1} + \frac{1}{\sin A + \cos A - 1} = \sec A + \operatorname{cosec} A$

$$\text{L.H.S.} = \frac{1}{\sin A + \cos A + 1} + \frac{1}{\sin A + \cos A - 1}$$

$$= \frac{\sin A + \cos A - 1 + \sin A + \cos A + 1}{(\sin A + \cos A + 1)(\sin A + \cos A - 1)}$$

$$= \frac{2(\sin A + \cos A)}{(\sin A + \cos A)^2 - (1)^2}$$

$$= \frac{2(\sin A + \cos A)}{\sin^2 A + \cos^2 A + 2\sin A\cos A - 1}$$

$$= \frac{2(\sin A + \cos A)}{1 + 2\sin A\cos A - 1}$$

$$= \frac{2(\sin A + \cos A)}{2\sin A\cos A}$$

$$= \frac{\sin A + \cos A}{\sin A\cos A} = \frac{\sin A}{\sin A\cos A} + \frac{\cos A}{\sin A\cos A}$$

$$= \frac{1}{\cos A} + \frac{1}{\sin A}$$

$$= \sec A + \operatorname{cosec} A = \text{R.H.S.}$$

**8. (i) + sin θ cos θ = 1**

**(ii) $(\sec A - \tan A)^2 (1 + \sin A) = 1 - \sin A$.**

**Solution:**



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(i) $\frac{sin^3\theta+cos^3\theta}{sin\theta cos\theta}+sin\theta cos\theta=1$

$$\text{LHS} = \frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta} + \sin\theta\cos\theta$$

$$= \frac{(\sin\theta+\cos\theta)(\sin^2\theta-\sin\theta\cos\theta+\cos^2\theta)}{(\sin\theta+\cos\theta)} + \sin\theta\cos\theta$$

$$= \sin^2\theta + \cos^2\theta - \sin\theta\cos\theta + \sin\theta\cos\theta$$

$$= 1 = \text{R.H.S.}$$

(*ii*) $(\sec A - \tan A)^2 (1 + \sin A) = 1 - \sin A$

$$\text{L.H.S.} = \left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right)^2 (1 + \sin A)$$

$$= \left(\frac{1-\sin A}{\cos A}\right)^2 (1 + \sin A)$$

$$= \frac{(1-\sin A)^2}{\cos^2 A}(1 + \sin A)$$

$$= \frac{(1-\sin A)^2(1+\sin A)}{1-\sin^2 A}$$

$$= \frac{(1-\sin A)^2(1+\sin A)}{(1-\sin A)(1+\sin A)}$$

$$= 1 - \sin A = \text{R.H.S.}$$

(i) $\frac{sin^3\theta+cos^3\theta}{sin\theta cos\theta}+sin\theta cos\theta=1$

$$\text{LHS} = \frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta} + \sin\theta\cos\theta$$

$$= \frac{(\sin\theta+\cos\theta)(\sin^2\theta-\sin\theta\cos\theta+\cos^2\theta)}{(\sin\theta+\cos\theta)} + \sin\theta\cos\theta$$

$$= \sin^2\theta+\cos^2\theta-\sin\theta\cos\theta+\sin\theta\cos\theta$$

$$= 1 = \text{R.H.S.}$$

(*ii*) $(\sec A-\tan A)^2(1+\sin A) = 1-\sin A$

$$\text{L.H.S.} = \left(\frac{1}{\cos A}-\frac{\sin A}{\cos A}\right)^2(1+\sin A)$$

$$= \left(\frac{1-\sin A}{\cos A}\right)^2(1+\sin A)$$

$$= \frac{(1-\sin A)^2}{\cos^2 A}(1+\sin A)$$

$$= \frac{(1-\sin A)^2(1+\sin A)}{1-\sin^2 A}$$

$$= \frac{(1-\sin A)^2(1+\sin A)}{(1-\sin A)(1+\sin A)}$$

$$= 1-\sin A = \text{R.H.S.}$$

**9. (i)  = sin A + cos A**

**(ii) (sec A – cosec A) (1 + tan A + cot A) = tan A sec A – cot A cosec A**

**(iii)**

**Solution:**

(i) $\frac{\cos A}{1-\tan A} - \frac{\sin^2 A}{\cos A-\sin A} = \sin A + \cos A$

L.H.S = $\frac{\cos A}{1-\tan A} - \frac{\sin^2 A}{\cos A-\sin A}$

$$= \frac{\cos A}{1-\frac{\sin A}{\cos A}} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos A \times \cos A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$

$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{(\cos A - \sin A)} = \cos A + \sin A = \text{R.H.S.}$$

(*ii*) (sec A – cosec A) (1 + tan A + cot A) = tan A sec A – cot A cosec A

L.H.S. = (sec A – cosec A) (1 + tan A + cot A)

$$= \left(\frac{1}{\cos A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= \frac{\sin A - \cos A}{\sin A \cos A} \times \frac{\sin A \cos A + \sin^2 A + \cos^2 A}{\sin A \cos A}$$

$$= \frac{\sin A - \cos A}{\sin A \cos A} \times \frac{\sin A \cos A + 1}{\sin A \cos A}$$

$$= \frac{(\sin A - \cos A)(\sin A \cos A + 1)}{\sin^2 A \cos^2 A}$$

R.H.S. = tan A sec A – cot A cosec A

$$= \frac{\sin A}{\cos A} \cdot \frac{1}{\cos A} - \frac{\cos A}{\sin A} \cdot \frac{1}{\sin A}$$

$$= \frac{\sin A}{\cos^2 A} - \frac{\cos A}{\sin^2 A} = \frac{\sin^3 A - \cos^3 A}{\sin^2 A \cos^2 A}$$

$$= \frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cos A)}{\sin^2 A \cos^2 A}$$

$$= \frac{(\sin A - \cos A)(1 + \sin A \cos A)}{\sin^2 A \cos^2 A}$$

$$= \frac{(\sin A - \cos A)(\sin A \cos A + 1)}{\sin^2 A \cos^2 A}$$

∴ L.H.S. = R.H.S.

(*iii*) $\frac{\tan^2\theta}{\tan^2\theta - 1} + \frac{\operatorname{cosec}^2\theta}{\sec^2\theta - \operatorname{cosec}^2\theta} = \frac{1}{\sin^2\theta - \cos^2\theta}$

$$\text{L.H.S.} = \frac{\tan^2\theta}{\tan^2\theta - 1} + \frac{\operatorname{cosec}^2\theta}{\sec^2\theta - \operatorname{cosec}^2\theta}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta} - 1} + \frac{\frac{1}{\sin^2\theta}}{\frac{1}{\cos^2\theta} - \frac{1}{\sin^2\theta}} = \frac{\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{\sin^2\theta - \cos^2\theta} + \frac{\frac{1}{\sin^2\theta}}{\frac{\sin^2\theta - \cos^2\theta}{\sin^2\theta\cos^2\theta}}$$

$$= \frac{\sin^2\theta}{\sin^2\theta - \cos^2\theta} + \frac{\sin^2\theta\cos^2\theta}{\sin^2\theta(\sin^2\theta - \cos^2\theta)}$$

$$= \frac{\sin^2\theta}{\sin^2\theta - \cos^2\theta} + \frac{\cos^2\theta}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta - \cos^2\theta} = \frac{1}{\sin^2\theta - \cos^2\theta} = \text{R.H.S.}$$

**10.**

**Solution:**

L.H.S = $\frac{sin A+cos A}{sin A-cos A}+\frac{sin A-cos A}{sin A+cos A}$

$= \frac{(\sin A+\cos A)^2+(\sin A-\cos A)^2}{\sin^2 A-\cos^2 A}$

$= \frac{2(\sin^2 A+\cos^2 A)}{\sin^2 A-\cos^2 A}$

$= \frac{2\times 1}{\sin^2 A-\cos^2 A} = \frac{2}{\sin^2 A-\cos^2 A}$

= R.H.S.

Now, we have

$= \frac{2}{\sin^2 A-\cos^2 A}$

$= \frac{2}{1-\cos^2 A-\cos^2 A} = \frac{2}{1-2\cos^2 A}$

= R.H.S.

Now, we have

$= \frac{2}{\sin^2 A-\cos^2 A}$

$= \frac{2}{\sin^2 A-(1-\sin^2 A)}$

$= \frac{2}{\sin^2 A-1+\sin^2 A}$

$= \frac{2}{\sin^2 A-\cos^2 A} = \frac{\frac{2}{\cos^2 A}}{\frac{\sin^2 A}{\cos^2 A}-\frac{\cos^2 A}{\cos^2 A}}$

$= \frac{2\sec^2 A}{\tan^2 A-1}$ = R.H.S.

**Hence,**

$\frac{sin A+cos A}{sin A-cos A}+\frac{sin A-cos A}{sin A+cos A}=\frac{2}{sin^2 A-cos^2 A}=\frac{2}{1-2cos^2 A}=\frac{2sec^2 A}{tan^2 A-1}$

**11. 2 $(\sin^6 \theta + \cos^6 \theta) - 3 (\sin^4 \theta + \cos^4 \theta) + 1 = 0$**

**Solution:**

Given,

$2 (\sin^6 \theta + \cos^6 \theta) - 3 (\sin^4 \theta + \cos^4 \theta) + 1 = 0$

L.H.S. = $2 (\sin^6 \theta + \cos^6 \theta) - 3 (\sin^4 \theta + \cos^4 \theta) + 1$

$= 2 [(\sin^2 \theta)^3 + (\cos^2 \theta)^3] - 3 (\sin^4 \theta + \cos^4 \theta) + 1$

$= 2 [(\sin^2 \theta + \cos^2 \theta) (\sin^4 \theta + \cos^4 \theta - \sin^2 \theta \cos^2 \theta)] - 3 (\sin^4 \theta + \cos^4 \theta) + 1$

$= 2 (\sin^4 \theta + \cos^4 \theta - \sin^2 \theta \cos^2 \theta) - 3 (\sin^4 \theta + \cos^4 \theta) + 1$

$= 2 \sin^4 \theta + 2 \cos^4 \theta - 2 \sin^2 \theta \cos^2 \theta - 3 \sin^4 \theta - 3 \cos^4 \theta + 1$

$= 1 - \sin^4 \theta - \cos^4 \theta - 2 \sin^2 \theta \cos^2 \theta$

$= 1 - [\sin^4 \theta + \cos^4 \theta + 2 \sin^2 \theta \cos^2 \theta]$

$= 1 - 1$

$= 0$ = R.H.S.

**12. If $\cot \theta + \cos \theta = m$, $\cot \theta - \cos \theta = n$, then prove that $(m^2 - n^2)^2 = 16$.**

**Solution:**

Given,

$\cot \theta + \cos \theta = m$ … (i)

$\cot \theta - \cos \theta = n$ … (ii)

Adding (i) and (ii), we get

$$2 \cot \theta = m + n \Rightarrow \cot \theta = \frac{m+n}{2}$$

$$\therefore \tan \theta = \frac{2}{m+n} \qquad \text{... (iii)}$$

Subtracting (*ii*) from (*i*),

$$2 \cos \theta = m - n \Rightarrow \cos \theta = \frac{m-n}{2}$$

$$\therefore \sec \theta = \frac{2}{m-n} \qquad \text{... (iv)}$$

**Now, squaring and subracting (iii) from (iv), we have**

$$\sec^2 \theta - \tan^2 \theta = \left(\frac{2}{m-n}\right)^2 - \left(\frac{2}{m+n}\right)^2$$

$$1 = \frac{4}{(m-n)^2} - \frac{4}{(m+n)^2}$$

$$\Rightarrow 4\left[\frac{1}{(m-n)^2} - \frac{1}{(m+n)^2}\right] = 1$$

$$4\left[\frac{(m+n)^2 - (m-n)^2}{(m+n)^2 (m-n)^2}\right] = 1$$

$$\frac{4(4\ mn)}{(m^2 - n^2)^2} = 1$$

$$\frac{16mn}{(m^2 - n^2)^2} = 1$$

$$\therefore (m^2 - n^2)^2 = 16\ mn.$$

$$2\cot\theta = m + n \Rightarrow \cot\theta = \frac{m+n}{2}$$

$$\therefore \tan\theta = \frac{2}{m+n} \quad \text{... (iii)}$$

Subtracting (*ii*) from (*i*),

$$2\cos\theta = m - n \Rightarrow \cos\theta = \frac{m-n}{2}$$

$$\therefore \sec\theta = \frac{2}{m-n} \quad \text{... (iv)}$$

**Now, squaring and subracting (iii) from (iv), we have**

$$\sec^2\theta - \tan^2\theta = \left(\frac{2}{m-n}\right)^2 - \left(\frac{2}{m+n}\right)^2$$

$$1 = \frac{4}{(m-n)^2} - \frac{4}{(m+n)^2}$$

$$\Rightarrow 4\left[\frac{1}{(m-n)^2} - \frac{1}{(m+n)^2}\right] = 1$$

$$4\left[\frac{(m+n)^2 - (m-n)^2}{(m+n)^2(m-n)^2}\right] = 1$$

$$\frac{4(4\,mn)}{(m^2-n^2)^2} = 1$$

$$\frac{16mn}{(m^2-n^2)^2} = 1$$

$$\therefore (m^2-n^2)^2 = 16\,mn.$$

**13. If sec θ + tan θ = p, prove that sin θ = (p² – 1)/ (p² + 1)**

**Solution:**

Given, sec θ + tan θ = p

$$\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = p$$

$$\frac{1+\sin\theta}{\cos\theta} = p$$

Squaring on both sides,

$$\frac{(1+\sin\theta)^2}{\cos^2\theta} = p^2 \Rightarrow \frac{(1+\sin\theta)^2}{1-\sin^2\theta} = p^2$$

$$\frac{(1+\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)} = p^2$$

$$\Rightarrow \frac{1+\sin\theta}{1-\sin\theta} = \frac{p^2}{1}$$

Applying componendo and dividendo,

$$\frac{1+\sin\theta+1-\sin\theta}{1+\sin\theta-1+\sin\theta} = \frac{p^2+1}{p^2-1}$$

$$\frac{2}{2\sin\theta} = \frac{p^2+1}{p^2-1} \Rightarrow \frac{1}{\sin\theta} = \frac{p^2+1}{p^2-1}$$

$$\therefore \sin\theta = \frac{p^2-1}{p^2+1}$$

**14. If tan A = n tan B and sin A = m sin B, prove that cos² A = (m² – 1)/ (n² – 1)**

**Solution:**

Given,

tan A = n tan B and sin A = m sin B

n = tan A/ tan B

m = sin A/ sin B

$$\frac{1}{\sin B} = \frac{m}{\sin A} \Rightarrow \text{cosec } B = \frac{m}{\sin A}$$

$$\frac{1}{\tan B} = \frac{n}{\tan A} \Rightarrow \cot B = \frac{n}{\tan A}$$

Now, $\text{cosec}^2 B - \cot^2 B = 1$

$$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1$$

$$\frac{m^2}{\sin^2 A} - \frac{n^2 \cos^2 A}{\sin^2 A} = 1$$

$$m^2 - n^2 \cos^2 A = \sin^2 A$$

$$m^2 - n^2 \cos^2 A = 1 - \cos^2 A$$

$$m^2 - 1 = \cos^2 A + n^2 \cos^2 A$$

$$m^2 - 1 = (n^2 - 1) \cos^2 A$$

$$\Rightarrow \cos^2 A = \frac{m^2 - 1}{n^2 - 1}$$

**15. If sec A = x + 1/4x, then prove that sec A + tan A = 2x or 1/2x**

**Solution:**

Given, sec A = x + 1/4x

We know that,

$$\tan A = \pm\sqrt{\sec^2 A - 1}$$

$$= \pm\sqrt{\left(x + \frac{1}{4x}\right)^2 - 1}$$

$$= \pm\sqrt{x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1}$$

$$= \pm\sqrt{x^2 + \frac{1}{16x^2} - \frac{1}{2}}$$

$$= \pm\left(x - \frac{1}{4x}\right)$$

$$\therefore\ \sec A + \tan A = x + \frac{1}{4x} + x - \frac{1}{4x} = 2x$$

$$\text{or } x + \frac{1}{4x} - x + \frac{1}{4x} = \frac{1}{2x}$$

$$\tan A = \pm\sqrt{\sec^2 A - 1}$$

$$= \pm\sqrt{\left(x + \frac{1}{4x}\right)^2 - 1}$$

$$= \pm\sqrt{x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1}$$

$$= \pm\sqrt{x^2 + \frac{1}{16x^2} - \frac{1}{2}}$$

$$= \pm\left(x - \frac{1}{4x}\right)$$

$$\therefore\ \sec A + \tan A = x + \frac{1}{4x} + x - \frac{1}{4x} = 2x$$

$$\text{or } x + \frac{1}{4x} - x + \frac{1}{4x} = \frac{1}{2x}$$

**16. When 0° < θ < 90°, solve the following equations:**

**(i) 2 $\cos^2$ θ + sin θ – 2 = 0**

**(ii) 3 cos θ = 2 $\sin^2$ θ**

**(iii) $\sec^2$ θ – 2 tan θ = 0**

**(iv) $\tan^2$ θ = 3 (sec θ – 1).**

**Solution:**

Given, 0° < θ < 90°

(i) 2 $\cos^2$ θ + sin θ – 2 = 0

2 (1 – $\sin^2$ θ) + sin θ – 2 = 0

$2 - 2\sin^2\theta + \sin\theta - 2 = 0$

$-2\sin^2\theta + \sin\theta = 0$

$\sin\theta(1 - 2\sin\theta) = 0$

So, either $\sin\theta = 0$ or $1 - 2\sin\theta = 0$

If $\sin\theta = 0$

$\Rightarrow \theta = 0^\circ$

And, if $1 - 2\sin\theta = 0$

$\sin\theta = \frac{1}{2}$

$\Rightarrow \theta = 30^\circ$

Thus, $\theta = 0^\circ$ or $30^\circ$

(ii) $3\cos\theta = 2\sin^2\theta$

$3\cos\theta = 2(1 - \cos^2\theta)$

$3\cos\theta = 2 - 2\cos^2\theta$

$2\cos^2\theta + 3\cos\theta - 2 = 0$

$2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0$

$2\cos\theta(\cos\theta + 2) - 1(\cos\theta + 2)$

$(2\cos\theta - 1)(\cos\theta + 2) = 0$

So, either $2\cos\theta - 1 = 0$ or $\cos\theta + 2 = 0$

If $2\cos\theta - 1 = 0$

$\cos\theta = \frac{1}{2}$

$\Rightarrow \theta = 60^\circ$

And, for $\cos\theta + 2 = 0$

$\Rightarrow \cos\theta = -2$ which is not possible being out of range.

Thus, $\theta = 60^\circ$

(iii) $\sec^2\theta - 2\tan\theta = 0$

$(1 + \tan^2 \theta) – 2 \tan \theta = 0$

$\tan^2 \theta – 2 \tan \theta + 1 = 0$

$(\tan \theta – 1)^2 = 0$

$\tan \theta – 1 = 0$

$\Rightarrow \tan \theta = 1$

Thus, $\theta = 45^o$

(iv) $\tan^2 \theta = 3 (\sec \theta – 1)$

$(\sec^2 \theta – 1) = 3 \sec \theta – 3$

$\sec^2 \theta – 1 – 3 \sec \theta + 3 = 0$

$\sec^2 \theta – 3 \sec \theta + 2 = 0$

$\sec^2 \theta – 2 \sec \theta – \sec \theta + 2 = 0$

$\sec \theta (\sec \theta – 2) – 1 (\sec \theta = 2) = 0$

$(\sec \theta – 1) (\sec \theta – 2) = 0$

So, either $\sec \theta – 1 = 0$ or $\sec \theta – 2 = 0$

If $\sec \theta – 1 = 0$

$\sec \theta = 1$

$\Rightarrow \theta = 0^o$

And, if $\sec \theta – 2 = 0$

$\sec \theta = 2$

$\Rightarrow \theta = 60^o$

Thus, $\theta = 0^o$ or $60^o$

# ML Aggarwal Solutions Class 10 Maths

## About ML Aggarwal

M. L. Aggarwal, is an Indian mechanical engineer, educator. His achievements include research in solutions of industrial problems related to fatigue design. Recipient Best Paper award, Manipal Institute of Technology, 2004. Member of TSTE.

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