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MTHP-5 Advanced Topology

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Q-1 Which of the following is true?
(a) Every second countable space is first countable. (b) Every First countable space is second countable.
(c) Every separable space is second countable. (d) Every Lindelöf space is second countable.

Q-2 Does product of two Lindelöf space is Lindelöf?
(a) Yes (b) No (c) May be (d) Always

Q-3 Which of the following property space $\mathbb{R}_l$ does not have?
(a) First Countable (b) Second Countable (c) Lindelöf (d) Separable

Q-4 Which of the following is true?
(a) Finite product of Lindelöf spaces is Lindelöf. (b)Arbitrary product of Lindelöf spaces is Lindelöf.
(c) Every metrizable Lindelöf space is second countable. (d) Subspace of Lindelöf space is Lindelöf

Q-5 Every metric space is
(a) Lindelöf (b) First Countable (c) Second Countable (d) Separable.

Q-6 Let X is separable space, then every collection of disjoint open sets in X is
(a) Finite (b) Countable (c) Uncountable (d) Connected.

Q-7 Let $f: X \to Y$ be a continuous function. Which of the following is false?
(a) If X is Lindelöf then $f(X)$ is Lindelöf. (b) If X is Lindelöf then Y is Lindelöf.
(c) If X is first countable then $f(X)$ is first countable. (d) If X is first countable then Y is first countable.

Q-8 Let A be a subspace of X, then
(a) If X is Lindelöf then A is Lindelöf. (b) If X is Lindelöf and A is closed then A is Lindelöf.
(c) If X is separable then A is separable. (d) If X is separable and A is closed then A is separable.

Q-9 Let X have a countable basis and A is uncountable subset of X, what can be said about limit points of A?
(a) Only finitely many points of A are limit points of A (b) Only countable many points of A are limit points of A.
(c) Uncountable many points of A are limit points of A (d) None of these.

Q-10 Let $f: X \to Y$ be the function, X is first countable then $f(X)$ is first countable if____
(a) f is open map. (b) f is homeomorphism (c) f is closed map. (d) f is continuous function.

Q-11 Which of the following is not a separation axiom?
(a) Hausdorff (b) Regular (c) Lindelöf (d) Normal

Q-12 Which of the following is false?
(a) Subspace of normal space is normal (b) Subspace of regular space is regular.
(c) Subspace of Hausdorff space is Hausdorff. (d) Closed subspace of normal space is normal.

Q-13 Let X be a regular space, then
(a) Every pair of points of Xhave neighborhoods whose interior are disjoint.
(b) Every pair of points of Xhave neighborhoods whose closures are disjoint.
(c) Every pair of points of X have disjoint neighborhoods are closed sets
(d) Closure of every open set is open.

Q-14 Let $f, g: X \to Y$ be continuous and Y is Hausdorff. Then the set $\{x: f(x) = g(x)\}$ is _____in X
(a) Open (b) Closed (c) Open as well as closed (d) Neither open nor closed.

Q-15 Which of the following condition is not sufficient for space to be normal?
(a) Regular and first countable. (b) Regular and second countable.
(c) Hausdorff and compact. (d) Metrizable.

Q-16 Every well order set X is _____ in ordered topology.
(a) Hausdorff (b) Regular (c) Normal (d) All of these.

Q-17 A connected normal space having more than one point is _____
(a) Finite (b) Countable (c) Uncountable (d) Can't say.

Q-18 Which of the following space is not completely regular
(a) Locally compact Hausdorff space (b) Regular (c) Normal (d) Perfectly normal

Q-19 Which of the following is true?
(a) Tietze extension theorem ⇒ Urysohn's lemma (b) Urysohn's lemma ⇒Tietze extension theorem
(c) Tietze extension theorem ⇒Tychonoff theorem (d) Urysohn's lemma ⇒Tychonoff theorem

Q-20 Let X be a metrizable limit point compact, then
(a) $X$is bounded under every metric that gives the topology of X.
(b) Every continuous function $f: X \to \mathbb{R}$are bounded.
(c) Both A and B
(d) None of these.

Q-21 2-manifold is known as
(a) Curve (b) Surface (c) Plane (d) Sphere

Q-22 A Hausdorff space X is called $m-$ manifold if
(a) X is second countable (b) Each $x \in X$ has a neighborhood that is homeomorphic with an open subset of $\mathbb{R}^m$.
(c) Both A and B (d) Either A or B

Q-23 If X is a compact $m-$ manifold, then
(a) Xcan be imbedded in $\mathbb{R}^n$ for some $n \in \mathbb{N}$. (b) Xcan be imbedded in $\mathbb{R}^n$ for some $n \in \mathbb{Z}$.
(c) Xsatisfies the T_1 axioms but not Hausdorff. (d) $X$is not second countable.

Q-24 Statement of Tychonoff theorem is
(a) A finite product of compact spaces is compact in product topology.
(b) A countable product of compact spaces is compact in product topology.
(c) An uncountable product of compact spaces is compact in product topology.
(d) An arbitrary product of compact spaces is compact in the product topology.

Q-25 Which of the following is essential condition for Y to be a compactification of X?
(a) Y is compact (b) Y is Hausdorff (c) $\bar{X} = Y$ (d) All of these.

Q-26 What is the condition for space X to have a compactification Y?
(a) Regular (b) Completely regular (c) Normal (d) Hausdorff

Q-27 Let $\{U_1, U_2, \dots, U_n\}$ be a finite indexed open covering of the space X. An indexed family of functions $\emptyset_i: X \to [0,1]$ for $i = 1,2, \dots, n$, is said to be a partition of unity dominated by $\{U_i\}$ if
(a) Each $\emptyset_i$ is continuous (b) (support $\emptyset_i$)$\subset U_i$ for each i
(c) $\sum_{i=1}^{n} \emptyset_i(x) = 1$ for each x (d) All of these.

Q-28 Statement of Existence of finite partitions of unity is
(a) Let $\{U_1, U_2, \dots, U_n\}$ be a finite open covering of the space X, then there exist a partition of unity dominated by $\{U_i\}$.
(b) Let $\{U_1, U_2, \dots, U_n\}$ be a finite covering of the regular space X, then there exist a partition of unity dominated by $\{U_i\}$.
(c) Let $\{U_1, U_2, \dots, U_n\}$ be a finite open covering of the normal space X, then there exist a partition of unity dominated by $\{U_i\}$.
(d) Let $\{U_1, U_2, \dots, U_n\}$ be a finite open covering of the regular space X, then there exist a partition of unity dominated by $\{U_i\}$.

Q-29 Two compactificationsY_1 and Y_2 of space X are said to be equivalent if
(a) There is a homeomorphism $h: Y_1 \to Y_2$ such that $h(x) = x$ for every $x \in X$.
(b) $Y_1 \cap Y_2 = \emptyset$ and $Y_1 \cup Y_2 = X$
(c) $Y_1 = Y_2$
(d) $Y_1 \subset Y_2$ and $Y_2 \subset Y_1$

Q-30 Let X be completely regular. X is completely regular iff
(a) Xhas one point compactification. (b) $\beta(X)$is connected. (c) $\beta(X)$is compact. (d) $\beta(X)$is second countable.

Q-31 Which of the following is true?
(a) Every Cauchy sequence is convergent. (b) Every Cauchy sequence is bounded.
(c) Every convergent sequence is Cauchy. (d) Every bounded sequence is Cauchy.

Q-32 Which of the following is not sufficient for completeness for the metric space (X, d) ?
(a) Every Cauchy sequence in X converges.
(b) Every open subset of X are compact.
(c) Every Cauchy sequence in X has a convergent subsequence.
(d) $(X, \bar{d})$is complete metric space, where $\bar{d}$ is the standard bounded metric corresponding to d.

Q-33 $\mathbb{R}^k$is complete metric space under which of the following metric?
(a) Usual metric (b) Euclidean metric (c) Square metric (d) All of these.

Q-34 Let X be a topological space and (Y, d) be a complete metric space, then which of the following is false?
(a) $(Y^X, \bar{\rho})$is non complete metric space. (b) $\mathcal{C}(X, Y)$ is a closed subspace of $(Y^X, \bar{\rho})$
(c) $\mathcal{C}(X, Y)$ is a complete subspace of $(Y^X, \bar{\rho})$ (d) $\mathcal{B}(X, Y)$ is a closed and complete subspace of $(Y^X, \bar{\rho})$

Q-35 Let (X, d) be a metric space, then
(a) There exists a homeomorphism from X to a complete metric space.
(b) There exists an isometric imbedding on X into a complete metric space.
(c) $(X, d)$is complete metric space.
(d) $(X, d)$is compact metric space.