

नोट : दोनों खण्डों से निर्देशानुसार उत्तर दीजिए। प्रश्नों के अंक उनके दाहिनी ओर अंकित हैं।

Note : Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.

खण्ड / Section-A

Answer the following questions:-

1x10=10

- Define simple graph.
- Define isolated vertex.
- Define spanning subgraph.
- If $P = \{S \rightarrow aCa, C \rightarrow aCa, C \rightarrow b\}$ is a production rule of a programmer G. Then identify the type of grammar.
- Define component of a graph.
- Define Euler graph.
- Define Spanning tree.
- Define K-equivalent states.
- What do you mean by a cut set of a connected graph?
- Define type -I grammar.

SECTION-A

2X5=10

2-Answer the following short answer type questions

- Show that K_5 is non-planar graph.
- Discuss the Konigsberg bridge problem.
- Let G be a connected graph with n vertices and e edges. Find the number of branches and chords of a spanning tree with respect to G.
- Define mealy machine.
- Define incidence matrix with an example.

SECTION-B

Answer the following questions

12X5=60

3-a- State and prove handshaking lemma.

b- show that the number of odd vertices in a graph is always even.

Or

- Define complement of a simple graph and self-complementary graph with an example.
- Prove that if G is self complementary then G has $4k$ or $4k + 1$ vertices, where K is an integer.

4- a- Define bipartite and complete bipartite graph with an example.

c- Show that the maximum number of edges in a complete bipartite graph of n vertices is $\frac{n^2}{4}$.

Or

Show that a connected graph G is an Euler graph iff the degree of every vertex in G is even.

5-a- Show that a graph G is a tree iff there is one and only one path between any two vertices of G.

b- Show that a tree with n vertices has $(n-1)$ edges.

Or

a- Prove that there are at least two pendant vertices in any tree with two or more vertices.

b- show that every tree has either one or two centers.

6- a- If every region of a simple planar graph with n vertices and e edges is bounded by k edges, show that

$$e = \frac{k(n-2)}{k-2}$$

b- If a connected simple planar graph G has $n \geq 3$ vertices e edges and f regions then show that

$$3f < 2e \text{ and } e < 3n - 6$$

Or

- a- Show that a Euler graph G can not have a cut set of odd number of edges.
- b- Show that every cut set in a connected graph G contains at least one branch of every spanning tree of G .

7-a- Design a finite state machine that receives the set $\{0,1,2,3\}$ as input and produces an output such that the output is equal to the modulo 3 sum of the digits in the input sequence.

b- Obtain grammar for language $L = \{a^m b^n : n \neq m > 0\}$

Or

- a- Design a finite state machine having an output of 1 exactly when the input sequence ends with the digits 101
- b- Minimize the machine whose state table is given below

State	Input		output
	0	1	
S_0	S_3	S_6	1
S_1	S_4	S_2	0
S_2	S_4	S_1	0
S_3	S_2	S_0	1
S_4	S_5	S_0	1
S_5	S_3	S_5	0
S_6	S_4	S_2	1