Paper Code - 5402

M.Sc. Semester-III (Physics) (Old Course)
July-September-2021
MTHP-3 Complex Analysis

કુલ સમય :– : ૩૦ મીનીટ **કુલ પ્રશ્નો : ૩૫** **વિદ્યાર્થીએ ટીક કરવાના પ્રશ્નો : ૧૮**

નોંધ : વિદ્યાર્થીએ માત્ર ૧૮ પ્રશ્નો જ ટીક કરવાના રહેશે. જો વધારે ટીક કરેલ હશે તો પ્રથમ ૧૮ પ્રશ્નો જ ગણતરીમાં લેવામાં આવશે.

Q-1 For any two complex number z_1 and z_2, $|z_1| > |z_2|$ represents
(a) z_1 is farther from the origin than the point z_2. (b) z_2 is farther from the origin than the point z_1.
(c) Meaningless statement. (d) None of these.

Q-2 The value of $\sqrt{\cos 3\pi + i \sin 3\pi}$ is _____.
(a) i (b) 1 (c) $-i$ (d) None of these

Q-3 The complex conjugate of $\frac{i}{1-i}$ is
(a) $-\frac{i}{1+i}$ (b) $\frac{1-i}{i}$ (c) $\frac{i-1}{2}$ (d) None of these.

Q-4 If $z = \cos\theta + i\sin\theta$ then $\sin n\theta$ = _____
(a) $\frac{z^n+z^{-n}}{2}$ (b) $\frac{z^n-z^{-n}}{2i}$ (c) $\frac{z^n-z^{-n}}{2}$ (d) $\frac{z^n+z^{-n}}{2i}$

Q-5 If $z = \sin\theta + i\cos\theta$ then z^n =_____
(a) $\sin n\theta + i\cos n\theta$ (b) $\cos n\theta + i\sin n\theta$
(c) $\cos\left(\frac{n\pi}{2} + n\theta\right) + i\sin\left(\frac{n\pi}{2} + n\theta\right)$ (d) $\cos\left(\frac{n\pi}{2} - n\theta\right) + i\sin\left(\frac{n\pi}{2} - n\theta\right)$

Q-6 The function $f(z) = z^2$ is
(a) Nowhere analytic (b) Everywhere analytic
(c) Analytic only at origin (d) Analytic everywhere other than origin.

Q-7 $\lim_{z\to 0}\left(\frac{z}{\bar{z}}\right) =$
(a) 0 (b) 1 (c) -1 (d) Does not exist

Q-8 A function $f(z)$ is continuous and non zero at z_0, then
(a) $f(z)$is entire function. (b) $f(z)$is analytic at z_0.
(c)$f(z) \neq 0$throughout some neighborhood of z_0. (d) $f(z)$is differentiable at z_0.

Q-9 Let $f(z) = |z|^2$, then
(a) $f(z)$is entire function. (b) $f(z)$is analytic everywhere except at $z = 0$.
(c) $f(z)$is no where differentiable. (d) $f(z)$is differentiable only at $z = 0$.

Q-10 The Cauchy-Riemann equations are
(a) $u_x = v_y$ and $u_y = v_x$ (b) $u_x = v_y$ and $u_y = -v_x$ (c) $u_x = v_x$ and $u_y = v_y$ (d) $u_x = v_x$ and $u_y = -v_y$

Q-11 What is the value of $\log -8$?
(a)$\log 8 + 2\pi i$ (b) $\log 64 + \pi i$ (c) $\log 64 + 2\pi i$ (d) $\log 8 + \pi i$

Q-12 The value of the integral $\oint_C \frac{3z^2+7z+1}{z+1}dz, C: |z| = \frac{1}{2}$ is
(a) $2\pi i$ (b) 0 (c) $\frac{\pi i}{2}$ (d) πi

Q-13 Which of the following is the statement of Cauchy-Goursat theorem?
(a) If $f(z)$ is analytic at all points inside and on a simple closed curve C contained in a simply connected domain D then $\oint_C f(z)dz = 0$,
(b) If $f(z)$ is continuous at all points inside and on a simple closed curve C contained in a simply connected domain D then $\oint_C f(z)dz = 0$,
(c) If $f(z)$ is analytic at all points inside and on any closed curve C contained in a simply connected domain D then $\oint_C f(z)dz = 0$,
(d) If $f(z)$ is analytic at all points inside and on a simple curve C contained in a connected domain D then $\oint_C f(z)dz = 0$,

Q-14 When the line integral is said to be independent of the path?
(a) If the function is analytic in the multiply connected region.
(b) If function is continuous everywhere.
(c) If the function is continuous in the domain.
(d) If the function is analytic in the simply connected region.

Q-15 Find $\int_C Re\, z\, dz$, where C is the shortest path from $1 + i$ to $3 + 2i$.
(a) $4 - 2i$ (b) $4 + 2i$ (c) $2i - 4$ (d) None of these

Q-16 Evaluate $\int_0^{1+i} x + y\, dz$
(a) $1 - i$ (b) 0 (c) $i - 1$ (d) $1 + i$

Q-17 By integral inequality, $\left|\int_C f(z)dz\right| \leq$ _____

(a) $\int_C |f(z)|dz$ (b) $\int_C f(z)dz$ (c) $\int_C |f(z)||dz|$ (d) $\int_C |f(z)dz|$

Q-18 $\int_a^b f(z)\,dz =$ _____

(a) $-\int_b^a f(z)dz$ (b) $\int_b^a f(z)dz$ (c) $-\int_b^{-a} f(z)dz$ (d) $\int_{-a}^{-b} f(z)dz$

Q-19 If the curve C consists of the curves C_1 and C_2 then $\int_C f(z)dz =$ _____

(a) $\int_{C_1}^{C_2} f(z)dz$ (b) $\int_{C_1} f(z)dz + \int_{C_2} f(z)dz$

(c) $\int_{C_1} f(z)dz - \int_{C_2} f(z)dz$ (d) $\int_{C_1} f(z)dz \int_{C_2} f(z)dz$

Q-20 $\int_1^i z dz =$ _____

(a) 0 (b) -2 (c) -1 (d) 1

Q-21 $\int_{C:|z|=1} e^{1/z} dz =$

(a) 0 (b) $2\pi i$ (c) $-\pi i$ (d) πi

Q-22 The value of the integral $\oint_C \frac{e^{iz}}{z+3i} dz, C: |z+3i| = 1$ is

(a) $2\pi i$ (b) 0 (c) $2\pi i e^{-3}$ (d) $2\pi i e^{3}$

Q-23 The value of the integral $\oint_C \frac{zdz}{z^2+1}, C: |z+i| = \frac{1}{2}$ is

(a) πi (b) 0 (c) $-\pi i$ (d) $2\pi i$

Q-24 Let C be the circle $|z-1| = 3$, then $\oint_C \frac{\cos z}{z-\pi} dz$

(a) πi (b) $-\pi i$ (c) $-2\pi i$ (d) $2\pi i$

Q-25 Find $\oint_C \frac{dz}{z-a}$, where $C: |z-a| = r$

(a) π (b) $2\pi i$ (c) 0 (d) πi

Q-26 If $f'(z) = 0$ everywhere in a domain D, then

(a) $f(z)$is an identity function. (b) $f(z)$is an hyperbolic function.

(c) $f(z)$is an trigonometric function. (d) $f(z)$is an constant function.

Q-27 What is the harmonic conjugate of $u(x,y) = x^2 - y^2$?

(a) $v(x,y) = xy$ (b) $v(x,y) = 2xy$ (c) $v(x,y) = x^2 + y^2$ (d) $v(x,y) = (x-y)^2$

Q-28 For a function $f(z) = u(x,y) + iv(x,y)$, which of the following condition is not sufficient for analyticity of f in domain D?

(a) $f$satisfies CR-equations.

(b) First order partial derivatives of u and v exists and continuous and satisfy CR-equations.

(c) $v$is harmonic conjugate of u.

(d) $f'(z)$exists at neighborhood of each point of D.

Q-29 Log $i^3 =$

(a) 3Log i (b) $\frac{3\pi i}{2}$ (c) iLog 3 (d) None of these

Q-30 Evaluate $\int_0^{1+i} z^2\,dz$ along the line $y = \frac{x}{2}$.

(a) $\frac{1}{3}\left(\frac{1}{4} + \frac{11}{8}i\right)$ (b) $\frac{2i+2}{3}$ (c) $\frac{2i-3}{3}$ (d) $\frac{3i-2}{2}$

Q-31 If C is any simple closed contour, in either direction then $\int_C e^{z^2} dz =$ _____

(a) 1 (b) 4 (c) 2 (d) 0

Q-32 What is the formula to find residue at simple pole$a$?

(a) $Res = \lim_{z\to -a}(z+a)f(z)$ (b) $Res = \lim_{z\to -a}(z-a)f(z)$

(c) $Res = \lim_{z\to a}(z+a)f(z)$ (d) $Res = \lim_{z\to a}(z-a)f(z)$

Q-33 Which of the following term is used to find residue at $z = a$ in Laurent's series

$$f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n + \sum_{n=1}^{\infty} \frac{b_n}{(z-a)^n}$$

(a) b_1 (b) b_2 (c) a_0 (d) a_1

Q-34 Let $f(z) = \frac{z}{(z-3)^4(z-5)(z-2)^2}$ then which of the following is simple pole?

(a) 3 (b) 2 (c) 5 (d) 0

Q-35 Find the residue of $f(z) = \frac{z}{z^2+1}$about $z = i$.

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 0