

AJ-1540 CV-19

M.A./M.Sc. (Previous) Term End Examination, 2021-22

MATHEMATICS (Paper-II)

(REAL ANALYSIS AND MEASURE THEORY)

Time : Three hours]

[Maximum Marks : 100

Note: Answer any five questions. All questions carry equal marks.

- 1.(a) State and prove fundamental theorem of calculus.
(b) Let $\alpha, f, \alpha: [a, b] \rightarrow \mathbb{R}$ be bounded functions and α be monotone increasing. If P^* is a refinement of the partition P of the interval $[a, b]$. The prove that $L(C, f\alpha) \leq LCP^* f, \alpha$ and $U(P^*, f, \alpha) \leq U(Pf\alpha)$
2. (a) Show that the sequence $\{f_n\}$ where $f_n(x) = nx(1-x)^n$ does not coverage uniformly on $[0, 1]$.
(b) The sum to n terms of a series is $f_n(x) = \frac{n^2 x}{1 + n^4 x^2}$ show that it converges non- uniformly in the interval $[0, 1]$.
- 3.(a) Obtain the radius of convergence R of the power series $\sum c_n z^n$ where.
(i) $C_n = \frac{1}{n!}$ (ii) $C_n = \frac{n^3}{3^n}$
(b) State and prove Dirichlet,s test for uniform convergence of the series.
- 4.(a) State and prove stokes theorem.
(b) State and prove uniqueness for power series.
- 5.(a) If $A, B \in L(\mathbb{R}^n, \mathbb{R}^m)$ and e is a scalar, then show that.
 $\|A + B\| \leq \|A\| + \|B\|, \|cA\| = |c| \|A\|$ with the distance between A and B defined as $\|A - B\|, L(\mathbb{R}^n, \mathbb{R}^m)$ is a metric space.
(b) Let f maps a convex open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^m, f$ be differentiable in E and there be a real number M such that. $\|f'(x)\| \leq M$ for every $x \in E$. Then prove that $|f(b) - f(a)| \leq M|b - a|$ for all $a \in E, b \in E$.
- 6.(a) Find the maximum and minimum values of the function. $f(x, y) = 2x^2 - 3y^2 - 2x$ subject to the constraint $x^2 + y^2 \leq 1$.
(b) Find the rectangle of perimeters which has maximum area.
- 7.(a) If $x = r \sin \theta \cos \phi, y = r \sin \theta - \sin \phi, z = r \cos \theta$ then show that: $\frac{\partial(c, y, z)}{\partial(r, \alpha, \phi)} = r^2 \sin \theta$
(b) Prove that the outer measure of an interval is its length.
- 8.(a) Let f be a non-negative function which is integrable over a set E . Then given $\epsilon > 0$ there is a $\delta > 0$ such that for every set $A \subset E$ with $m(A) < \delta$, we have $\int_A f < \epsilon$
(b) A function of bounded variation is necessarily bounded, but not conversely.
- 9.(a) Prove that the union of two mo-measurable (or outer measurable) set is also mo-measurable (outer measurable)
(b) If $f \in L^p[a, b], p > 1$, then $f \in L[a, b]$
- 10.(a) State and prove Minkowski's Inequality.
(b) State and prove Holder's Inequality for L^p - spaces.