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BH-5313

M.Sc. (Ist Semester) Examination, 2022

MATHEMATICS

(Topology)

Time Allowed : Three Hours

Maximum Marks : 70

Note : Attempt all sections as per instructions given.

SECTION - A

(Objective Type / Multiple Choice / True or False)

Q. 1. Attempt all questions : **1×10=10**

(i) The set N × N is countable. (True/False).

(ii) The set of all rational numbers is denumerable. (True/False)

(iii) Usual metric on $X=\left\{1/n : n \in N\right\}$ induces a :

(a) Indiscrete topology on X

(b) Discrete topology on X

(c) Co-finite topology on X

(d) Co-countable topology on X

(iv) $\overline{A \cap B}$ is a proper subset of :

(a) $A \cup B$

(b) $\overline{A} \cup \overline{B}$

(c) $\overline{A} \cap \overline{B}$

(d) $\overline{A \cup B}$

(v) First countability implies second countability. (True/False).

(vi) Every compact subset of a Hausdorff space is not closed. (True/False).

(vii) A compact subset of a metric space is :

(a) Open and bounded

(b) Closed

(c) Bounded

(d) Both (b) and (c)

(viii) "A subset of R is compact if and only if it is bounded and closed" this theorem is known as :

(a) Heine Borel theorem

(b) Lindelof theorem

(c) Tietze extension theorem

(d) Urysohn's lemma

(ix) Every T_1-space is T_2-space. (True/False).

(x) Every compact Hausdorff space is a T_3-space. (True/False).

SECTION - B

(Very Short Answer Type Questions)

Q. 2. Answer the following questions in two or three sentences. (Attempt any five) : **2×5=10**

(i) Define T_3-space.

(ii) State Tietze extension theorem.

(iii) Give an example of a T_2-space.

(iv) Define sequential compactness.

(v) Give an example of a second countable space.

(vi) Write Kuratowski's closure axiom.

(vii) Find a mapping of X into R which is not T-U continuous.

SECTION - C

(Short Answer Type Questions)

Q. 3. Attempt any five questions : **4×5=20**

(i) Show that :

$$(A \cap B)^{o} = A^{o} \cap B^{o}$$

(ii) "A subset of a topological space is open if and only if it is a neighbourhood of each of its points". Prove it.

(iii) Let X = {a, b, c} and let

T = {ϕ, {a}, {a, b}, {a, c}, X}

Find all limit points of the set A = {b, c}.

(iv) Give an example of a T_2-space which is not T_3.

(v) Prove that "Every closed subsets of compact sets are compact".

(vi) Show by means of an example that locally compact space need not be compact.

(vii) Let A be a subset of a topological space. Then show that $\overline{A}$ is the smallest closed set containing A.

SECTION - D

(Long Answer Type Questions)

Q. 4. Attempt any three questions : **3×10=30**

(i) "A topological space X is compact if and only if every collection of closed subsets of X with the finite intersection property is fixed, that is, has a non-empty intersection." Prove it.

(ii) A topological space X is regular if and only if every point $x \in X$ and every neighbourhood N of x, there exists a neighbourhood M of x such that $\overline{M} \subset N$.

(iii) Show that "Every compact Hausdorff space is normal (T_4)."

(iv) Let (X, $\mathcal{T}$) and (Y, V) be topological spaces and let f be a bijective mapping of X into Y. Then the following statements are equivalent :

(a) f is a homeomorphism

(b) f is continuous and open

(c) f is continuous and closed