

AJ-1541
M.A./M.Sc. (Previous)
Term End Examination, 2021-22
MATHEMATICS (Paper-III)
(TOPOLOGY)

Time : Three hours]

[Maximum Marks : 100

Note: Answer any five questions. All questions carry equal marks.

1. (a) Prove that the intersection of two topologies for X is also a topology for x .
(b) Prove that $\tilde{A} = AUD(A)$
2. (a) Let x and y be two topological spaces. A mapping $f: x \rightarrow y$ is continuous iff the inverse image under f of every open set in y is open in x .
(b) If F_1 and F_2 be two closed subsets of a topological space X . Then show that $F_1 \cup F_2$ is also closed set.
3. (a) Show that a subset of a topological space is open if and only if it is a neighbourhood of each of its points.
(b) Define the following:
(i) Local Base at a point, (ii) Base for topology,
(iii) Neighbourhood, (iv) Net
(v) Filter
4. (a) Show that every discrete space is locally connected.
(b) Show that every compact subset A of a Hausdorff space x is closed.
5. (a) Show that the property of a space being a regular space is topological property.
(b) Show that Normality is a topological property.
6. State and prove Urysohn's Lemma.
7. Show that a topological space X is completely regular if and only if for every $x \in X$ and every open set G containing x , there exists a continuous mapping f of X into $[0, 1]$ such that $f(x) = 0$ and $f(y) = 1 \quad \forall \quad y \in X - G$.
8. Show that the product space $X \times Y$ is connected if and only if X and Y are connected.
9. Show that Each projection map π_x is an open map.
10. Show that, the product space $x = \times \{x_\lambda : x \in \lambda\}$ is Hausdorff iff each coordinate space x_λ is Hausdorff.