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H-5344

M.A./M.Sc. (Semester-IV) July 2022

MATHEMATICS

(Numerical Analysis-II)

Time Allowed : Three Hours

Maximum Marks : 70

Minimum Passing Marks : 25

Note : This question paper is divided into **four** sections. **All** sections are **compulsory**. Attempt questions as per instructions given in each section.

Section-A

Note : Attempt **any ten** questions. Each question carries **1** mark. Choose correct answer. [1×10=10]

1. (i) The bisection method is applied to compute a zero of the function $f(x) = x^4 - x^3 - x^2 - 4$

in the interval [1, 9]. The method converges to a solution after.....iterations.

(a) 1

(b) 3

(c) 5

(d) 7

(ii) In which of the following categories can we put Bisection method?

(a) Bracket solutions

(b) Graphical solutions

(c) Empirical solutions

(d) Trial solutions

(iii) The convergence of a bisection method is :

(a) Very slow

(b) Exponential

(c) Quadratic

(d) Very fast

(iv) The second approximation to a root of the equation $x^3 - x - 1 = 0$ in the interval (1, 2) by Bisection method will be :

(a) 1.15

(b) 1.25

(c) 1.35

(d) 1.45

(v) With initial approximation of $x_1 = x_2 = x_3 = 0$, what is the next value of x_1 in the following set of simultaneous equations?

$27x_1 + 6x_2 - x_3 = 81$

$6x_1 + 15x_2 + 2x_3 = 75$

$x_1 + x_2 + 50x_3 = 110$

(a) 2.25

(b) 3.0

(c) 3.25

(d) 4.0

(vi) The procedure adopted in the Gauss-Jordan method in solving linear simultaneous equation is :

(a) It is required to assume initial approximate value of the variables.

(b) It reduces the given system of equations to a diagonal matrix.

(c) It reduces the given system of equations to an equivalent triangular system.

(d) The given matrix is factored into lower and upper triangular matrices.

(vii) The Wronskian of the differential equation

$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{-e^x}{1+e^x}$ is :

(a) e^x

(b) $-e^{3x}$

(c) e^{2x}

(d) e^{5x}

(viii) Consider the following differential equation

$$x(ydx+xdy)\cos\frac{y}{x}=y(xdy-ydx)\sin\frac{y}{x}.$$

Which of the following is the solution of the above equation? (c is an arbitrary constant)

(a) $\frac{x}{y}\cos\frac{y}{x}=c$

(b) $\frac{x}{y}\sin\frac{y}{x}=c$

(c) $xy\cos\frac{y}{x}=c$

(d) $xy\sin\frac{y}{x}=c$

(ix) The LU method of factorization was introduced by the mathematician:

(a) Alan Tango

(b) David Hilbert

(c) G.W. Leibniz

(d) Alex. Grothendieck

(x) What is the other name for factorization method?

(a) Doolittle's method

(b) Lin Bairstow method

(c) Muller's method

(d) Decomposition method

Section-B

(Very Short Answer Type Questions)

Note : Attempt **any five** questions. Each question carries **2** marks. [2×5=10]

2. Answer the following questions in 2-3 sentences :

(a) Establish whether the system :

$$1.01x+2y=2.01$$
$$x+2y=2$$

is well conditioned or not?

(b) Write the important method of solving ordinary differential equation of first order numerically.

(c) Write the theorem of Descartes' rule of signs.

(d) Separate the real roots of the equation :

$$x^5 - 80x + 6 = 0$$

(e) Find the upper limit of positive roots of the following equation :

$$x^6 - 13x^5 - 14x^4 + x^2 - 6x - 20 = 0$$

(f) Apply Gauss-Jordan method to find the inverse of :

$$\begin{bmatrix} 2 & 6 & 6 \\ 2 & 8 & 6 \\ 2 & 6 & 8 \end{bmatrix}$$

Section-C

(Short Answer Type Questions)

Note : Attempt **any five** questions. Each question carries **4** marks. [4×5=20]

3. (a) Apply Jordan's method to solve :

$$10x - 7y + 3z + 5u = 6$$
$$-6x + 8y - z - 4u = 5$$
$$3x - 9y - 2z + 4u = 7$$

(b) Find a real root of the equation $f(x) = x^3 + x^2 - 1 = 0$ by using iteration method.

(c) Find the root of $x^2 - 5x + 2 = 0$ correct to five decimal places by Newton-Raphson method.

(d) Find solution of the simultaneous equations

$$\frac{dx}{dt} = xy + 2t$$

$$\frac{dy}{dt} = 2ty + x$$

Subject to the initial conditions

$$x = 1, y = -1, t = 0$$

(e) Using Taylor's series. Find the solution of the differential equation $xy' = x - y$; $y(2) = 2$ at $t = 2.1$ correct to five places of decimal.

(f) Use Picard's method to approximate y when $x = 0.1$ given that $y = 1$ when $x = 0$ and

$$\frac{dy}{dx} = \frac{y - x}{y + x}$$

Section-D

(Long Answer Type Questions)

Note : Attempt **any three** questions. Each question carries **10** marks. [3×10=30]

4. (i) Given that $\frac{dy}{dx} = x + y$ with the initial condition that $y = 1$ when $x = 0$. Find y for $x = 0.05$ and $x = 0.1$. Give the correct result upto four decimal places.

(ii) Solve the equations :

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8$$

by the factorization method.

(iii) Factorize the matrix :

$$A = \begin{bmatrix} -2 & 4 & 8 \\ -4 & 18 & -16 \\ -6 & 2 & -20 \end{bmatrix}$$

in the form LDU and use the result $A^{-1} = (U)^{-1} D^{-1} L^{-1}$ to obtain A^{-1}.

(iv) Apply Crout's method to obtain the inverse of

$$\begin{pmatrix} 2 & -2 & 4 \\ 2 & 3 & 2 \\ -1 & 1 & -1 \end{pmatrix}$$

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