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K-1044

M.A./M.Sc. (Previous) Examination, 2022

MATHEMATICS

(Advance Abstract Algebra)

[Paper : First]

Time Allowed : Three Hours

Maximum Marks : 100

Minimum Passing Marks : 33

Note : Attempt **any five** questions. All questions carry **equal** marks.

1. Let N be a subgroup of a group G. Then prove that the following are equivalent :

   (i) $N \lhd G$

   (ii) $xNx^{-1} = N$ for every $x \in G$

(iii) $xN = Nx$ for every $x \in G$

(iv) $(xN)(yN) = xyN$ for all $x, y \in G$

2. Prove that any two subnormal series of a group have equivalent refinements.

3. (a) Prove that a group G is solvable if and only if $G^{(k)} = e$ for some integer k.

(b) Prove that a group G is nilpotent if and only if G has a normal series $(e) = G_0 \subset G_1 \subset G_2 \subset \ldots\ldots \subset G_m \supset G$, such that $G_i / G_{i-1} \subset Z(G/G_{i-1})$ for all $i = 1, 2, \ldots.m$.

4. (a) Prove that an ideal S in the ring of integer I is maximal if and only if S can be generated by integer.

(b) Prove that an ideal I of R a commutative ring with unity is maximal if and only if R/I is field.

5. Let $F \subseteq E \subseteq K$ be field. If K is finite extension of E and E is a finite extension of F, then K is a finite extension of F, and $[K : F] = [K : E][E : F]$.

6. Let F be a field. Then there exists an algebrically closed field K containing F as a subfield.

7. (a) State and prove uniqueness of splitting field.

   (b) Let E be a finite extension of F. Then prove that E is a normal extension of F if and only if E is a splitting field of some polynomials over F.

8. If E be a finite extension of F, then prove that E is simple extension of F if and only if there are only a finite number of subfields of E containing F.

9. State and prove fundamental theorem of Galois theorem.

10. State and prove Hilbert's basis theorem.

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