Total Pages : 4

K-1045

M.A./M.Sc. (Previous) Examination, 2022

MATHEMATICS

(Real Analysis and Measure Theory)

[Paper : Second]

Time Allowed : Three Hours

Maximum Marks : 100

Minimum Passing Marks : 33

Note : Attempt **any five** questions. All questions carry **equal** marks.

1. Define Riemann-Stieltjes integral and prove that if $f_1 \in \Re(\alpha)$ and $f_2 \in \Re(\alpha)$ on $[a,b]$; then $f_1 + f_2 \in \Re(\alpha)$ and

$$\int_a^b (f_1 + f_2)\, d\alpha = \int_a^b f_1\, d\alpha + \int_a^b f_2\, d\alpha$$

2. Show that :

(a) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - ..... = \log 2$

(b) $1 - \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{6} - \frac{1}{8} + \frac{1}{5} - ..... = \frac{1}{2}\log 2$

3. State and prove Abel's test for uniform convergence.

4. State and prove Weierstrass's approximation theorem.

5. Prove that :

If $\lim_{n\to\infty} n\, a_n = 0$ and $f(x) = \sum a_n x^n$ with $\lim_{n\to 1-0}$ $f(x) = s$, then the series $\sum a_n$ converges to the sum s.

6. Find the maximum and minimum values of the function

$$f(x, y) = 2x^2 - 3y^2 - 2x$$

subject to the constraint $x^2 + y^2 \leq 1$.

7. Prove that : The outer measure of an interval is its length.

8. Show that if $\alpha > 1$,

$$\int_0^1 \frac{x \sin x}{1+(nx)^{\alpha}} dx = 0\left(n^{-1}\right) as\ n \to \infty$$

9. Prove that : An outer measure is monotonic and σ-subadditive.

10. State and prove Minkowski's Inequality.

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