

AJ-1549
M.A./M.Sc. (Final) Mathematics
Term End Examination, 2021-22
(Paper-V)
(Difference Equation)

Time : 3 hrs.]

[Maximum Marks : 100

Note : Answer any five questions. All questions carry equal marks.

1. (a) Show that :

$${}^{n+1}C_1 U_0 + {}^{n+1}C_2 \Delta U_0 + {}^{n+1}C_3 \Delta^2 U_0 + \dots + \Delta^n U_0 = U_0 + U_1 + U_2 + \dots + U_n$$

(b) Express $f(x) = x^4 - 4x^3 + 7x^2 + 3x - 6$ in terms of the factorial notations.2. (a) Solve $3y_{x+1} + 2y_x = 1$, when $y_0 = 1$ (b) If $\langle y_x \rangle$ is the solution sequence of the difference equation $y_{x+1} = y_{x+B}$, $x = 0, 1, 2$ with y_0 prescribed then $\langle y_x \rangle$ is a constant sequence, if $B = 0$ it diverges to $+\infty$, if $B > 0$ and diverges to $-\infty$, if $B < 0$

3. (a) Solve :

$$y_{x+1} + y_x = 2 \text{ when } x = 0, 1, 2$$

(b) Solve :

$$\Delta y_x + \Delta^2 y_x = \sin x$$

4. (a) If $Z[\langle f(x) \rangle] = F(z)$ then

$$Z[\langle x f(x) \rangle] = -z \frac{df(z)}{dz}.$$

(b) Using the z-transform solve the equation

$$f_{n+z} - 2f_{n+1} + f_n = 2n + 5$$

5. (a) Use generating function to show that :

$$\sum_{k=0}^n c(n, k)^2 = c(2n, n) \text{ where } n \text{ is a positive integer.}$$

(b) Solve the difference equation $a_r = 8a_{r-1} + 10^{r-1}$, with initial condition $a_0 = 1$

6. (a) For the system of equations

$$\frac{dx}{dt} = x - y + xy, \quad \frac{dy}{dt} = 3x - 2y - xy$$

(b) Find the critical points of the system $\frac{dx}{dt} = -y$, $\frac{dy}{dt} = x$ and discuss its nature and stability. Also find the general solution of the system and sketch the trajectories.

7. (a) State and prove that Sturm separation theorem.

(b) Let f and g be real Linearly independent solution of $\frac{d}{dx} \left[P(x) \frac{dy}{dx} \right] + Q(x)y = 0$ on $x \in [a, b]$. Then between any two consecutive zeros of f , there is precisely one zero of g .

8. (a) Define a Sturm-Liouville problem of boundary value problem with example.
 (b) Find the formal expansion of the function f equation where $f(x) = \pi x - x^2$, $x \in [0, \pi]$ in the series of orthonormal characteristics function $\{g_n\}$ of the Sturm-Liouville problem.
9. (a) Show that the shortest curve joining two fixed points is a straight line.
 (b) On which curve the functional

$$I[y(x)] = \int_0^\pi [y'^2 - y^2 + 4y \cos x] dx$$

$$y(0) = 0, y(\pi) = 0 \text{ be extremalized}$$

10. (a) Explain the uniqueness of characteristic values of the boundary value problem.
 (b) Transform the equation

$$y'' + \frac{1}{z} y' + \left(x - \frac{N^2}{Z^2} \right) y = 0$$

into normal form.