

PE-255-CV-19
M.A./M.Sc. (2nd Semester)
Examination, June 2022
MATHEMATICS
Paper-III
TOPOLOGY (II)

Time : Three Hours]

[Maximum Marks : 80
[Minimum Pass Marks : 29

Note : Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. Answer the following questions:

1x10=10

- (a) Define Separated sets.
- (b) Define connected and disconnected space.
- (c) Define open cover and sub cover.
- (d) Define compact space.
- (e) Define Net.
- (f) Define Filter.
- (g) Define Ultra-Filter.
- (h) Define Ultra-Net.
- (i) Define locally connected space.
- (j) Define projection mapping.

2. Answer the following questions:

2x5=10

Answer the following short answer type questions:-

- (a) If $A =]2, 3[$, $B =]3, 4[$ [Show that A & B are disjoint but not separated.
- (b) If A & B are separated subsets of a space X & $C \subset A$ & $D \subset B$, then D are also Separated.
- (c) Show that closed subsets of compact sets are compact.
- (d) Define cluster point of a Net, with an example.
- (e) Show that every filter base on an set X is contained in an ultrafilter on X.

Section-B

12x5=60

Answer the Questions from the following:

- 3. a). Two closed (open) subsets A, B of a topological space are separated if and only if they are disjoint.
b). Show that continuous image of a connected space is connected.
OR
a). Show that, Every component of a topological space (X, J) is closed.
b). Show that the components of a totally disconnected space (X,J) are its points.
- 4. a). Show that $C^* = \{(-n, n): n \in \mathbb{N}\}$ is a U-open cover of $\mathbb{R}$.
b). Show that every compact subset A of a Hausdorff space X is closed.
OR
a). Show that closed subset of a compact space are compact.
b). Show that every closed and bounded interval on $\mathbb{R}$ is compact where $\mathbb{R}$ has the usual topology U.
- 5. State and prove Heine-Borel theorem.

OR

A topological space X is disconnected if and only if there exists a non-empty proper subset of X which is both open and closed.

6. Let Y be a subspace of topological space X and let $A \subset Y$. Then show that A is compact relative to X if and only if A is compact relative to Y .

OR

Prove that product of non-empty closed compact spaces is compact.

7. State and prove Tychonoff Embedding theorem.

OR

Let $(X, \mathcal{J})$ be a topological space and let $Y \subset X$. Then show that Y is $\mathcal{J}$ -Closed if and only if no net in Y converges to a point in $X - Y$.