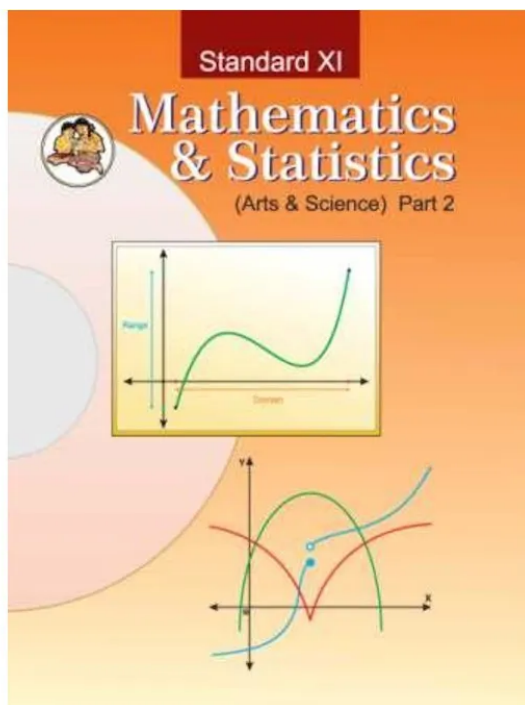


Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 2- Sequences and Series

Class 11 - Chapter 2 Sequences and Series



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Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 2- Sequences and Series

Class 11: Maths Chapter 2 solutions. Complete Class 11 Maths Chapter 2 Notes.

Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 2- Sequences and Series

Maharashtra Board 11th Maths Chapter 2, Class 11 Maths Chapter 2 solutions

Ex. 2.1

Question 1.

Check whether the following sequences are G.P. If so, write t_n .

(i) 2, 6, 18, 54,

Solution:

2, 6, 18, 54, ...

$t_1 = 2, t_2 = 6, t_3 = 18, t_4 = 54, \dots$

Here, $\frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = 3$

$\therefore$ the ratio of any two consecutive terms is a constant, the given sequence is a Geometric progression.

Here, $a = 2, r = 3$

$t_n = ar^{n-1}$

$\therefore t_n = 2(3^{n-1})$

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(ii) 1, -5, 25, -125,

Solution:

$$1, -5, 25, -125, \dots$$

$$t_1 = 1, t_2 = -5, t_3 = 25, t_4 = -125$$

$$\text{Here, } \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = -5$$

$\therefore$ the ratio of any two consecutive terms is a constant, the given sequence is a Geometric progression.

$$\text{Here, } a = 1, r = -5$$

$$t_n = ar^{n-1}$$

$$\therefore t_n = (-5)^{n-1}$$

(iii) $\sqrt{5}, \frac{1}{\sqrt{5}}, \frac{1}{5\sqrt{5}}, \frac{1}{25\sqrt{5}}, \dots$

Solution:

$$\sqrt{5}, \frac{1}{\sqrt{5}}, \frac{1}{5\sqrt{5}}, \frac{1}{25\sqrt{5}}, \dots$$

$$t_1 = \sqrt{5}, t_2 = \frac{1}{\sqrt{5}}, t_3 = \frac{1}{5\sqrt{5}}, t_4 = \frac{1}{25\sqrt{5}}, \dots$$

$$\text{Here, } \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = \frac{1}{5}$$

$\therefore$ the ratio of any two consecutive terms is a constant, the given sequence is a Geometric progression.

$$\text{Here, } a = \sqrt{5}, r = \frac{1}{5}$$

$$t_n = ar^{n-1}$$

$$\therefore t_n = \sqrt{5} \left(\frac{1}{5}\right)^{n-1}$$

(iv) 3, 4, 5, 6,

Solution:

$$3, 4, 5, 6, \dots$$

$$t_1 = 3, t_2 = 4, t_3 = 5, t_4 = 6$$

$$\text{Here, } \frac{t_2}{t_1} = \frac{4}{3}, \frac{t_3}{t_2} = \frac{5}{4}, \frac{t_4}{t_3} = \frac{6}{5}$$

$\therefore \frac{t_2}{t_1} \neq \frac{t_3}{t_2} \neq \frac{t_4}{t_3}$, the given sequence is not a Geometric progression.

(v) 7, 14, 21, 28,

Solution:

7, 14, 21, 28, ...

$t_1 = 7, t_2 = 14, t_3 = 21, t_4 = 28$

Here, $\frac{t_2}{t_1} = 2, \frac{t_3}{t_2} = \frac{3}{2}, \frac{t_4}{t_3} = \frac{4}{3}$

$\therefore \frac{t_2}{t_1} \neq \frac{t_3}{t_2} \neq \frac{t_4}{t_3}$, the given sequence is not a Geometric progression.

Question 2.

For the G.P.

(i) If $r = \frac{1}{3}$, $a = 9$, find t_7 .

Solution:

Given, $r = \frac{1}{3}$, $a = 9$

$t_n = ar^{n-1}$

$\therefore t_7 = 9 \times \left(\frac{1}{3}\right)^{7-1}$

$$= \frac{9}{3^6}$$

$$= \frac{1}{81}$$

(ii) If $a = \frac{7}{243}$, $r = 3$, find t_6 .

Solution:

$$\text{Given, } a = \frac{7}{243}, r = 3$$

$$t_n = ar^{n-1}$$

$$\therefore t_6 = \frac{7}{243} \times (3)^{6-1}$$

$$= \frac{7}{243} \times 3^5$$

$$= 7$$

(iii) If $r = -3$ and $t_6 = 1701$, find a .

Solution:

$$\text{Given, } r = -3, t_6 = 1701$$

$$t_n = ar^{n-1}$$

$$\therefore t_6 = a(-3)^{6-1}$$

$$\therefore 1701 = a(-3)^5$$

$$\therefore a = \frac{1701}{-243}$$

$$\therefore a = -7$$

(iv) If $a = \frac{2}{3}$, $t_6 = 162$, find r .

Solution:

$$\text{Given, } a = \frac{2}{3}, t_6 = 162$$

$$t_n = ar^{n-1}$$

$$\therefore t_6 = \left(\frac{2}{3}\right)(r^{6-1})$$

$$\therefore 162 = \frac{2}{3} r^5$$

$$\therefore r^5 = 162 \times \frac{3}{2}$$

$$\therefore r^5 = 3^5$$

$$\therefore r = 3$$

Question 3.

Which term of the G. P. 5, 25, 125, 625, is 5^{10} ?

Solution:

Here, $t_1 = a = 5$, $r = \frac{t_2}{t_1} = \frac{25}{5} = 5$, $t_n = 5^{10}$

$$\begin{aligned} t_n &= ar^{n-1} \\ \therefore 5^{10} &= 5 \times 5^{(n-1)} \\ \therefore 5^{10} &= 5^{(1+n-1)} \\ \therefore 5^{10} &= 5^n \\ \therefore n &= 10 \\ \therefore 5^{10} &\text{ is the } 10^{\text{th}} \text{ term of the G.P.} \end{aligned}$$

Alternate Method:

$$\begin{aligned} t_1 &= 5, t_2 = 25 = 5^2, t_3 = 125 = 5^3, t_4 = 625 = 5^4, \\ \therefore t_{10} &= 5^{10} \end{aligned}$$

Question 4.

For what values of x , the terms $\frac{4}{3}$, x , $\frac{4}{27}$ are in G. P.?

Solution:

$$\begin{aligned} \frac{4}{3}, x, \frac{4}{27} &\text{ are in Geometric progression.} \\ \therefore \frac{t_2}{t_1} &= \frac{t_3}{t_2} \\ \therefore \frac{x}{\frac{4}{3}} &= \frac{\frac{4}{27}}{x} \\ \therefore x^2 &= \frac{4}{3} \times \frac{4}{27} \\ \therefore x^2 &= \frac{16}{81} \end{aligned}$$

Question 5.

If for a sequence, $t_n = \frac{5^{n-3}}{2^{n-3}}$, show that the sequence is a G. P. Find its first term and the common ratio.

Solution:

The sequence (t_n) is a G.P. if $\frac{t_{n+1}}{t_n} = \text{constant}$ for

all $n \in \mathbb{N}$.

$$\text{Now, } t_n = \frac{5^{n-3}}{2^{n-3}}$$

$$t_{n+1} = \frac{5^{n+1-3}}{2^{n+1-3}} = \frac{5^{n-2}}{2^{n-2}}$$

$$\begin{aligned}\therefore \frac{t_{n+1}}{t_n} &= \frac{5^{n-2}}{2^{n-2}} \times \frac{2^{n-3}}{5^{n-3}} \\ &= (5)^{(n-2)-(n-3)} \times (2)^{(n-3)-(n-2)} \\ &= (5)^1 \times (2)^{-1} \\ &= \frac{5}{2}, \text{ which is a constant, for all } n \in \mathbb{N}.\end{aligned}$$

$$\therefore r = \frac{5}{2}$$

$\therefore$ The sequence is a G.P. with common ratio $\frac{5}{2}$.

$$\text{First term } t_1 = \frac{5^{1-3}}{2^{1-3}} = \frac{5^{-2}}{2^{-2}} = \frac{2^2}{5^2} = \frac{4}{25}$$

Question 6.

Find three numbers in G. P. such that their sum is 21 and the sum of their squares is 189.

Solution:

Let the three numbers in G. P. be $\frac{a}{r}$, a , ar .

According to the given conditions,

$$\frac{a}{r} + a + ar = 21$$

$$\therefore \frac{1}{r} + 1 + r = \frac{21}{a}$$

$$\therefore \frac{1}{r} + r = \frac{21}{a} - 1 \quad \dots(i)$$

$$\text{and } \frac{a^2}{r^2} + a^2 + a^2 r^2 = 189$$

$$\therefore \frac{1}{r^2} + 1 + r^2 = \frac{189}{a^2}$$

$$\therefore \frac{1}{r^2} + r^2 = \frac{189}{a^2} - 1 \quad \dots(ii)$$

On squaring equation (i), we get

$$\frac{1}{r^2} + r^2 + 2 = \frac{441}{a^2} - \frac{42}{a} + 1$$

$$\therefore \left(\frac{189}{a^2} - 1 \right) + 2 = \frac{441}{a^2} - \frac{42}{a} + 1 \quad \dots[\text{From (ii)}]$$

$$\therefore \frac{189}{a^2} + 1 = \frac{441}{a^2} - \frac{42}{a} + 1$$

$$\therefore \frac{441}{a^2} - \frac{189}{a^2} = \frac{42}{a}$$

$$\therefore \frac{441}{a^2} - \frac{189}{a^2} = \frac{42}{a}$$

$$\therefore \frac{252}{a^2} = \frac{42}{a}$$

$$\therefore 252 = 42a$$

$$\therefore a = 6$$

Substituting the value of a in (i), we get

$$\frac{1}{r} + r = \frac{21}{6} - 1$$

$$\therefore \frac{1+r^2}{r} = \frac{15}{6}$$

$$\therefore \frac{1+r^2}{r} = \frac{5}{2}$$

$$\therefore 2r^2 - 5r + 2 = 0$$

$$\therefore 2r^2 - 4r - r + 2 = 0$$

$$\therefore (2r-1)(r-2) = 0$$

$$\therefore r = \frac{1}{2} \text{ or } 2$$

$$\text{When } a = 6, r = \frac{1}{2},$$

$$\frac{a}{r} = 12, a = 6, ar = 3$$

When $a = 6, r = 2,$

$$\frac{a}{r} = 3, a = 6, ar = 12$$

Hence, the three numbers in G.P. are 12, 6, 3 or 3, 6, 12.

Check:

If sum of the three numbers is 21 and sum of their squares is 189, then our answer is correct.

$$\text{Sum of the numbers} = 12 + 6 + 3 = 21$$

$$\text{Sum of the squares of the numbers} = 12^2 + 6^2 + 3^2$$

$$= 144 + 36 + 9$$

$$= 189$$

Thus, our answer is correct.

Question 7.

Find four numbers in G. P. such that the sum of the middle two numbers is $\frac{10}{3}$ and their product is 1.

Solution:

Let the four numbers in G.P. be $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

According to the given conditions,

$$\frac{a}{r^3} \left(\frac{a}{r} \right) (ar)(ar^3) = 1$$

$$\therefore a^4 = 1$$

$$\therefore a = 1 \quad \dots(i)$$

$$\text{and } \frac{a}{r} + ar = \frac{10}{3}$$

$$\therefore \frac{1}{r} + (1)r = \frac{10}{3} \quad \dots[\text{From (i)}]$$

$$\therefore \frac{1+r^2}{r} = \frac{10}{3}$$

$$\therefore 3 + 3r^2 = 10r$$

$$\therefore 3r^2 - 10r + 3 = 0$$

$$\therefore (r - 3)(3r - 1) = 0$$

$$\therefore r = 3 \text{ or } r = \frac{1}{3}$$

When $a = 1, r = 3,$

$$\frac{a}{r^3} = \frac{1}{(3)^3} = \frac{1}{27}, \frac{a}{r} = \frac{1}{3}, ar = 1(3) = 3 \text{ and}$$

$$ar^3 = 1(3)^3 = 27$$

When $a = 1, r = \frac{1}{3},$

$$\frac{a}{r^3} = \frac{1}{\left(\frac{1}{3}\right)^3} = 27, \frac{a}{r} = \frac{1}{\left(\frac{1}{3}\right)} = 3,$$

$$ar = 1\left(\frac{1}{3}\right) = \frac{1}{3} \text{ and } ar^3 = 1\left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

Hence, the four numbers in G.P. are

$$\frac{1}{27}, \frac{1}{3}, 3, 27 \text{ or } 27, 3, \frac{1}{3}, \frac{1}{27}.$$

Question 8.

Find five numbers in G. P. such that their product is 1024 and the fifth term is square of the third term.

Solution:

Let the five numbers in G. P. be

$$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$$

According to the given conditions,

$$\frac{a}{r^2} \times \frac{a}{r} \times a \times ar \times ar^2 = 1024$$

$$\therefore a^5 = 4^5$$

$$\therefore a = 4 \quad \dots(i)$$

$$\text{and } ar^2 = a^2$$

$$\therefore r^2 = a$$

$$\therefore r^2 = 4 \quad \dots[\text{From (i)}]$$

$$\therefore r = \pm 2$$

When $a = 4, r = 2,$

$$\frac{a}{r^2} = 1, \frac{a}{r} = 2, a = 4, ar = 8, ar^2 = 16$$

When $a = 4, r = -2,$

$$\frac{a}{r^2} = 1, \frac{a}{r} = -2, a = 4, ar = -8, ar^2 = 16$$

Hence, the five numbers in G.P. are

1, 2, 4, 8, 16 or 1, -2, 4, -8, 16.

Question 9.

The fifth term of a G. P. is x, the eighth term of a G.P. is y and the eleventh term of a G.P. is z, verify whether $y^2 = xz$.

Solution:

$$\text{Given, } t_5 = x, t_8 = y, t_{11} = z$$

$$\text{Since } t_n = ar^{n-1},$$

$$t_5 = ar^4, t_8 = ar^7, t_{11} = ar^{10}$$

Consider,

$$\text{L.H.S.} = y^2$$

$$= (t_8)^2 = (ar^7)^2 = a^2 r^{14}$$

$$\text{R.H.S.} = xz$$

$$= t_5 \cdot t_{11} = ar^4 \cdot ar^{10} = a^2 r^{14}$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$$\therefore y^2 = xz$$

Question 10.

If p, q, r, s are in G.P., show that $p + q, q + r, r + s$ are also in G. P.

Solution:

p, q, r, s are in G.P.

$$\therefore \frac{q}{p} = \frac{r}{q} = \frac{s}{r}$$

$$\text{Let } \frac{q}{p} = \frac{r}{q} = \frac{s}{r} = k$$

$$\therefore q = pk, r = qk, s = rk$$

We have to prove that $p + q, q + r, r + s$ are in G.P.

i.e., to prove that $\frac{q+r}{p+q} = \frac{r+s}{q+r}$

$$\text{L.H.S.} = \frac{q+r}{p+q} = \frac{q+qk}{p+pk} = \frac{q(1+k)}{p(1+k)} = \frac{q}{p} = k$$

$$\text{R.H.S.} = \frac{r+s}{q+r} = \frac{r+rk}{q+qk} = \frac{r(1+k)}{q(1+k)} = \frac{r}{q} = k$$

$$\therefore \frac{q+r}{p+q} = \frac{r+s}{q+r}$$

$\therefore p + q, q + r, r + s$ are in G.P.

Question 11.

The number of bacteria in a culture doubles every hour. If there were 50 bacteria originally in the culture, how many bacteria will be there at the end of the 5th hour?

Solution:

Since the number of bacteria in culture doubles every hour, increase in number of bacteria after

every hour is in G.P.

$$\therefore a = 50, r = \frac{100}{50} = 2$$

$$t_n = ar^{n-1}$$

To find the number of bacteria at the end of the 5th hour.

(i.e., to find the number of bacteria at the beginning of the 6th hour, i.e., to find t_6 .)

$$\therefore t_6 = ar^5$$

$$= 50 \times (2^5)$$

$$= 50 \times 32$$

$$= 1600$$

Question 12.

A ball is dropped from a height of 80 ft. The ball is such that it rebounds $\left(\frac{3}{4}\right)^{\text{th}}$ of the height it has fallen. How high does the ball rebound on the 6th bounce? How high does the ball rebound on the n^{th} bounce?

Solution:

Since the ball rebounds $\left(\frac{3}{4}\right)^{\text{th}}$ of the height it has fallen, the height in successive bounce is in G.P.

$$\text{1st height in the bounce} = 80 \times \frac{3}{4}$$

$$a = 80 \times \frac{3}{4}, r = \frac{3}{4}$$

$$t_n = ar^{n-1}$$

$$\text{Height in the 6th bounce} = 80 \times \frac{3}{4} \times \left(\frac{3}{4}\right)^5$$

$$= 80 \times \left(\frac{3}{4}\right)^6$$

$$\text{Height in the } n^{\text{th}} \text{ bounce} = 80 \left(\frac{3}{4}\right)^n$$

Question 13.

The numbers 3, x and x + 6 are in G. P. Find

(i) x

(ii) 20th term

(iii) nth term.

Solution:

(i) 3, x and x + 6 are in G. P.

$$\frac{x}{3} = \frac{x+6}{x}$$

$$x^2 = 3x + 18$$

$$x^2 - 3x - 18 = 0$$

$$(x - 6)(x + 3) = 0$$

$$x = 6, -3$$

$$\text{ii. } r = \frac{6}{3} = 2 \quad \text{or } r = \frac{-3}{3} = -1$$

$$t_n = ar^{n-1}$$

$$\therefore t_{20} = 3(2^{19}) \quad \text{or } t_{20} = 3(-1)^{19}$$

$$\therefore t_{20} = 3(2^{19}) \quad \text{or } t_{20} = -3$$

$$\text{iii. } t_n = 3(2)^{n-1} \quad \text{or } t_n = 3(-1)^{n-1}$$

Question 14.

Mosquitoes are growing at a rate of 10% a year. If there were 200 mosquitoes in the beginning,

write down the number of mosquitoes after

(i) 3 years

(ii) 10 years

(iii) n years

Solution:

$$a = 200, r = 1 + \frac{10}{100} = \frac{11}{10}$$

$$\text{Mosquitoes at the end of 1st year} = 200 \times \frac{11}{10}$$

(i) Number of mosquitoes after 3 years

$$= 200 \times \frac{11}{10} \times \left(\frac{11}{10}\right)^2$$

$$= 200 \left(\frac{11}{10}\right)^3$$

$$= 200 (1.1)^3$$

$$(ii) \text{ Number of mosquitoes after 10 years} = 200 (1.1)^{10}$$

$$(iii) \text{ Number of mosquitoes after } n \text{ years} = 200 (1.1)^n$$

Question 15.

The numbers $x - 6$, $2x$ and x^2 are in G. P. Find

(i) x

(ii) 1st term

(iii) n th term

Solution:

(i) $x - 6$, $2x$ and x are in Geometric progression.

$$\therefore \frac{2x}{x-6} = \frac{x^2}{2x}$$

$$4x^2 = x^2(x - 6)$$

$$4 = x - 6$$

$$x = 10$$

$$(ii) t_1 = x - 6 = 10 - 6 = 4$$

$$iii. \quad a = 4, r = \frac{2x}{x-6} = \frac{2(10)}{4} = 5$$

$$t_n = ar^{n-1}$$

$$\therefore t_n = 4(5^{n-1})$$

Ex 2.2

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Question 1.

For the following G.P.s, find S_n .

(i) 3, 6, 12, 24,

Solution:

$$3, 6, 12, 24, \dots$$

$$\text{Here, } a = 3, r = \frac{6}{3} = 2 > 1$$

$$S_n = \frac{a(r^n - 1)}{r - 1}, \text{ for } r > 1$$

$$\therefore S_n = \frac{3(2^n - 1)}{2 - 1}$$

$$\therefore S_n = 3(2^n - 1)$$

(ii) $p, q, \frac{q^2}{p}, \frac{q^3}{p^2}, \dots$

Solution:

$$p, q, \frac{q^2}{p}, \frac{q^3}{p^2}, \dots$$

$$\text{Here, } a = p, r = \frac{q}{p}$$

$$\text{Let } \frac{q}{p} < 1$$

$$S_n = \frac{a(1 - r^n)}{1 - r}, \text{ for } r < 1$$

$$\therefore S_n = \frac{p \left[1 - \left(\frac{q}{p} \right)^n \right]}{1 - \frac{q}{p}}$$

$$\therefore S_n = \frac{p^2}{p - q} \left[1 - \left(\frac{q}{p} \right)^n \right]$$

Let $\frac{q}{p} > 1$

$$S_n = \frac{a(r^n - 1)}{r - 1}, \text{ for } r > 1$$

$$\therefore S_n = \frac{p \left[\left(\frac{q}{p} \right)^n - 1 \right]}{\frac{q}{p} - 1} = \frac{p^2}{q - p} \left[\left(\frac{q}{p} \right)^n - 1 \right]$$

(iii) 0.7, 0.07, 0.007,

Solution:

$$a = 0.7 = \frac{7}{10}, r = \frac{0.07}{0.7} = \frac{1}{10} < 1$$

$$\begin{aligned} S_n &= \frac{a(1-r^n)}{1-r}, \text{ for } r < 1 \\ &= \frac{\frac{7}{10} \left[1 - \left(\frac{1}{10} \right)^n \right]}{1 - \frac{1}{10}} = \frac{7}{9} \left(1 - \frac{1}{10^n} \right) \end{aligned}$$

(iv) $\sqrt{5}, -5, 5\sqrt{5}, -25, \dots$

Solution:

$$a = \sqrt{5}, r = \frac{-5}{\sqrt{5}} = -\sqrt{5} < 1$$

$$S_n = \frac{a(1-r^n)}{1-r}, \text{ for } r < 1$$

$$= \frac{\sqrt{5} [1 - (-\sqrt{5})^n]}{1 - (-\sqrt{5})}$$

$$= \frac{\sqrt{5}}{1 + \sqrt{5}} [1 - (-\sqrt{5})^n]$$

$$= \frac{-\sqrt{5}}{(\sqrt{5} + 1)} [(-\sqrt{5})^n - 1]$$

Question 2.

For a G.P.

(i) $a = 2$, $r = -\frac{2}{3}$, find S_6 .

Solution:

$$a = 2, r = -\frac{2}{3}$$

$$S_n = \frac{a(1-r^n)}{1-r}, \text{ for } r < 1$$

$$S_6 = \frac{2 \left[1 - \left(-\frac{2}{3} \right)^6 \right]}{1 - \left(-\frac{2}{3} \right)}$$

$$= \frac{2 \left[1 - \left(\frac{2}{3} \right)^6 \right]}{\frac{5}{3}}$$

$$= \frac{6}{5} \left[\frac{729 - 64}{3^6} \right]$$

$$= \frac{6}{5} \left[\frac{665}{729} \right]$$

$$S_6 = \frac{266}{243}$$

(ii) If $S_5 = 1023$, $r = 4$, find a .

Solution:

$$r = 4, S_5 = 1023$$

$$S_n = a \left(\frac{r^n - 1}{r - 1} \right), \text{ for } r > 1$$

$$\therefore S_5 = a \left(\frac{4^5 - 1}{4 - 1} \right)$$

$$\therefore 1023 = a \left(\frac{1024 - 1}{3} \right)$$

$$\therefore 1023 = \frac{a}{3} (1023)$$

$$\therefore a = 3$$

Question 3.

For a G.P.

(i) If $a = 2$, $r = 3$, $S_n = 242$, find n .

Solution:

$$a = 2, r = 3, S_n = 242$$

$$S_n = a \left(\frac{r^n - 1}{r - 1} \right), \text{ for } r > 1$$

$$\therefore 242 = 2 \left(\frac{3^n - 1}{3 - 1} \right)$$

$$\therefore 242 = 3^n - 1$$

$$\therefore 3^n = 243$$

$$\therefore 3^n = 3^5$$

$$\therefore n = 5$$

(ii) For a G.P. sum of the first 3 terms is 125 and the sum of the next 3 terms is 27, find the value of r .

Solution:

$$S_3 = 125, S_6 = 125 + 27 = 152$$

$$S_n = a \left(\frac{1-r^n}{1-r} \right)$$

$$\therefore S_3 = a \left(\frac{1-r^3}{1-r} \right)$$

$$\therefore 125 = a \left(\frac{1-r^3}{1-r} \right) \quad \dots(i)$$

$$\text{Also, } S_6 = a \left(\frac{1-r^6}{1-r} \right)$$

$$\therefore 152 = a \left(\frac{1-r^6}{1-r} \right) \quad \dots(ii)$$

Dividing (ii) by (i), we get

$$\frac{152}{125} = \frac{1-r^6}{1-r^3}$$

$$\therefore \frac{152}{125} = \frac{(1+r^3)(1-r^3)}{(1-r^3)}$$

$$\therefore 1+r^3 = \frac{152}{125}$$

$$\therefore r^3 = \frac{152}{125} - 1$$

$$\therefore r^3 = \frac{27}{125}$$

$$\therefore r = \frac{3}{5}$$

$$\therefore r^3 = \left(\frac{3}{5}\right)^3$$

Question 4.

For a G.P.

(i) If $t_3 = 20$, $t_6 = 160$, find S_7 .

Solution:

$$t_3 = 20, t_6 = 160$$

$$t_n = ar^{n-1}$$

$$\therefore t_3 = ar^{3-1} = ar^2$$

$$\therefore ar^2 = 20$$

$$\therefore a = \frac{20}{r^2} \quad \dots(i)$$

$$\text{Also, } t_6 = ar^5$$

$$\therefore ar^5 = 160$$

$$\therefore \left(\frac{20}{r^2}\right)r^5 = 160 \quad \dots[\text{From (i)}]$$

$$\therefore r^3 = \frac{160}{20} = 8$$

$$\therefore r = 2$$

Substituting the value of r in (i) we get

$$a = \frac{20}{2^2} = 5$$

$$\text{Now, } S_n = \frac{a(r^n - 1)}{r - 1}, \text{ for } r > 1$$

$$\therefore S_7 = \frac{5(2^7 - 1)}{2 - 1} = 5(128 - 1) = 635$$

(ii) If $t_4 = 16$, $t_9 = 512$, find S_{10} .

Solution:

$$t_4 = 16, t_9 = 512$$

$$t_n = ar^{n-1}$$

$$\therefore t_4 = ar^{4-1} = ar^3$$

$$\therefore ar^3 = 16$$

$$\therefore a = \frac{16}{r^3} \quad \dots(i)$$

$$\text{Also, } t_9 = ar^8$$

$$\therefore ar^8 = 512$$

$$\therefore \frac{16}{r^3} \times r^8 = 512$$

$$\therefore r^5 = 32$$

$$\therefore r = 2$$

Substituting $r = 2$ in (i), we get

$$a = \frac{16}{2^3} = \frac{16}{8} = 2$$

$$\text{Now, } S_n = \frac{a(r^n - 1)}{r - 1}, \text{ for } r > 1$$

$$\therefore S_{10} = \frac{2(2^{10} - 1)}{2 - 1} = 2(1024 - 1) = 2046$$

Question 5.

Find the sum to n terms

(i) $3 + 33 + 333 + 3333 + \dots$

Solution:

$$\begin{aligned} S_n &= 3 + 33 + 333 + \dots \text{ upto } n \text{ terms} \\ &= 3(1 + 11 + 111 + \dots \text{ upto } n \text{ terms}) \\ &= \frac{3}{9}(9 + 99 + 999 + \dots \text{ upto } n \text{ terms}) \\ &= \frac{3}{9}[(10 - 1) + (100 - 1) + (1000 - 1) + \dots \text{ upto } n \text{ terms}] \\ &= \frac{3}{9}[(10 + 100 + 1000 + \dots \text{ upto } n \text{ terms}) - (1 + 1 + 1 + \dots n \text{ times})] \end{aligned}$$

But 10, 100, 1000, n terms are in G.P. with

$$a = 10, r = \frac{100}{10} = 10$$

$$\therefore S_n = \frac{3}{9} \left[10 \left(\frac{10^n - 1}{10 - 1} \right) - n \right] = \frac{3}{9} \left[\frac{10}{9} (10^n - 1) - n \right]$$

$$\therefore S_n = \frac{3}{81} [10(10^n - 1) - 9n]$$

(ii) $8 + 88 + 888 + 8888 + \dots$

Solution:

$$\begin{aligned} S_n &= 8 + 88 + 888 + \dots \text{ upto } n \text{ terms} \\ &= 8(1 + 11 + 111 + \dots \text{ upto } n \text{ terms}) \\ &= \frac{8}{9}(9 + 99 + 999 + \dots \text{ upto } n \text{ terms}) \end{aligned}$$

$$= \frac{80}{9} [(10 - 1) + (100 - 1) + (1000 - 1) + \dots \text{ upto } n \text{ terms}]$$
$$= \frac{80}{9} [(10 + 100 + 1000 + \dots \text{ upto } n \text{ terms}) - (1 + 1 + 1 + \dots n \text{ times})]$$

But 10, 100, 1000, ... n terms are in G.P. with

$$a = 10, r = \frac{100}{10} = 10$$

$$\therefore S_n = \frac{8}{9} \left[10 \left(\frac{10^n - 1}{10 - 1} \right) - n \right]$$

$$= \frac{8}{9} \left[\frac{10}{9} (10^n - 1) - n \right]$$

$$\therefore S_n = \frac{8}{81} [10(10^n - 1) - 9n]$$

Question 6.

Find the sum to n terms

(i) $0.4 + 0.44 + 0.444 + \dots$

Solution:

$$S_n = 0.4 + 0.44 + 0.444 + \dots \text{ upto } n \text{ terms}$$

$$= 4(0.1 + 0.11 + 0.111 + \dots \text{ upto } n \text{ terms})$$

$$= \frac{4}{9} (0.9 + 0.99 + 0.999 + \dots \text{ upto } n \text{ terms})$$

$$= \frac{4}{9} [(1 - 0.1) + (1 - 0.01) + (1 - 0.001) \dots \text{ upto } n \text{ terms}]$$

$$= \frac{4}{9} [(1 + 1 + 1 + \dots n \text{ times}) - (0.1 + 0.01 + 0.001 + \dots \text{ upto } n \text{ terms})]$$

But 0.1, 0.01, 0.001, ... n terms are in G.P. with

$$a = 0.1, r = \frac{0.01}{0.1} = 0.1$$

$$\therefore S_n = \frac{4}{9} \left\{ n - 0.1 \left[\frac{1 - (0.1)^n}{1 - 0.1} \right] \right\}$$

$$\therefore S_n = \frac{4}{9} \left\{ n - \frac{0.1}{0.9} [1 - (0.1)^n] \right\}$$

$$\therefore S_n = \frac{4}{9} \left[n - \frac{1}{9} (1 - (0.1)^n) \right]$$

$$\therefore S_n = \frac{4}{81} \left[9n - \left(1 - \frac{1}{10^n} \right) \right]$$

(ii) $0.7 + 0.77 + 0.777 + \dots$

Solution:

$S_n = 0.7 + 0.77 + 0.777 + \dots$ upto n terms

$= 7(0.1 + 0.11 + 0.111 + \dots$ upto n terms)

$= \frac{7}{9}(0.9 + 0.99 + 0.999 + \dots$ upto n terms)

$= \frac{7}{9} [(1 - 0.1) + (1 - 0.01) + (1 - 0.001) + \dots$ upto n terms]

$= \frac{7}{9} [(1 + 1 + 1 + \dots n \text{ times}) - (0.1 + 0.01 + 0.001 + \dots \text{ upto } n \text{ terms})]$

But $0.1, 0.01, 0.001, \dots$ n terms are in G.P. with

$a = 0.1, r = \frac{0.01}{0.1} = 0.1$

$$\therefore S_n = \frac{7}{9} \left\{ n - 0.1 \left[\frac{1 - (0.1)^n}{1 - 0.1} \right] \right\}$$

$$= \frac{7}{9} \left\{ n - \frac{0.1}{0.9} [1 - (0.1)^n] \right\}$$

$$= \frac{7}{9} \left\{ n - \frac{1}{9} [1 - (0.1)^n] \right\}$$

$$\therefore S_n = \frac{7}{81} \left[9n - \left(1 - \frac{1}{10^n} \right) \right]$$

Question 7.

Find the sum to n terms of the sequence

(i) $0.5, 0.05, 0.005, \dots$

Solution:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-2-sequences-and-series/>

Here, $t_1 = 0.5$, $t_2 = 0.05$, $t_3 = 0.005$

$$\therefore \frac{t_2}{t_1} = \frac{0.05}{0.5} = 0.1 \text{ and } \frac{t_3}{t_2} = \frac{0.005}{0.05} = 0.1$$

$\therefore$ The given sequence is a G.P.

$\therefore a = 0.5$ and $r = 0.1$

$$\therefore S_n = \frac{a(1-r^n)}{1-r}, \text{ for } r < 1$$

$$= \frac{0.5[1-(0.1)^n]}{1-0.1}$$

$$= \frac{0.5}{0.9} [1-(0.1)^n]$$

$$= \frac{5}{9} \left[1 - \left(\frac{1}{10} \right)^n \right]$$

(ii) 0.2, 0.02, 0.002,

Solution:

Here, $t_1 = 0.2$, $t_2 = 0.02$, $t_3 = 0.002$

$$\therefore \frac{t_2}{t_1} = \frac{0.02}{0.2} = 0.1 \text{ and } \frac{t_3}{t_2} = \frac{0.002}{0.02} = 0.1$$

$\therefore$ The given sequence is a G.P.

$\therefore a = 0.2$ and $r = 0.1$

$$\therefore S_n = \frac{a(1-r^n)}{1-r}, \text{ for } r < 1$$

$$= \frac{0.2[1-(0.1)^n]}{1-0.1}$$

$$= \frac{0.2}{0.9} [1-(0.1)^n]$$

$$= \frac{2}{9} \left[1 - \left(\frac{1}{10} \right)^n \right]$$

Question 8.

For a sequence, if $S_n = 2(3^n - 1)$, find the n th term, hence showing that the sequence is a G.P.

Solution:

$$\begin{aligned} S_n &= 2(3^n - 1) \\ \therefore S_{n-1} &= 2(3^{n-1} - 1) \\ \text{But } t_n &= S_n - S_{n-1} \\ &= 2(3^n - 1) - 2(3^{n-1} - 1) \\ &= 2(3^n - 1 - 3^{n-1} + 1) \\ &= 2(3^n - 3^{n-1}) = 2(3^{n-1+1} - 3^{n-1}) \\ \therefore t_n &= 2 \cdot 3^{n-1}(3 - 1) = 4 \cdot 3^{n-1} \\ \therefore t_{n+1} &= 4 \cdot 3^{(n+1)-1} \\ &= 4(3)^n \end{aligned}$$

The sequence (t_n) is a G.P., if $\frac{t_{n+1}}{t_n} = \text{constant}$

for $n \in \mathbb{N}$.

$$\therefore \frac{t_{n+1}}{t_n} = \frac{4(3)^n}{4(3)^{n-1}} = 3 = \text{constant, for } n \in \mathbb{N}$$

$$\therefore r = 3$$

$$\therefore \text{the sequence is a G.P. with } t_n = 4(3)^{n-1}$$

Question 9.

If S , P , R are the sum, product, and sum of the reciprocals of n terms of a G.P, respectively, then verify that $[SR]^n = P^2$.

Solution:

Let a be the 1st term and r be the common ratio of the G.P.

$$\therefore \text{the G.P. is } a, ar, ar^2, ar^3, \dots, ar^{n-1}$$

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$$\therefore S = a + ar + ar^2 + \dots + ar^{n-1} = a \left(\frac{r^n - 1}{r - 1} \right)$$

$$P = a(ar) (ar^2) \dots (ar^{n-1})$$

$$= a^n \cdot r^{1+2+3+\dots+(n-1)}$$

$$= a^n \cdot r^{\frac{n(n-1)}{2}}$$

$$\therefore P^2 = a^{2n} \cdot r^{n(n-1)} \quad \dots(i)$$

$$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \dots + \frac{1}{ar^{n-1}}$$

$$= \frac{r^{n-1} + r^{n-2} + r^{n-3} + \dots + r^2 + r + 1}{a \cdot r^{n-1}}$$

$$= \frac{1 + r + r^2 + \dots + r^{n-2} + r^{n-1}}{a \cdot r^{n-1}}$$

$1, r, r^2, \dots, r^{n-1}$ are in G.P., with $a = 1, r = r$

$$\therefore 1 + r + r^2 + \dots + r^{n-1} = 1 \cdot \left(\frac{r^n - 1}{r - 1} \right)$$

$$\therefore R = \frac{1}{a r^{n-1}} \left(\frac{r^n - 1}{r - 1} \right) = \frac{1}{a^2 \cdot r^{n-1}} \times a \times \left(\frac{r^n - 1}{r - 1} \right)$$

$$\therefore R = \frac{1}{a^2 \cdot r^{n-1}} S$$

$$\therefore a^2 \cdot r^{n-1} = \frac{S}{R}$$

$$\therefore (a^2 \cdot r^{n-1})^n = \left(\frac{S}{R} \right)^n$$

$$\therefore a^{2n} \cdot r^{n(n-1)} = \left(\frac{S}{R} \right)^n$$

$$\therefore P^2 = \left(\frac{S}{R} \right)^n \quad \dots[\text{From (i)}]$$

Question 10.

If S_n, S_{2n}, S_{3n} are the sum of $n, 2n, 3n$ terms of a G.P. respectively, then verify that $S_n(S_{3n} - S_{2n}) = (S_{2n} - S_n)^2$.

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Solution:

Let a and r be the 1st term and common ratio of the G.P. respectively.

$$\therefore S_n = a \left(\frac{r^n - 1}{r - 1} \right), S_{2n} = a \left(\frac{r^{2n} - 1}{r - 1} \right), S_{3n} = a \left(\frac{r^{3n} - 1}{r - 1} \right)$$

$$\therefore S_{2n} - S_n = a \left(\frac{r^{2n} - 1}{r - 1} \right) - a \left(\frac{r^n - 1}{r - 1} \right)$$

$$= \frac{a}{r - 1} (r^{2n} - 1 - r^n + 1)$$

$$= \frac{a}{r - 1} (r^{2n} - r^n)$$

$$= \frac{a r^n}{r - 1} (r^n - 1)$$

$$\therefore S_{2n} - S_n = r^n \cdot \frac{a(r^n - 1)}{r - 1} \quad \dots(i)$$

$$S_{3n} - S_{2n} = a \left(\frac{r^{3n} - 1}{r - 1} \right) - a \left(\frac{r^{2n} - 1}{r - 1} \right)$$

$$= \frac{a}{r - 1} (r^{3n} - 1 - r^{2n} + 1)$$

$$= \frac{a}{r - 1} (r^{3n} - r^{2n})$$

$$= \frac{a}{r - 1} \cdot r^{2n} (r^n - 1)$$

$$= a \cdot \left(\frac{r^n - 1}{r - 1} \right) \cdot r^{2n}$$

$$\therefore S_n(S_{3n} - S_{2n}) = \left[a \cdot \left(\frac{r^n - 1}{r - 1} \right) \right] \left[a \cdot \left(\frac{r^n - 1}{r - 1} \right) r^{2n} \right]$$

$$= \left[r^n \cdot \frac{a(r^n - 1)}{r - 1} \right]^2$$

$$\therefore S_n(S_{3n} - S_{2n}) = (S_{2n} - S_n)^2 \quad \dots[\text{From (i)}]$$

Question 11.

Find

(i) $\sum_{r=1}^{10} (3 \times 2^r)$

Solution:

$$\sum_{r=1}^{10} 3 \times 2^r$$

$$= 3 \times 2 + 3 \times 2^2 + 3 \times 2^3 + \dots + 3 \times 2^{10}$$

$$= 3 [2 + 2^2 + 2^3 + \dots + 2^{10}]$$

Here, $2, 2^2, 2^3, \dots, 2^{10}$ are the terms in G.P. with $a = 2, r = 2$

$$\sum_{r=1}^{10} 3 \times 2^r = 3 \cdot \frac{2[2^{10} - 1]}{2 - 1}$$

$$= 6 (1024 - 1)$$

$$= 6 (1023) = 6138$$

(ii) $\sum_{r=1}^{10} 5 \times 3^r$

Solution:

$$\sum_{r=1}^{10} 5 \times 3^r$$

$$= 5 \times 3 + 5 \times 3^2 + 5 \times 3^3 + \dots + 5 \times 3^{10}$$

$$= 5 [3 + 3^2 + 3^3 + \dots + 3^{10}]$$

Here, $3, 3^2, 3^3, \dots, 3^{10}$ are the terms in G.P. with

$$a = 3, r = 3$$

$$\sum_{r=1}^{10} 5 \times 3^r = 5 \cdot \frac{3[3^{10} - 1]}{3 - 1} = \frac{15}{2} (3^{10} - 1)$$

Question 12.

The value of a house appreciates 5% per year. How much is the house worth after 6 years if its current worth is Rs. 15 Lac. [Given: $(1.05)^5 = 1.28$, $(1.05)^6 = 1.34$]

Solution:

The value of a house is Rs. 15 Lac.

$$\text{Appreciation rate} = 5\% = \frac{5}{100} = 0.05$$

$$\text{Value of house after 1st year} = 15(1 + 0.05) = 15(1.05)$$

$$\text{Value of house after 6 years} = 15(1.05)(1.05)^5$$

$$= 15(1.05)^6$$

$$= 15(1.34)$$

$$= 20.1 \text{ lac.}$$

Question 13.

If one invests Rs. 10,000 in a bank at a rate of interest 8% per annum, how long does it take to double the money by compound interest? [$(1.08)^5 = 1.47$]

Solution:

Amount invested = Rs. 10000

$$\text{Interest rate} = \frac{8}{100} = 0.08$$

$$\text{amount after 1st year} = 10000(1 + 0.08) = 10000(1.08)$$

Value of the amount after n years

$$= 10000(1.08) \times (1.08)^{n-1}$$

$$= 10000(1.08)^n$$

$$= 20000$$

$$\therefore (1.08)^n = 2$$

$$\therefore (1.08)^5 = 1.47 \dots\dots [\text{Given}]$$

$$\therefore n = 10 \text{ years, (approximately)}$$

Ex 2.3

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-2-sequences-and-series/>

Question 1.

Determine whether the sum to infinity of the following G.P.s exist, if exists find them.

(i) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$

Solution:

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$$

$$\text{Here, } a = \frac{1}{2}, r = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}, |r| < 1$$

$\therefore$ Sum to infinity exists.

$$\begin{aligned}\therefore \text{Sum to infinity} &= \frac{a}{1-r} \\ &= \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1\end{aligned}$$

(ii) $2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}, \dots$

Solution:

$$2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}, \dots$$

$$\text{Here, } a = 2, r = \frac{\frac{4}{3}}{2} = \frac{2}{3}, |r| < 1$$

$\therefore$ Sum to infinity exists.

$$\begin{aligned}\therefore \text{Sum to infinity} &= \frac{a}{1-r} \\ &= \frac{2}{1-\frac{2}{3}} = 6\end{aligned}$$

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(v) 9, 8.1, 7.29,

Solution:

9, 8.1, 7.29,

Here, $a = 9$, $r = \frac{8.1}{9} = 0.9$, $|r| < 1$

$\therefore$ Sum to infinity exists.

$\therefore$ Sum to infinity = $\frac{a}{1-r}$

$$= \frac{9}{1-0.9}$$

$$= \frac{9}{0.1}$$

$$= 90$$

Question 2.

Express the following recurring decimals as rational numbers.

(i) $0.\overline{7}$

(ii) $2.\overline{4}$

(iii) $2.\overline{35}$

(iv) $51.0\overline{2}$

Solution:

(i) $0.\overline{7} = 0.7777... = 0.7 + 0.07 + 0.007 +$

The terms 0.7, 0.07, 0.007,... are in G.P.

$$\therefore a = 0.7, r = \frac{0.07}{0.7} = 0.1, |r| = |0.1| < 1$$

$\therefore$ Sum to infinity exists.

$\therefore$ Sum to infinity

$$\begin{aligned} &= \frac{a}{1-r} = \frac{0.7}{1-0.1} \\ &= \frac{0.7}{0.9} \\ &= \frac{7}{9} \end{aligned}$$

(ii) $2.\overline{4} = 2.444... = 2 + 0.4 + 0.04 + 0.004 + ...$

The terms 0.4, 0.04, 0.004,... are in G.P.

$$\therefore a = 0.4, r = \frac{0.04}{0.4} = 0.1, |r| = 0.1 < 1$$

$\therefore$ Sum to infinity exists.

$\therefore$ Sum to infinity

$$\begin{aligned} &= 2 + \frac{a}{1-r} = 2 + \frac{0.4}{1-0.1} \\ &= 2 + \frac{0.4}{0.9} \\ &= 2 + \frac{4}{9} = \frac{22}{9} \end{aligned}$$

(iii) $2.\overline{35} = 2.3555... = 2.3 + 0.05 + 0.005 + 0.0005 + ...$

The terms 0.05, 0.005, 0.0005, ... are in G.P.

$$\therefore a = 0.05, r = \frac{0.005}{0.05} = 0.1, |r| = |0.1| < 1$$

$\therefore$ Sum to infinity exists.

$\therefore$ Sum to infinity

$$\begin{aligned} &= 2.3 + \frac{a}{1-r} \\ &= 2.3 + \frac{0.05}{1-0.1} \\ &= 2.3 + \frac{0.05}{0.9} \\ &= \frac{23}{10} + \frac{5}{90} \\ &= \frac{212}{90} = \frac{106}{45} \end{aligned}$$

(iv) $51.\overline{02} = 51.0222 = 51 + 0.02 + 0.002 + 0.0002 +$

The terms 0.02, 0.002, 0.0002, ... are in G.P.

$$\therefore a = 0.02, r = \frac{0.002}{0.02} = 0.1, |r| = |0.1| < 1$$

$\therefore$ Sum to infinity exists.

$\therefore$ Sum to infinity

$$\begin{aligned} &= 51 + \frac{a}{1-r} = 51 + \frac{0.02}{1-0.1} \\ &= 51 + \frac{0.02}{0.9} \\ &= 51 + \frac{2}{90} \\ &= 51 + \frac{1}{45} \\ &= \frac{2296}{45} \end{aligned}$$

Question 3.

If the common ratio of a G.P. is $\frac{2}{3}$ and the sum to infinity is 12, find the first term.

Solution:

$$r = \frac{2}{3}, \text{ sum to infinity} = 12 \dots\dots [\text{Given}]$$

$$\text{Sum to infinity} = \frac{a}{1-r}$$

$$12 = \frac{a}{1-\frac{2}{3}}$$

$$a = 12 \times \frac{1}{3}$$

$$\therefore a = 4$$

Question 4.

If the first term of the G.P. is 6 and its sum to infinity is $\frac{96}{17}$, find the common ratio.

Solution:

$$a = 6, \text{ sum to infinity} = \frac{96}{17} \dots\dots[\text{Given}]$$

$$\text{Sum to infinity} = \frac{a}{1-r}$$

$$\therefore \frac{96}{17} = \frac{6}{1-r}$$

$$\therefore \frac{16}{17} = \frac{1}{1-r}$$

$$\therefore 16 - 16r = 17$$

$$\therefore 16r = 16 - 17$$

$$\therefore r = \frac{-1}{16}$$

Question 5.

The sum of an infinite G.P. is 5 and the sum of the squares of these terms is 15, find the G.P.

Solution:

Let the required G.P. be $a, ar, ar^2, ar^3, \dots$

Sum to infinity of this G.P. = 5

$$\therefore 5 = \frac{a}{1-r}$$

$$\therefore a = 5(1-r) \quad \dots(i)$$

Also, the sum of the squares of the terms is 15.

$$\therefore (a^2 + a^2r^2 + a^2r^4 + \dots) = 15$$

$$\therefore 15 = \frac{a^2}{1-r^2}$$

$$\therefore 15(1-r^2) = a^2$$

$$\therefore 15(1-r)(1+r) = 25(1-r)^2 \quad \dots[\text{From (i)}]$$

$$\therefore 3(1+r) = 5(1-r)$$

$$\therefore 3 + 3r = 5 - 5r$$

$$\therefore 8r = 2$$

$$\therefore r = \frac{1}{4}$$

$$\therefore a = 5 \left(1 - \frac{1}{4}\right) = 5 \left(\frac{3}{4}\right) = \frac{15}{4}$$

$\therefore$ Required G.P. is $a, ar, ar^2, ar^3, \dots$

$$\text{i.e. } \frac{15}{4}, \frac{15}{16}, \frac{15}{64}, \dots$$

Question 6.

Find

(i) $\sum_{r=1}^{\infty} 4(0.5)^r$

Solution:

$$\begin{aligned} & \sum_{r=1}^{\infty} 4(0.5)^r \\ &= 4(0.5) + 4(0.5)^2 + 4(0.5)^3 + \dots \\ &= 4 \left\{ \frac{5}{10} + \left(\frac{5}{10}\right)^2 + \left(\frac{5}{10}\right)^3 + \dots \right\} \\ &= 4 \left\{ \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \dots \right\} \end{aligned}$$

The terms $\frac{1}{2}, \left(\frac{1}{2}\right)^2, \left(\frac{1}{2}\right)^3, \dots$ are in G.P.

$$\therefore a = \frac{1}{2}, r = \frac{1}{2}, |r| = \left|\frac{1}{2}\right| < 1$$

$\therefore$ Sum to infinity exists.

$$\therefore \sum_{r=1}^{\infty} 4(0.5)^r = 4 \times \frac{a}{1-r} = 4 \times \frac{\frac{1}{2}}{1-\frac{1}{2}} = 4$$

(ii) $\sum_{r=1}^{\infty} \left(-\frac{1}{3}\right)^r$

Solution:

Solution:

$$\sum_{r=1}^{\infty} \left(-\frac{1}{3}\right)^r$$
$$= \left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^3 + \dots$$

The terms $\left(-\frac{1}{3}\right), \left(-\frac{1}{3}\right)^2, \left(-\frac{1}{3}\right)^3, \dots$ are in G.P.

$$\therefore a = -\frac{1}{3}, r = -\frac{1}{3}, |r| = \left|-\frac{1}{3}\right| < 1$$

$\therefore$ Sum to infinity exists.

$$\therefore \sum_{r=1}^{\infty} \left(-\frac{1}{3}\right)^r = \frac{a}{1-r} = \frac{-\frac{1}{3}}{1-\left(-\frac{1}{3}\right)} = \frac{-\frac{1}{3}}{\frac{4}{3}} = -\frac{1}{4}$$

$$(iii) \sum_{r=0}^{\infty} (-8) \left(-\frac{1}{2}\right)^r$$

Solution:

$$\sum_{r=1}^{\infty} (-8) \left(-\frac{1}{2}\right)^r$$
$$= (-8) \left[\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^3 + \dots \right]$$

The terms $-\frac{1}{2}, \left(-\frac{1}{2}\right)^2, \left(-\frac{1}{2}\right)^3, \dots$ are in G.P.

$$\therefore a = -\frac{1}{2}, r = -\frac{1}{2}, |r| = \left| -\frac{1}{2} \right| < 1$$

$\therefore$ Sum to infinity exists.

$$\begin{aligned}\therefore \sum_{r=1}^{\infty} (-8) \left(-\frac{1}{2} \right)^r &= (-8) \frac{a}{1-r} \\ &= (-8) \left[\frac{-\frac{1}{2}}{1 - \left(-\frac{1}{2} \right)} \right] \\ &= (-8) \left(\frac{-\frac{1}{2}}{\frac{3}{2}} \right) = \frac{8}{3}\end{aligned}$$

(iv) $\sum_{n=1}^{\infty} 0.4^n$

Solution:

$$\sum_{n=1}^{\infty} (0.4)^n$$

$$= 0.4 + (0.4)^2 + (0.4)^3 + \dots$$

The terms $0.4, (0.4)^2, (0.4)^3, \dots$ are in G.P.

$$\therefore a = 0.4, r = 0.4, |r| = |0.4| < 1$$

$\therefore$ Sum to infinity exists.

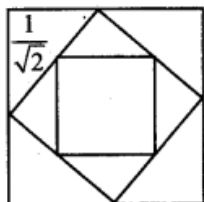
$$\therefore \sum_{n=1}^{\infty} (0.4)^n = \frac{a}{1-r} = \frac{0.4}{1-0.4} = \frac{0.4}{0.6} = \frac{2}{3}$$

Question 7.

The midpoints of the sides of a square of side 1 are joined to form a new square. This procedure is repeated indefinitely. Find the sum of

- (i) the areas of all the squares.
- (ii) the perimeters of all the squares.

Solution:



(i) Area of the 1st square = 1

Area of the 2nd square = $\left(\frac{1}{\sqrt{2}}\right)^2$

Area of the 3rd square = $\left(\frac{1}{2}\right)^2$

and so on.

∴ Sum of the areas of all the squares

$$= 1^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \dots$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + \dots$$

The terms $1, \frac{1}{2}, \frac{1}{4}, \dots$ are in G.P.

$$a=1, r=\frac{1}{2}, |r|=\left|\frac{1}{2}\right|<1$$

$\therefore$ Sum to infinity exists.

$$\therefore \text{Sum of the areas of all the squares} = \frac{1}{1 - \frac{1}{2}} = 2$$

(ii) Perimeter of 1st square = 4

$$\text{Perimeter of 2nd square} = 4 \left(\frac{1}{\sqrt{2}} \right)$$

$$\text{Perimeter of 3rd square} = 4 \left(\frac{1}{2} \right)$$

and so on.

Sum of the perimeters of all the squares

$$= 4 + 4 \left(\frac{1}{\sqrt{2}} \right) + 4 \left(\frac{1}{2} \right) + \dots$$

$$= 4 \left(1 + \frac{1}{\sqrt{2}} + \left(\frac{1}{\sqrt{2}} \right)^2 + \dots \right)$$

The terms $1, \frac{1}{\sqrt{2}}, \left(\frac{1}{\sqrt{2}} \right)^2, \dots$ are in G.P.

$$\therefore a = 1, r = \frac{1}{\sqrt{2}}, |r| = \left| \frac{1}{\sqrt{2}} \right| < 1$$

$\therefore$ Sum to infinity exists.

$\therefore$ Sum of the perimeters of all the squares

$$= 4 \cdot \left(\frac{1}{1 - \frac{1}{\sqrt{2}}} \right)$$

$$= \frac{4\sqrt{2}}{\sqrt{2} - 1}$$

Question 8.

A ball is dropped from a height of 10 m. It bounces to a height of 6m, then 3.6 m, and so on. Find the total distance travelled by the ball.

Solution:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-2-sequences-and-series/>

Here, on the first bounce, the ball will go 6 m and it will return 6 m.

On the second bounce, the ball will go 3.6 m and it will return 3.6 m, and so on.

Given that, a ball is dropped from a height of 10 m.

∴ Total distance travelled by the ball is = $10 + 2[6 + 3.6 + \dots]$

The terms 6, 3.6 ... are in G.P.

$a = 6$, $r = 0.6$, $|r| = |0.6| < 1$

∴ Sum to infinity exists.

∴ Total distance travelled by the ball

$$\begin{aligned} &= 10 + 2 \left[\frac{6}{1 - \frac{6}{10}} \right] = 10 + 2 \left[\frac{6}{\frac{10-6}{10}} \right] \\ &= 10 + 2 \left[\frac{6 \times 10}{4} \right] \\ &= 10 + 30 \\ &= 40 \text{ m} \end{aligned}$$

Ex 2.4

Question 1.

Verify whether the following sequences are H.P.

(i) $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \dots$

Solution:

$$\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \dots$$

Here, the reciprocal sequence is 3, 5, 7, 9,...

$$t_1 = 3, t_2 = 5, t_3 = 7, t_4 = 9, \dots$$

$$t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = 2 = \text{constant}$$

$\therefore$ The reciprocal sequence is an A.P.

$\therefore$ The given sequence is a H.P.

(ii) $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \dots$

Solution:

$$\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \dots$$

Here, the reciprocal sequence is 3, 6, 12, 24,...

$$t_1 = 3, t_2 = 6, t_3 = 12, \dots$$

$$t_2 - t_1 = 3, t_3 - t_2 = 6$$

$$t_2 - t_1 \neq t_3 - t_2$$

$\therefore$ The reciprocal sequence is not an A.P.

$\therefore$ The given sequence is not a H.P.

(iii) $5, \frac{10}{17}, \frac{10}{32}, \frac{10}{47}, \dots$

Solution:

$$5, \frac{10}{17}, \frac{10}{32}, \frac{10}{47}, \dots$$

Here, the reciprocal sequence is

$$\frac{1}{5}, \frac{17}{10}, \frac{32}{10}, \frac{47}{10}, \dots$$

$$t_1 = \frac{1}{5}, t_2 = \frac{17}{10}, t_3 = \frac{32}{10}, t_4 = \frac{47}{10}, \dots$$

$$t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = \frac{3}{2} = \text{constant}$$

$\therefore$ The reciprocal sequence is an A.P.

$\therefore$ The given sequence is a H.P.

Question 2.

Find the n th term and hence find the 8th term of the following HPs.

(i) $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \dots$

Solution:

$$\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \dots \text{ are in H.P.}$$

$$\therefore 2, 5, 8, 11, \dots \text{ are in A.P.}$$

$$\therefore a = 2, d = 3$$

$$t_n = a + (n - 1)d$$

$$= 2 + (n - 1)(3)$$

$$= 3n - 1$$

$$\therefore \text{nth term of H.P.} = \frac{1}{3n - 1}$$

(ii) $\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \dots$

Solution:

$\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \dots$ are in H.P.

$\therefore 4, 6, 8, 10, \dots$ are in A.P.

$$\therefore a = 4, d = 2$$

$$t_n = a + (n - 1)d$$

$$= 4 + (n - 1)(2)$$

$$= 2n + 2$$

$$\therefore \text{nth term of H.P.} = \frac{1}{2n+2}$$

$$\therefore \text{8th term of H.P.} = \frac{1}{2(8)+2} = \frac{1}{18}$$

(iii) $\frac{1}{5}, \frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \dots$

Solution:

$\frac{1}{5}, \frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \dots$ are in H.P.

$\therefore 5, 10, 15, 20, \dots$ are in A.P.

$$\therefore a = 5, d = 5$$

$$t_n = a + (n - 1)d$$

$$= 5 + (n - 1)(5)$$

$$= 5n$$

$$\therefore \text{nth term of H.P.} = \frac{1}{5n}$$

$$\therefore \text{8th term of H.P.} = \frac{1}{5(8)} = \frac{1}{40}$$

Question 3.

Find A.M. of two positive numbers whose G.M. and H.M. are 4 and $\frac{16}{5}$ respectively.

Solution:

$$\text{G.M.} = 4, \text{H.M.} = \frac{16}{5}$$

$$\text{Now, } (\text{G.M.})^2 = (\text{A.M.}) (\text{H.M.})$$

$$\therefore 4^2 = \text{A.M.} \times \frac{16}{5}$$

$$\therefore \text{A.M.} = 16 \times \frac{5}{16}$$

$$\therefore \text{A.M.} = 5$$

Question 4.

Find H.M. of two positive numbers whose A.M. and G.M. are $\frac{15}{2}$ and 6.

Solution:

$$\text{A.M.} = \frac{15}{2}, \text{G.M.} = 6$$

$$\text{Now, } (\text{G.M.})^2 = (\text{A.M.}) (\text{H.M.})$$

$$\therefore 6^2 = \frac{15}{2} \times \text{H.M.}$$

$$\therefore \text{H.M.} = 36 \times \frac{2}{15}$$

$$\therefore \text{H.M.} = \frac{24}{5}$$

Question 5.

Find G.M. of two positive numbers whose A.M. and H.M. are 75 and 48.

Solution:

$$\text{A.M.} = 75, \text{H.M.} = 48$$

$$\text{Now, } (\text{G.M.})^2 = (\text{A.M.}) (\text{H.M.})$$

$$\therefore (\text{G.M.})^2 = 75 \times 48$$

$$\therefore (\text{G.M.})^2 = 25 \times 3 \times 16 \times 3$$

$$\therefore (\text{G.M.})^2 = 5^2 \times 4^2 \times 3^2$$

$$\therefore \text{G.M.} = 5 \times 4 \times 3$$

$$\therefore \text{G.M.} = 60$$

Question 6.

Insert two numbers between $\frac{1}{4}$ and $\frac{1}{3}$ so that the resulting sequence is a H.P.

Solution:

Let the required numbers be $\frac{1}{H_1}$ and $\frac{1}{H_2}$.

$$\therefore \frac{1}{4}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{3} \text{ are in H.P.}$$

$$\therefore 4, H_1, H_2, 3 \text{ are in A.P.}$$

$$t_1 = 4, t_2 = H_1, t_3 = H_2, t_4 = 3$$

$$\therefore t_1 = a = 4, t_4 = 3$$

$$t_n = a + (n - 1)d$$

$$t_4 = 4 + (4 - 1)d$$

$$3 = 4 + 3d$$

$$3 = 4 + 3d$$

$$3d = -1$$

$$\therefore d = \frac{-1}{3}$$

$$H_1 = t_2 = a + d = 4 - \frac{1}{3} = \frac{11}{3}$$

$$H_2 = t_3 = a + 2d = 4 - \frac{2}{3} = \frac{10}{3}$$

$\therefore$ For resulting sequence to be H.P. we need to insert numbers $\frac{3}{11}$ and $\frac{3}{10}$.

Question 7.

Insert two numbers between 1 and -27 so that the resulting sequence is a G.P.

Solution:

Let the required numbers be G_1 and G_2 .

$\therefore 1, G_1, G_2, -27$ are in G.P.

$$t_1 = 1, t_2 = G_1, t_3 = G_2, t_4 = -27$$

$$\therefore t_1 = a = 1$$

$$t_n = ar^{n-1}$$

$$t_4 = (1) r^{4-1}$$

$$-27 = r^3$$

$$r^3 = (-3)^3$$

$$\therefore r = -3$$

$$\therefore G_1 = t_2 = ar = 1(-3) = -3$$

$$G_2 = t_3 = ar^2 = 1(-3)^2 = 9$$

$\therefore$ For resulting sequence to be G.P. we need to insert numbers -3 and 9.

Question 8.

If the A.M. of two numbers exceeds their G.M. by 2 and their H.M. by $\frac{18}{5}$, find the numbers.

Solution:

Let a and b be the two numbers.

$$A = \frac{a+b}{2}, G = \sqrt{ab}, H = \frac{2ab}{a+b}$$

According to the given conditions,

$$A = G + 2, A = H + \frac{18}{5}$$

$$\therefore G = A - 2, H = A - \frac{18}{5}$$

$$\text{Now, } G^2 = AH$$

$$\therefore (A - 2)^2 = A \left(A - \frac{18}{5} \right)$$

$$\therefore A^2 - 4A + 4 = A^2 - \frac{18}{5}A$$

$$\therefore \frac{18}{5}A - 4A = -4$$

$$\therefore \frac{-2A}{5} = -4$$

$$\therefore A = 10$$

$$\therefore \frac{a+b}{2} = 10$$

$$\therefore a + b = 20$$

$$\therefore b = 20 - a \quad \dots(i)$$

Consider, $G = A - 2 = 10 - 2 = 8$

$$\sqrt{ab} = 8$$

$$ab = 64$$

$$a(20 - a) = 64 \text{[From (i)]}$$

$$a^2 - 20a + 64 = 0$$

$$(a - 4)(a - 16) = 0$$

$$\therefore a = 4 \text{ or } a = 16$$

$$\text{When } a = 4, b = 20 - 4 = 16$$

$$\text{When } a = 16, b = 20 - 16 = 4$$

$\therefore$ The two numbers are 4 and 16.

Question 9.

Find two numbers whose A.M. exceeds their G.M. by $\frac{1}{2}$ and their H.M. by $\frac{25}{26}$.

Solution:

Let a and b be the two numbers.

$$A = \frac{a+b}{2}, G = \sqrt{ab}, H = \frac{2ab}{a+b}$$

According to the given conditions,

$$A = G + \frac{1}{2}, A = H + \frac{25}{26}$$

$$\therefore G = A - \frac{1}{2}, H = A - \frac{25}{26}$$

$$\text{Now, } G^2 = AH$$

$$\therefore \left(A - \frac{1}{2}\right)^2 = A\left(A - \frac{25}{26}\right)$$

$$\therefore A^2 - A + \frac{1}{4} = A^2 - \frac{25}{26}A$$

$$\therefore A - \frac{25}{26}A = \frac{1}{4}$$

$$\therefore \frac{1}{26}A = \frac{1}{4}$$

$$\therefore A = \frac{13}{2}$$

$$\therefore \frac{a+b}{2} = \frac{13}{2}$$

$$\therefore a + b = 13$$

$$\therefore b = 13 - a \quad \dots(i)$$

$$\text{Consider, } G = A - \frac{1}{2} = \frac{13}{2} - \frac{1}{2} = 6$$

$$\sqrt{ab} = 6$$

$$ab = 36$$

$$a(13 - a) = 36 \dots\dots[\text{From (i)}]$$

$$a^2 - 13a + 36 = 0$$

$$(a - 4)(a - 9) = 0$$

$$\therefore a = 4 \text{ or } a = 9$$

$$\text{When } a = 4, b = 13 - 4 = 9$$

$$\text{When } a = 9, b = 13 - 9 = 4$$

$\therefore$ The two numbers are 4 and 9.

Ex 2.5

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-2-chapter-2-sequences-and-series/>

Question 1.

Find S_n of the following arithmetico-geometric sequences.(i) $2, 4x, 6x^2, 8x^3, 10x^4, \dots$

Solution:

 $2, 4x, 6x^2, 8x^3, 10x^4, \dots$ Here, $2, 4, 6, 8, 10, \dots$ are in A.P.

$$\therefore a = 2, d = 2$$

$$\therefore n^{\text{th}} \text{ term} = a + (n - 1)d$$

$$= 2 + (n - 1)(2)$$

$$= 2n$$

Also, $1, x, x^2, x^3, \dots$ are in G.P.

$$\therefore a = 1, r = x$$

$$\therefore n^{\text{th}} \text{ term} = a.r^{n-1} \\ = x^{n-1}$$

$$\therefore n^{\text{th}} \text{ term of arithmetico-geometric sequence is} \\ t_n = 2n.x^{n-1}$$

$$\therefore S_n = 2 + 4x + 6x^2 + 8x^3 + 10x^4 + \dots + 2n.x^{n-1} \quad \dots(i)$$

Multiplying throughout by x , we get

$$x S_n = 2x + 4x^2 + 6x^3 + 8x^4 + 10x^5 + \dots + 2n.x^n \quad \dots(ii)$$

Equation (i) – equation (ii), we get

$$S_n - xS_n \\ = 2 + 2x + 2x^2 + 2x^3 + \dots + 2x^{n-1} - 2nx^n$$

$$\therefore (1 - x)S_n = 2 + 2x \left(\frac{1 - x^{n-1}}{1 - x} \right) - 2nx^n$$

$$\therefore (1 - x)S_n = 2(1 - nx^n) + \frac{2x(1 - x^{n-1})}{1 - x}$$

$$\therefore S_n = \frac{2(1 - nx^n)}{1 - x} + \frac{2x(1 - x^{n-1})}{(1 - x)^2}$$

(ii) $1, 4x, 7x^2, 10x^3, 13x^4, \dots$

Solution:

$1, 4x, 7x^2, 10x^3, 13x^4, \dots$

Here, $1, 4, 7, 10, 13, \dots$ are in A.P.

$$a = 1, d = 3$$

$$\therefore \text{nth term} = a + (n - 1)d$$

$$= 1 + (n - 1)(3)$$

$$= 3n - 2$$

Also, $1, x, x^2, x^3, \dots$ are in G.P.

$$\therefore a = 1, r = x,$$

$$\text{nth term} = ar^{n-1} = x^{n-1}$$

nth term of arithmetico-geometric sequence is

$$\begin{aligned} t_n &= (3n - 2)x^{n-1} \\ \therefore S_n &= 1 + 4x + 7x^2 + 10x^3 + \dots + (3n - 2)x^{n-1} \end{aligned} \quad \dots(i)$$

Multiplying throughout by x , we get

$$xS_n = x + 4x^2 + 7x^3 + 10x^4 + \dots + (3n - 2)x^n \quad \dots(ii)$$

Equation (i) – equation (ii), we get

$$\begin{aligned} S_n - xS_n &= 1 + 3x + 3x^2 + 3x^3 + \dots + 3x^{n-1} - (3n - 2)x^n \\ \therefore (1 - x)S_n &= 1 + 3x \left(\frac{1 - x^{n-1}}{1 - x} \right) - (3n - 2)x^n \\ \therefore (1 - x)S_n &= 1 - (3n - 2)x^n + \frac{3x(1 - x^{n-1})}{1 - x} \end{aligned}$$

(iii) $1, 2 \times 3, 3 \times 9, 4 \times 27, 5 \times 81, \dots$

Solution:

$1, 2 \times 3, 3 \times 9, 4 \times 27, 5 \times 81, \dots$

Here, $1, 2, 3, 4, 5, \dots$ are in A.P.

$$\therefore a = 1, d = 1$$

$$\therefore \text{nth term} = a + (n - 1)d$$

$$= 1 + (n - 1)1$$

$$= n$$

Also, $1, 3, 3^2, 3^3, 3^4, \dots$ are in G.P.

$$\therefore a = 1, r = 3$$

$$\therefore n^{\text{th}} \text{ term} = ar^{n-1}$$

$$= 1.(3^{n-1})$$

$$= 3^{n-1}$$

$\therefore n^{\text{th}}$ term of arithmetico-geometric sequence is

$$t_n = n.3^{n-1}$$

$$\begin{aligned} \therefore S_n &= 1 + 2 \times 3 + 3 \times 3^2 + 4 \times 3^3 + 5 \times 3^4 + \dots \\ &\quad + n.3^{n-1} \end{aligned} \quad \dots(i)$$

Multiplying throughout by 3, we get

$$\begin{aligned} 3S_n &= 1 \times 3 + 2 \times 3^2 + 3 \times 3^3 + 4 \times 3^4 + 5 \times 3^5 + \dots \\ &\quad + n.3^n \end{aligned} \quad \dots(ii)$$

Equation (i) – equation (ii), we get

$$\begin{aligned} S_n - 3S_n &= [1 - n.(3^n)] + [3 + 3^2 + 3^3 + \dots \\ &\quad + 3^{n-1}] \end{aligned}$$

$$\therefore -2S_n = [1 - n.(3^n)] + \frac{3(3^{n-1} - 1)}{3 - 1}$$

$$\therefore -2S_n = [1 - n \cdot (3^n)] + \frac{3}{2} (3^{n-1} - 1)$$

$$\therefore S_n = \frac{n(3^n) - 1}{2} - \frac{3}{4} (3^{n-1} - 1)$$

$$\therefore S_n = \frac{n(3^n) - 1}{2} + \frac{3 - 3^n}{4}$$

(iv) 3, 12, 36, 96, 240,

Solution:

3, 12, 36, 96, 240,

i.e., $1 \times 3, 2 \times 6, 3 \times 12, 4 \times 24, 5 \times 48, \dots$

Here, 1, 2, 3, 4, 5, are in A.P.

$\therefore$ nth term = n

Also, 3, 6, 12, 24, 48, are in G.P.

$\therefore a = 3, r = 2$

$\therefore$ nth term = $ar^{n-1} = 3 \cdot (2^{n-1})$

$\therefore$ nth term of arithmetico-geometric sequence is

$$\begin{aligned} t_n &= n \cdot 3(2^{n-1}) \\ \therefore S_n &= 1 \times 3 + 2 \times 6 + 3 \times 12 + 4 \times 24 + 5 \times 48 + \dots + n \cdot 3(2^{n-1}) \\ &\dots(i) \end{aligned}$$

Multiplying throughout by 2, we get

$$\begin{aligned} 2S_n &= 1 \times 6 + 2 \times 12 + 3 \times 24 + 4 \times 48 \\ &\quad + 5 \times 96 + \dots + n \cdot 3(2^n) \\ &\dots(ii) \end{aligned}$$

Equation (i) – equation (ii), we get

$$S_n - 2S_n = [3 - 3n(2^n)] + [6 + 12 + 24 + \dots + 3(2^{n-1})]$$

$$\therefore -S_n = [3 - 3n(2^n)] + \frac{6(2^{n-1} - 1)}{2 - 1}$$
$$= 3 - 3n(2^n) + 6(2^{n-1} - 1)$$

$$\therefore S_n = 3n(2^n) - 3 - 6(2^{n-1}) + 6$$
$$= 3n(2^n) - 3(2^n) + 3$$

$$\therefore S_n = (n - 1)3(2^n) + 3 = 3[(n - 1)2^n + 1]$$

Question 2.

Find the sum to infinity of the following arithmetico-geometric sequence.

(i) $1, \frac{2}{4}, \frac{3}{16}, \frac{4}{64}, \dots$

Solution:

$$S = 1 + \frac{2}{4} + \frac{3}{4^2} + \frac{4}{4^3} + \dots \quad \dots(i)$$

Multiplying (i) by $\frac{1}{4}$, we get

$$\frac{1}{4}S = \frac{1}{4} + \frac{2}{4^2} + \frac{3}{4^3} + \frac{4}{4^4} + \dots \quad \dots(ii)$$

Equation (i) – equation (ii), we get

$$\frac{3}{4}S = 1 + \left(\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \dots \right)$$

The terms $\frac{1}{4}, \frac{1}{4^2}, \frac{1}{4^3}, \dots$ are in G.P.

$$\therefore a = \frac{1}{4}, r = \frac{1}{4}, |r| = \left| \frac{1}{4} \right| < 1$$

$\therefore$ Sum to infinity exists.

$$\begin{aligned} \therefore \frac{3}{4}S &= 1 + \frac{\frac{1}{4}}{1 - \frac{1}{4}} \\ &= 1 + \frac{1}{3} \end{aligned}$$

$$\therefore \frac{3}{4}S = \frac{4}{3}$$

$$\therefore S = \frac{16}{9}$$

(ii) $3, \frac{6}{5}, \frac{9}{25}, \frac{12}{125}, \frac{15}{625}, \dots$

Solution:

$$S = 3 + \frac{6}{5} + \frac{9}{5^2} + \frac{12}{5^3} + \frac{15}{5^4} + \dots \quad \dots(i)$$

Multiplying (i) by $\frac{1}{5}$, we get

$$\frac{1}{5}S = \frac{3}{5} + \frac{6}{5^2} + \frac{9}{5^3} + \frac{12}{5^4} + \frac{15}{5^5} + \dots \quad \dots(ii)$$

Equation (i) – equation (ii), we get

$$\frac{4}{5}S = 3 + \left(\frac{3}{5} + \frac{3}{5^2} + \frac{3}{5^3} + \dots \right)$$

The terms $\frac{3}{5}, \frac{3}{5^2}, \frac{3}{5^3}, \dots$ are in G.P.

$$\therefore a = \frac{3}{5}, r = \frac{1}{5}, |r| = \left| \frac{1}{5} \right| < 1$$

$\therefore$ Sum to infinity exists.

$$\begin{aligned} \therefore \frac{4}{5}S &= 3 + \frac{\frac{3}{5}}{1 - \frac{1}{5}} \\ &= 3 + \frac{3}{4} \end{aligned}$$

$$\therefore \frac{4}{5}S = \frac{15}{4}$$

$$\therefore S = \frac{75}{16}$$

(iii) $1, \frac{-4}{3}, \frac{7}{9}, \frac{-10}{27}, \dots$

Solution:

$$S = 1 + \left(\frac{-4}{3}\right) + \frac{7}{(-3)^2} + \frac{10}{(-3)^3} + \dots \quad \dots(i)$$

Multiplying (i) by $-\frac{1}{3}$, we get

$$\frac{-1}{3} S = \frac{1}{(-3)} + \frac{4}{(-3)^2} + \frac{7}{(-3)^3} + \frac{10}{(-3)^4} + \dots \quad \dots(ii)$$

Equation (i) – equation (ii), we get

$$\frac{4}{3} S = 1 + \frac{3}{(-3)} + \frac{3}{(-3)^2} + \frac{3}{(-3)^3} + \dots$$

The terms $\frac{3}{(-3)}, \frac{3}{(-3)^2}, \frac{3}{(-3)^3}, \dots$ are in G.P.

$$\therefore a = \frac{-3}{3}, r = \frac{-1}{3}, |r| = \left|\frac{-1}{3}\right| < 1$$

$\therefore$ Sum to infinity exists.

$$\begin{aligned} \therefore \frac{4}{3} S &= 1 + \frac{\left(\frac{3}{-3}\right)}{1 - \left(\frac{-1}{3}\right)} \\ &= 1 + \frac{-1}{\frac{4}{3}} = 1 - \frac{3}{4} \end{aligned}$$

$$\therefore \frac{4}{3} S = \frac{1}{4}$$

$$\therefore S = \frac{3}{16}$$

Ex 2.6

Question 1.

Find the sum $\sum_{r=1}^n (r+1)(2r-1)$.

Solution:

$$\begin{aligned} & \sum_{r=1}^n (r+1)(2r-1) \\ &= \sum_{r=1}^n (2r^2 + r - 1) \\ &= 2 \sum_{r=1}^n r^2 + \sum_{r=1}^n r - \sum_{r=1}^n 1 \\ &= 2 \cdot \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} - n \\ &= \frac{n}{6} [2(2n^2 + 3n + 1) + 3(n+1) - 6] \\ &= \frac{n}{6} (4n^2 + 9n - 1) \end{aligned}$$

Question 2.

Find $\sum_{r=1}^n (3r^2 - 2r + 1)$

Solution:

$$\begin{aligned} & \sum_{r=1}^n (3r^2 - 2r + 1) \\ &= 3 \sum_{r=1}^n r^2 - 2 \sum_{r=1}^n r + \sum_{r=1}^n 1 \\ &= 3 \cdot \frac{n(n+1)(2n+1)}{6} - 2 \frac{n(n+1)}{2} + n \end{aligned}$$

$$\begin{aligned} &= \frac{n}{2} [(2n^2 + 3n + 1) - 2(n + 1) + 2] \\ &= \frac{n}{2} (2n^2 + n + 1) \end{aligned}$$

Question 3.

Find $\sum_{r=1}^n \left(\frac{1+2+3+\dots+r}{r} \right)$

Solution:

$$\begin{aligned} &\sum_{r=1}^n \left(\frac{1+2+3+\dots+r}{r} \right) \\ &= \sum_{r=1}^n \frac{r(r+1)}{2r} \\ &= \frac{1}{2} \sum_{r=1}^n (r+1) \\ &= \frac{1}{2} \left[\sum_{r=1}^n r + \sum_{r=1}^n 1 \right] \\ &= \frac{1}{2} \left[\frac{n(n+1)}{2} + n \right] \\ &= \frac{n}{4} [(n+1) + 2] \\ &= \frac{n}{4} (n+3) \end{aligned}$$

Question 4.

Find $\sum_{r=1}^n \left(\frac{1^3 + 2^3 + \dots + r^3}{r(r+1)} \right)$

Solution:

$$\begin{aligned} & \sum_{r=1}^n \frac{1^3 + 2^3 + \dots + r^3}{r(r+1)} \\ &= \sum_{r=1}^n \frac{r^2(r+1)^2}{4} \times \frac{1}{r(r+1)} \\ &= \frac{1}{4} \sum_{r=1}^n r(r+1) \\ &= \frac{1}{4} \sum_{r=1}^n (r^2 + r) \\ &= \frac{1}{4} \left[\sum_{r=1}^n r^2 + \sum_{r=1}^n r \right] \\ &= \frac{1}{4} \left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \right] \\ &= \frac{1}{4} \cdot \frac{n(n+1)}{6} [(2n+1) + 3] \\ &= \frac{n(n+1)}{24} 2(n+2) \\ &= \frac{n(n+1)(n+2)}{12} \end{aligned}$$

Question 5.

Find the sum $5 \times 7 + 9 \times 11 + 13 \times 15 + \dots$ upto n terms.

Solution:

$5 \times 7 + 9 \times 11 + 13 \times 15 + \dots$ upto n terms

Now, 5, 9, 13, ... are in A.P. with

r th term $= 5 + (r - 1)(4) = 4r + 1$

7, 11, 15, are in A.P. with

r th term $= 7 + (r - 1)(4) = 4r + 3$

$\therefore 5 \times 7 + 9 \times 11 + 13 \times 15 + \dots$ upto n terms

$$= \sum_{r=1}^n (4r+1)(4r+3)$$

$$= \sum_{r=1}^n (16r^2 + 16r + 3)$$

$$= 16 \sum_{r=1}^n r^2 + 16 \sum_{r=1}^n r + 3 \sum_{r=1}^n 1$$

$$= 16 \frac{n(n+1)(2n+1)}{6} + 16 \frac{n(n+1)}{2} + 3n$$

$$= \frac{n}{3} [8(2n^2 + 3n + 1) + 24(n + 1) + 9]$$

$$= \frac{n}{3} (16n^2 + 48n + 41)$$

Question 6.

Find the sum $2^2 + 4^2 + 6^2 + 8^2 + \dots$ upto n terms.

Solution:

$$\begin{aligned} & 2^2 + 4^2 + 6^2 + 8^2 + \dots \text{ upto } n \text{ terms} \\ &= (2 \times 1)^2 + (2 \times 2)^2 + (2 \times 3)^2 + (2 \times 4)^2 + \dots \\ &= \sum_{r=1}^n (2r)^2 \\ &= 4 \sum_{r=1}^n r^2 \\ &= \frac{4 \cdot n(n+1)(2n+1)}{6} \\ &= \frac{2n(n+1)(2n+1)}{3} \end{aligned}$$

Question 7.

Find $(70^2 - 69^2) + (68^2 - 67^2) + (66^2 - 65^2) + \dots + (2^2 - 1^2)$

Solution:

Let $S = (70^2 - 69^2) + (68^2 - 67^2) + \dots + (2^2 - 1^2)$

$\therefore S = (2^2 - 1^2) + (4^2 - 3^2) + \dots + (70^2 - 69^2)$

Here, 2, 4, 6, ..., 70 are in A.P. with r th term $= 2r$

and 1, 3, 5, ..., 69 are in A.P. with r th term $= 2r - 1$

$$\begin{aligned} S &= \sum_{r=1}^{35} [(2r)^2 - (2r-1)^2] \\ &= \sum_{r=1}^{35} [4r^2 - (4r^2 - 4r + 1)] \end{aligned}$$

$$\begin{aligned} &= \sum_{r=1}^{35} (4r-1) \\ &= 4 \sum_{r=1}^{35} r - \sum_{r=1}^{35} 1 \\ &= 4 \cdot \frac{35 \times 36}{2} - 35 \\ &= (72-1)(35) \\ &= 71 \times 35 \\ &= 2485 \end{aligned}$$

Question 8.

Find the sum $1 \times 3 \times 5 + 3 \times 5 \times 7 + 5 \times 7 \times 9 + \dots + (2n-1)(2n+1)(2n+3)$

Solution:

Let $S = 1 \times 3 \times 5 + 3 \times 5 \times 7 + \dots$ upto n terms

Here, 1, 3, 5, 7 ... are in A.P. with r th term $= 2r - 1$,

3, 5, 7, 9,... are in A.P. with r th term $= 2r + 1$,

5, 7, 9, 11,... are in A.P. with r th term $= 2r + 3$

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$$\begin{aligned} S &= \sum_{r=1}^n (2r-1)(2r+1)(2r+3) \\ &= \sum_{r=1}^n (4r^2-1)(2r+3) \\ &= \sum_{r=1}^n (8r^3+12r^2-2r-3) \\ &= 8\sum_{r=1}^n r^3 + 12\sum_{r=1}^n r^2 - 2\sum_{r=1}^n r - 3\sum_{r=1}^n 1 \\ &= 8 \cdot \frac{n^2(n+1)^2}{4} + 12 \frac{n(n+1)(2n+1)}{6} \\ &\quad - 2 \frac{n(n+1)}{2} - 3n \\ &= 2n^2(n+1)^2 + 2n(n+1)(2n+1) - n(n+1) - 3n \\ &= n(n+1) [2n(n+1) + 4n + 2 - 1] - 3n \\ &= n(n+1) (2n^2 + 6n + 1) - 3n \\ &= n(2n^3 + 8n^2 + 7n + 1 - 3) \\ &= n(2n^3 + 8n^2 + 7n - 2) \\ &= n(n+2) (2n^2 + 4n - 1) \end{aligned}$$

Question 9.

If $\frac{1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + \dots \text{upto } n \text{ terms}}{1 + 2 + 3 + 4 + \dots \text{upto } n \text{ terms}} = \frac{100}{3}$, find n .

Solution:

$$\frac{1 \times 2 + 2 \times 3 + 3 \times 4 + \dots \text{upto } n \text{ terms}}{1 + 2 + 3 + 4 + \dots \text{upto } n \text{ terms}} = \frac{100}{3}$$

$$\therefore \frac{\sum_{r=1}^n r(r+1)}{\sum_{r=1}^n r} = \frac{100}{3}$$

$$\therefore \frac{\sum_{r=1}^n r^2 + \sum_{r=1}^n r}{\sum_{r=1}^n r} = \frac{100}{3}$$

$$\therefore \frac{\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}}{\frac{n(n+1)}{2}} = \frac{100}{3}$$

$$\therefore \frac{\frac{n(n+1)}{6} [(2n+1) + 3]}{\frac{n(n+1)}{2}} = \frac{100}{3}$$

$$\therefore \frac{2(n+2)}{3} = \frac{100}{3}$$

$$\therefore n + 2 = 50$$

$$\therefore n = 48$$

Question 10.

If S_1 , S_2 and S_3 are the sums of first n natural numbers, their squares and their cubes respectively, then show that $9S_2^2 = S_3(1 + 8S_1)$.

Solution:

$$S_1 = 1 + 2 + 3 + \dots + n = \sum_{r=1}^n r = \frac{n(n+1)}{2}$$

$$S_2 = 1^2 + 2^2 + 3^2 + \dots + n^2 \\ = \sum_{r=1}^n r^2 = \frac{n(n+1)(2n+1)}{6}$$

$$S_3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \sum_{r=1}^n r^3 = \frac{n^2(n+1)^2}{4}$$

$$\begin{aligned} \text{R.H.S.} &= S_3(1 + 8S_1) \\ &= \frac{n^2(n+1)^2}{4} \left[1 + 8 \cdot \frac{n(n+1)}{2} \right] \\ &= \frac{n^2(n+1)^2}{4} (1 + 4n^2 + 4n) \\ &= \frac{n^2(n+1)^2}{4} (2n+1)^2 \\ &= \frac{9n^2(n+1)^2(2n+1)^2}{36} \\ &= 9 \left[\frac{n(n+1)(2n+1)}{6} \right]^2 \\ &= 9S_2^2 \\ &= \text{L.H.S.} \end{aligned}$$



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Class 11 Arts & Science Maths

(Part 2)

- Chapter 1- Complex Numbers
- Chapter 2- Sequences and Series
- Chapter 3- Permutations and Combination
- Chapter 4- Methods of Induction and Binomial Theorem
- Chapter 5- Sets and Relations
- Chapter 6- Functions
- Chapter 7- Limits
- Chapter 8- Continuity
- Chapter 9- Differentiation

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