



CLASS X MATHEMATICS NOTES

INTRODUCTION TO TRIGONOMETRY

Key Notes and Important Questions with
Answers

Introduction to Trigonometry

Key Points

1. Trigonometric ratio : In $\triangle ABC$, $\angle B = 90^\circ$. For $\angle A$,

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\text{adjacent side}}{\text{Hypotenuse}}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{\text{Opposite side}}{\text{adjacent side}}$$

$$\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{\text{adjacent side}}{\text{opposite side}}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{\text{Hypotenuse}}{\text{adjacent side}}$$

$$\operatorname{cosec} A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{\text{Hypotenuse}}{\text{Opposite side}}$$

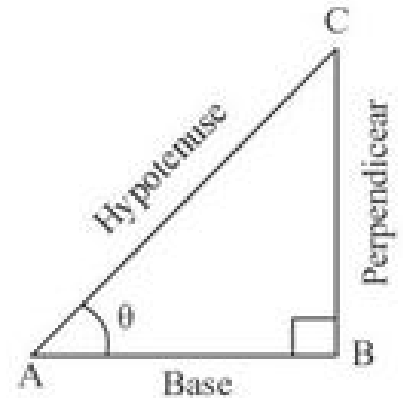
2. Opposites

$$\sin \theta = \frac{1}{\operatorname{cosec} \theta}, \operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}, \sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}, \cot \theta = \frac{1}{\tan \theta}$$

3. $\tan \theta = \frac{\sin \theta}{\cos \theta}, \cot \theta = \frac{\cos \theta}{\sin \theta}$



4. Identities

$$\sin^2 \theta + \cos^2 = 1 \Rightarrow \sin^2 \theta = 1 - \cos^2 \theta \text{ and } \cos^2 \theta = 1 - \sin^2 \theta$$

$$1 + \tan^2 \theta = \sec^2 \theta \Rightarrow \tan^2 \theta = \sec^2 \theta - 1 \text{ and } \sec^2 \theta - \tan^2 \theta = 1$$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta \Rightarrow \cot^2 \theta = \operatorname{cosec}^2 \theta - 1 \text{ and } \operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

5. Trigonometric ratios of some specific angles

$\angle A$	0°	30°	45°	60°	90°
$\sin A$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos A$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan A$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\cot A$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec A$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\operatorname{cosec} A$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

6. Trigonometric ratios of complimentary angles

$$\sin (90^\circ - \theta) = \cos \theta$$

$$\cos (90^\circ - \theta) = \sin \theta$$

$$\tan (90^\circ - \theta) = \cot \theta$$

$$\cot (90^\circ - \theta) = \tan \theta$$

$$\sec (90^\circ - \theta) = \operatorname{cosec} \theta$$

$$\operatorname{cosec} (90^\circ - \theta) = \sec \theta$$

Important Questions with Solutions

VERY SHORT ANSWER TYPE QUESTIONS

1. If $\sin \theta = \cos \theta$, find the value of θ
2. If $\tan \theta = \cot (30^\circ + \theta)$, find the value of θ
3. If $\sin \theta = \cos (\theta - 6^\circ)$, find the value of θ

4. If $\cos A = \frac{7}{25}$, find the value of $\tan A + \cot A$
5. If $\tan \theta = \frac{4}{3}$ then find the value of $\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta}$
6. If $3x = \operatorname{cosec} \theta$ and $\frac{3}{x} = \cot \theta$ then find $3\left(x^2 - \frac{1}{x^2}\right)$
7. If $x = a \sin \theta$ and $y = a \cos \theta$ then find the value of $x^2 + y^2$
8. Find the value of $\operatorname{cosec} 70^\circ - \sec 20^\circ$
9. If $5x = \sec \theta$ and $\frac{5}{x} = \tan \theta$ then find the value of $5\left(x^2 - \frac{1}{x^2}\right)$
10. Find the value of $9 \sec^2 A - 9 \tan^2 A$
11. Express $\sec \theta$ in terms of $\cot \theta$
12. Find the value of $\cos \theta \cos (90^\circ - \theta) - \sin \theta \sin (90^\circ - \theta)$
13. If $\sin (20^\circ + \theta) = \cos 30^\circ$ then find the value of θ .
14. Find the value of $\frac{1 + \tan^2 \theta}{1 + \cot^2 \theta}$
15. Find the value of $\frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}}$

SHORT ANSWER TYPE (I) QUESTIONS

Prove that :

16. $\sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2 \theta$
17. $\sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}} = \tan \theta + \sec \theta$
18. If $x = p \sec \theta + q \tan \theta$ & $y = p \tan \theta + q \sec \theta$ then prove that $x^2 - y^2 = p^2 - q^2$
19. If $7 \sin^2 \theta + 3 \cos^2 \theta = 4$ then show that $\tan \theta = \frac{1}{\sqrt{3}}$

20. If $\sin(A - B) = \frac{1}{2}$, $\cos(A + B) = \frac{1}{2}$ then find the value of A and B.

21. Find the value of $\frac{\cos^2 20^\circ + \cos^2 70^\circ}{\sin^2 59^\circ + \sin^2 31^\circ}$.

22. Prove that : $\tan 1^\circ \tan 11^\circ \tan 21^\circ \tan 69^\circ \tan 79^\circ \tan 89^\circ = 1$

23. If $\sec 4A = \operatorname{cosec}(A - 20^\circ)$ then find the value of A.

24. If $3 \cot A = 4$, find the value of $\frac{\operatorname{Cosec}^2 A + 1}{\operatorname{Cosec}^2 A - 1}$.

25. If $\tan(3x - 15^\circ) = 1$ then find the value of x.

SHORT ANSWER TYPE QUESTIONS

Prove that :

26. $\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \frac{1 + \sin A}{\cos A}$

27. $\frac{1}{\sec x - \tan x} - \frac{1}{\cos x} = \frac{1}{\cos x} - \frac{1}{\sec x + \tan x}$

28. $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \tan \theta + \cot \theta = \sec \theta \operatorname{cosec} \theta + 1$

29. $(\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2 = 7 + \tan^2 \theta + \cot^2 \theta$

30. $\sec A (1 - \sin A) (\sec A + \tan A) = 1$

31. If $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$ then show that $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$

32. If $\tan \theta + \sin \theta = m$, $\tan \theta - \sin \theta = n$ then show that $m^2 - n^2 = 4 \sqrt{mn}$.

33. If $\sec \theta = x + \frac{1}{4x}$, prove that $\sec \theta + \tan \theta = 2x$ or $\frac{1}{2x}$

34. If $\sin \theta + \sin^2 \theta = 1$, prove that $\cos^2 \theta + \cos^4 \theta = 1$

35. Without using trigonometric table, the value of

$$\cot \theta \tan(90^\circ - \theta) - \sec(90^\circ - \theta) \operatorname{cosec} \theta + \sin^2 65^\circ + \sin^2 25^\circ + \sqrt{3} \tan 5^\circ \tan 85^\circ.$$

36. Prove that : $\frac{\cot(90^\circ - \theta)}{\tan \theta} + \frac{\operatorname{cosec}(90^\circ - \theta) \sin \theta}{\tan(90^\circ - \theta)} = \sec^2 \theta$

37. Find the value of :

$$\frac{\cos 20^\circ + \cos^2 70^\circ}{\sec^2 50^\circ - \cot^2 40^\circ} + 2 \operatorname{Cosec}^2 58^\circ - 2 \cot 58^\circ \tan 32^\circ - 4 \tan 13^\circ \tan 37^\circ \tan 77^\circ \tan 45^\circ \tan 53^\circ.$$

38. If A, B, C are the angles of ΔABC then prove that $\operatorname{cosec}^2 \left(\frac{B+C}{2} \right) - \tan^2 \frac{A}{2} = 1$

39. Find the value of $\sec^2 10^\circ - \cot^2 80^\circ + \frac{\sin 15^\circ \cos 75^\circ + \cos 15^\circ \sin 75^\circ}{\cos \theta \sin (90^\circ - \theta) + \sin \theta \cos (90^\circ - \theta)}$.

40. Prove that : $\frac{\tan \theta - \cot \theta}{\sin \theta \cos \theta} = \tan^2 \theta - \cot^2 \theta$.

LONG ANSWER TYPE QUESTIONS

Prove That:

41.
$$\frac{\sec \theta + \tan \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{\cos \theta}{1 - \sin \theta}$$

42.
$$\left(1 + \frac{1}{\tan^2 \theta} \right) \left(1 + \frac{1}{\cot^2 \theta} \right) = \frac{1}{\sin^2 \theta - \sin^4 \theta}$$

43. $2 (\sin^6 \theta + \cos^6 \theta) - 3 (\sin^4 \theta + \cos^4 \theta) + 1 = 0$

44. $(1 + \cot A + \tan A) (\sin A - \cos A) = \sin A \tan A - \cot A \cos A$

45. If $\sin \theta + \cos \theta = m$ and $\sec \theta + \operatorname{cosec} \theta = n$ then show that $n(m^2 - 1) = 2m$

46. find the value of:

$$\frac{\cot (90^\circ - \theta) \tan \theta - \operatorname{Cosec} (90^\circ - \theta) \sec \theta}{\sin 12^\circ \cos 15^\circ \sec 78^\circ \operatorname{Cosec} 75^\circ} + \frac{\cos^2 (50^\circ + \theta) \tan^2 (40^\circ - \theta)}{\tan 15^\circ \tan 37^\circ \tan 53^\circ \tan 75^\circ}$$

47. Prove that :

$$\frac{1}{\operatorname{Cosec} \theta + \cot \theta} - \frac{1}{\sin \theta} = \frac{1}{\sin \theta} - \frac{1}{\operatorname{Cosec} \theta - \cot \theta}$$

48. If $\frac{\cos \alpha}{\cos \beta} = m$ and $\frac{\cos \alpha}{\sin \beta} = n$, then prove that $(m^2 + n^2) \cos^2 \beta = n^2$

49. If $\tan \theta + \sin \theta = m$, $\tan \theta - \sin \theta = n$, then prove that $m^2 - n^2 = 4\sqrt{mn}$

50. Prove that :

$$\sec^2 \theta - \frac{\sin^2 \theta - 2\sin^4 \theta}{2\cos^4 \theta - \cos^2 \theta} = 1$$

51. $\cot \theta \tan (90^\circ - \theta) - \sec (90^\circ - \theta) \operatorname{cosec} \theta + \sqrt{3} \tan 12^\circ \tan 60^\circ \tan 78^\circ$ find its value.

52. Find the value of —

$$\frac{\sec (90^\circ - \theta) \operatorname{cosec} \theta - \tan (90^\circ - \theta) \cot \theta + \cos^2 25^\circ + \cos^2 65^\circ}{3 \tan 27^\circ \tan 63^\circ}$$

ANSWERS

1. 45°

2. 30°

3. 24°

4. $\frac{625}{168}$

5. 7

6. $\frac{1}{3}$

7. a

8. 0

9. $\frac{1}{5}$

10. 9

11. $\frac{\sqrt{1 + \cos^2 \theta}}{\cot \theta}$

12. 0°

13. 50°

14. $\tan^2 \theta$

15. $\tan \theta$

20. $A = 45^\circ, B = 15^\circ$

21. 1

23. 22°

24. $\frac{17}{8}$

25. 20°

35. $\sqrt{3}$

37. -1

39. 2

46. 0

51. 2

52. $\frac{2}{3}$

Practice-Test

Introduction to Trigonometry

Time : 50 Minutes

M.M.: 20

SECTION-A

1. If $\sin \theta = \frac{4}{5}$ what is the value of $\cos \theta$. 1
2. Write the value of $\sin (45^\circ + \theta) - \cos (45^\circ - \theta)$. 1

SECTION-B

3. If $5 \tan \theta = 4$ then find the value of $\frac{5 \sin \theta - 3 \cos \theta}{5 \sin \theta + 2 \cos \theta}$ 2
4. Find the value of $\tan 35^\circ \tan 40^\circ \tan 45^\circ \tan 50^\circ \tan 55^\circ$ 2

SECTION-C

5. Prove that $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \operatorname{Cosec} \theta$ 3
6. Prove that $\frac{\cos A}{1 - \tan A} - \frac{\sin^2 A}{\cos A - \sin A} = \sin A + \cos A$ 3

SECTION-D

7. If $\tan (A + B) = \sqrt{3}$ and $\tan (A - B) = \frac{1}{\sqrt{3}}$ then find the value of A & B. 4
8. Prove that $\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{\cos \theta}{1 - \sin \theta}$ 4