

ખૂણાનું માપન

- (i) $1 \text{ રેડિયન} = \frac{180^\circ}{\pi} = 57^\circ 16' 22''$ (લગભગ)
- (ii) $1^\circ = \frac{\pi}{180}$ રેડિયન, $1^\circ = (60' \text{ મિ}), 1' = 60''$ (60 સે)
- (iii) r ત્રિજ્યાવાળા વર્તુળના ચાપની લંબાઈ

(i) $l = \frac{\pi r \alpha}{180}$, જ્યાં $r =$ વર્તુળની ત્રિજ્યા

$\alpha =$ ચાપે કેન્દ્ર આગળ આંતરેલા ખૂણાનું અંશ માપ

- (iii) $l = r\theta$, જ્યાં $\theta =$ ચાપે કેન્દ્ર આગળ આંતરેલા ખૂણાનું રેડિયન માપ

(iv) કલાકકાંટા (x) અને મિનિટકાંટા (y) વચ્ચે બનતા ખૂણાનું અંશ માપ $\alpha = \frac{|60x - 11y|}{2}$

(v) n બાજુવાળા નિયમિત બહુકોણ દ્વારા બનતા અંતર્ગત ખૂણાના માપનો સરવાળો $= (n - 2)\pi$ છે.

(1) ત્રિકોણમિતિય વિધેયોના પ્રદેશ વિસ્તાર, શૂન્યોનો ગણ અને મુખ્ય આવર્તમાન :

વિધેય	પ્રદેશ	વિસ્તાર	શૂન્યોનો ગણ	મુખ્ય આવર્તમાન
$\sin$	$\mathbb{R}$	$[-1, 1]$	$\{k\pi k \in \mathbb{Z}\}$	2π
$\cos$	$\mathbb{R}$	$[-1, 1]$	$\{(2k+1)\frac{\pi}{2} k \in \mathbb{Z}\}$	2π
$\tan$	$\mathbb{R} - \{(2k+1)\frac{\pi}{2} k \in \mathbb{Z}\}$	$\mathbb{R}$	$\{k\pi k \in \mathbb{Z}\}$	π
$\cot$	$\mathbb{R} - \{k\pi k \in \mathbb{Z}\}$	$\mathbb{R}$	$\{(2k+1)\frac{\pi}{2} k \in \mathbb{Z}\}$	π
$\sec$	$\mathbb{R} - \{(2k+1)\frac{\pi}{2} k \in \mathbb{Z}\}$	$\mathbb{R} - (-1, 1)$	$\emptyset$	2π
$\csc$	$\mathbb{R} - \{k\pi k \in \mathbb{Z}\}$	$\mathbb{R} - (-1, 1)$	$\emptyset$	2π

(2) વધતું-ઘટતું વિધેય :

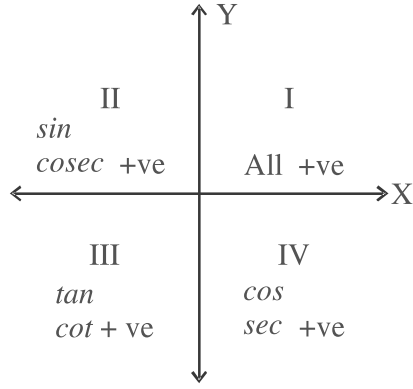
વિધેય/ચરણ	I	II	III	IV
$\cos$	↓	↓	↑	↑
$\sin$	↑	↓	↓	↑
$\tan$	↑	↑	↑	↑
$\cot$	↓	↓	↓	↓
$\sec$	↑	↑	↓	↓
$\csc$	↓	↑	↑	↓

અહીં,

↑ = વધતું વિધેય

↓ = ઘટતું વિધેય દર્શાવે છે.

(3) જુદા-જુદા ચરણમાં ત્રિકોણમિતિય વિધેયનાં મૂલ્યનાં ચિહ્ન :



આકૃતિ 12.1

(4) ત્રિકોણમિતિય વિધેયનાં મૂલ્યો :

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{3\pi}{2}$	2π
$\sin$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	0	-1	0
$\cos$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	0	1
$\tan$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	અવ્યાખ્યાયિત	$-\sqrt{3}$	0	અવ્યાખ્યાયિત	0

(5) વિશિષ્ટ માપના ખૂણા માટે ત્રિવિધેયોનાં મૂલ્યો :

ત્રિ-વિધેયો $\rightarrow$ ખૂણા $\downarrow$	$\sin$	$\cos$	$\tan$	$\cot$
$15^\circ = \frac{\pi}{12}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$2 - \sqrt{3}$	$2 + \sqrt{3}$
$75^\circ = \frac{5\pi}{12}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$2 + \sqrt{3}$	$2 - \sqrt{3}$
$18^\circ = \frac{\pi}{10}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$	$\sqrt{5+2\sqrt{5}}$
$72^\circ = \frac{2\pi}{5}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\sqrt{5+2\sqrt{5}}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$
$36^\circ = \frac{\pi}{5}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\sqrt{5-2\sqrt{5}}$	$\frac{\sqrt{25+10\sqrt{5}}}{5}$
$54^\circ = \frac{3\pi}{10}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{25+10\sqrt{5}}}{5}$	$\sqrt{5-2\sqrt{5}}$
$22\frac{1}{2}^\circ = \frac{\pi}{8}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\sqrt{2} - 1$	$\sqrt{2} + 1$
$67\frac{1}{2}^\circ = \frac{3\pi}{8}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\sqrt{2} + 1$	$\sqrt{2} - 1$

સરવાળાં સૂત્રો :

- (1) $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$
- (2) $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$
- (3) $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$
- (4) $\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$
- (5) $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$
- (6) $\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$
- (7) $\sin(\alpha + \beta) \sin(\alpha - \beta) = \sin^2\alpha - \sin^2\beta$
- (8) $\cos(\alpha + \beta) \cos(\alpha - \beta) = \cos^2\alpha - \sin^2\beta$
- (9) $\cot(\alpha + \beta) = \frac{\cot\alpha \cot\beta - 1}{\cot\beta + \cot\alpha}$
- (10) $\cot(\alpha - \beta) = \frac{\cot\alpha \cot\beta + 1}{\cot\beta - \cot\alpha}$

- (11) $\sin(-\theta) = -\sin\theta$ (અયુગ્મ વિધેય)
- $\cos(-\theta) = \cos\theta$ (યુગ્મ વિધેય)

નોંધ : $\cos$ તથા $\sec$ સિવાયના તમામ ત્રિકોણમિતીય વિધેયો અયુગ્મ વિધેયો છે.

$$(12) \sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta \text{ તથા } \cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta$$

અવયવ સૂત્રો :

- (1) $2\sin\alpha \cos\beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$
- (2) $2\cos\alpha \sin\beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$
- (3) $2\cos\alpha \cos\beta = \cos(\alpha + \beta) + \cos(\alpha - \beta)$
- (4) $2\sin\alpha \sin\beta = -\cos(\alpha + \beta) + \cos(\alpha - \beta)$
- (5) $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$
- (6) $\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$
- (7) $\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$
- (8) $\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$

ગુણિત-ઉપગુણિત સંખ્યાઓ માટે ત્રિકોણમિતીય વિધેયનાં મૂલ્યો :

$$(1) \sin 2\theta = 2\sin\theta \cos\theta = \frac{2\tan\theta}{1 + \tan^2\theta}$$

$$(2) \cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1 = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}$$

$$(3) \tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$(4) \cos^2\theta = \frac{1 + \cos 2\theta}{2}, \sin^2\theta = \frac{1 - \cos 2\theta}{2}, \tan^2\theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

$$(5) \sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

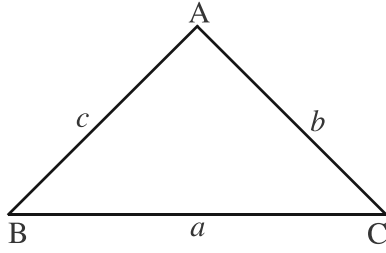
$$(6) \cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$(7) \tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

$$(8) \sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$$

$$(9) \cos^3\theta = \frac{3\cos\theta + \cos 3\theta}{4}$$

ત્રિકોણના ગુણધર્મો



આકૃતિ 12.2

(1) sine સૂત્ર

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

(2) cosine સૂત્ર

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \cos B = \frac{a^2 + c^2 - b^2}{2ac}, \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

(3) પ્રક્ષેપ સૂત્ર

$$a = b \cos C + c \cos B, b = a \cos C + c \cos A, c = a \cos B + b \cos A$$

(4) અર્ધકોણવાળા સૂત્ર

$$(i) \sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}, \sin \frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{ac}}, \sin \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$$

$$(ii) \cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}, \cos \frac{B}{2} = \sqrt{\frac{s(s-b)}{ac}}, \cos \frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$$

$$(iii) \tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}, \tan \frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}, \tan \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

(5) ત્રિકોણનું ક્ષેત્રફળ

$$\Delta = \frac{1}{2}ab \sin C = \frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \frac{abc}{4R}$$

(6) અંતઃત્રિજ્યાના સૂત્રો

$$(i) r = \frac{\Delta}{s}$$

$$(ii) r = (s - a)\tan \frac{A}{2}$$

$$r = (s - b)\tan \frac{B}{2}$$

$$r = (s - c)\tan \frac{C}{2}$$

$$\cot A = \frac{b^2 + c^2 - a^2}{4\Delta}$$

$$\cot B = \frac{a^2 + c^2 - b^2}{4\Delta}$$

$$\cot C = \frac{a^2 + b^2 - c^2}{4\Delta}$$

$$(iii) r = 4R \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

ત્રિકોણમિતિય સમીકરણ :

$$(i) \cos \theta = a, |a| \leq 1 \text{ નો ઉકેલ}$$

$$\theta = 2k\pi \pm \cos^{-1}a, k \in Z$$

$$(ii) \sin \theta = a, |a| \leq 1 \text{ નો ઉકેલ}$$

$$\theta = k\pi + (-1)^k \sin^{-1}a, k \in Z$$

$$(iii) \tan \theta = a, a \in R \text{ નો ઉકેલ}$$

$$\theta = k\pi + \tan^{-1}a, k \in Z$$

$$(iv) a \cos \theta + b \sin \theta = c \text{ ના ઉકેલ માટેની શરત } \left| \frac{c}{r} \right| \leq 1, \text{ જ્યાં } r = \sqrt{a^2 + b^2}$$

$$\theta = 2k\pi \pm \cos^{-1} \frac{c}{r} - \beta, k \in Z, \text{ જ્યાં } \cos \beta = \frac{a}{r}, \sin \beta = \frac{b}{r}.$$

નોંધ : જો $c^2 > a^2 + b^2$ એટલે કે $c^2 > r^2$ તો ઉકેલગણ $\emptyset$ થાય.

ત્રિકોણમિતિય પ્રતિવિધેયનાં સૂત્રો

1. (i) $\sin^{-1}(-x) = -\sin^{-1}x, |x| \leq 1$
- (ii) $\cos^{-1}(-x) = \pi - \cos^{-1}x, |x| \leq 1$
- (iii) $\tan^{-1}(-x) = -\tan^{-1}x, x \in R$
- (iv) $\cot^{-1}(-x) = \pi - \cot^{-1}x, x \in R$
- (v) $\sec^{-1}(-x) = \pi - \sec^{-1}x, |x| \geq 1$
- (iv) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x, |x| \geq 1$

$$2. (i) \sin^{-1} \frac{1}{x} = \operatorname{cosec}^{-1}x; |x| \geq 1$$

$$(ii) \cos^{-1} \frac{1}{x} = \sec^{-1}x, |x| \geq 1$$

$$(iii) \tan^{-1} \frac{1}{x} = \begin{cases} \cot^{-1}x, & x > 0 \\ \cot^{-1}x - \pi, & x < 0 \end{cases}$$

$$3. (i) \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \quad |x| \leq 1$$

$$(ii) \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2} \quad x \in R$$

$$(iii) \sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2} \quad |x| \geq 1$$

4. જો $x > 0, y > 0$, તો

$$(i) \quad \tan^{-1}x + \tan^{-1}y = \begin{cases} \tan^{-1}\left(\frac{x+y}{1-xy}\right) & xy < 1 \\ \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right) & xy > 1 \\ \frac{\pi}{2} & xy = 1 \end{cases}$$

$$(ii) \quad \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$$

$$(iii) \quad \tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$$

નોંધ : (i) જો $xy + yz + zx = 1$ તો $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$

(ii) જો $x + y + z = xyz$ તો $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$

5. આંતરસંબંધ : $0 < x < 1$

$$(a) \quad \sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$$

$$(b) \quad \cos^{-1}x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{\sqrt{1-x^2}}{x}$$

$$(c) \quad \tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = \cos^{-1}\frac{1}{\sqrt{1+x^2}}$$

વિશિષ્ટ સૂત્રો :

1. ત્રણ ખૂણાના સરવાળાનાં સૂત્રો :

$$(1) \quad \sin(A + B + C) = \sin A \cos B \cos C + \cos A \sin B \cos C + \cos A \cos B \sin C - \sin A \sin B \sin C$$

$$(2) \quad \cos(A + B + C) = \cos A \cos B \cos C - \sin A \sin B \cos C - \sin A \cos B \sin C - \cos A \sin B \sin C \\ = \cos A \cos B \cos C (1 - \tan A \tan B - \tan B \tan C - \tan A \tan C)$$

$$(3) \quad \tan(A + B + C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan A \tan C}$$

$$2. (i) \quad \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = \sec\theta + \tan\theta$$

$$(ii) \quad \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) = \sec\theta - \tan\theta$$

$$(iii) \quad \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \cot \frac{\theta}{2} = \operatorname{cosec}\theta + \cot\theta$$

$$(iv) \quad \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \tan \frac{\theta}{2} = \operatorname{cosec}\theta - \cot\theta$$

$$(v) \quad \cos\theta \cos 2\theta \cos 2^2\theta \dots \cos 2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n \sin\theta}, \quad n \in \mathbb{Z} \quad (\theta \neq n\pi)$$

$$(vi) \cos A + \cos(A + B) + \cos(A + 2B) + \dots + \cos(A + (n - 1)B)$$

$$= \frac{\sin \frac{nB}{2}}{\sin \frac{B}{2}} \cos \left\{ A + (n - 1) \frac{B}{2} \right\}$$

$$(vii) \tan \theta \tan \left(\frac{\pi}{3} - \theta \right) \tan \left(\frac{\pi}{3} + \theta \right) = \tan 3\theta$$

$$(viii) A + B + C = \pi \Rightarrow \tan A + \tan B + \tan C = \tan A \tan B \tan C$$

બહુવિકલ્પી પ્રશ્નો

- (1) બે નિયમિત બહુકોણની બાજુઓની સંખ્યાનો ગુણોત્તર 5 : 4 છે અને તેમની બાજુઓના અંતર્ગત ખૂણાઓના રેડિયન માપ વચ્ચે તફાવત $\frac{\pi}{30}$ છે, તો બંને બહુકોણની બાજુઓની સંખ્યા થાય.

(A) 15, 12

(B) 20, 16

(C) 10, 8

(D) 25, 20

ઉકેલ : N બાજુવાળા નિયમિત બહુકોણના અંતર્ગત ખૂણાના (ધારો $N > n$) માપનો સરવાળો $= (N - 2)\pi$

$$\therefore \text{અંતર્ગત એક ખૂણાનું માપ} = \frac{(N - 2)\pi}{N}$$

$$n \text{ બાજુવાળા નિયમિત બહુકોણના અંતર્ગત એક ખૂણાનું માપ} = \frac{(n - 2)\pi}{n}$$

$$\therefore \text{તેમની વચ્ચે તફાવત} = \frac{\pi}{30}$$

$$\therefore \frac{(N - 2)\pi}{N} - \frac{(n - 2)\pi}{n} = \frac{\pi}{30}$$

$$\Rightarrow \frac{2}{n} - \frac{2}{N} = \frac{1}{30}$$

$$\text{તથા } \frac{N}{n} = \frac{5}{4} \text{ આથી } N = \frac{5}{4}n$$

$$\therefore \frac{2}{n} - \frac{2}{\left(\frac{5}{4}\right)n} = \frac{1}{30}$$

$$\therefore \frac{2}{n} - \frac{8}{5n} = \frac{1}{30}$$

$$\therefore \frac{2}{5n} = \frac{1}{30}$$

$$\therefore n = 12 \text{ તથા } N = \frac{5}{4}n = 15 \quad \text{જવાબ : (A)}$$

- (2) એક સમબાજુ ત્રિકોણની બાજુનું માપ 4 સેમી છે. તેનું એક શિરોબિંદુ વર્તુળનું કેન્દ્ર છે. એ વર્તુળની એક ચાપ ત્રિકોણને બે એકરૂપ ભાગમાં વિભાજિત કરે છે. આ વર્તુળની ત્રિજ્યા હોય.

(A) $\sqrt{12\sqrt{3}}$ સેમી

(B) $\frac{\sqrt{12\sqrt{3}}}{\pi}$ સેમી

(C) $\sqrt{\frac{\pi}{12\sqrt{3}}}$ સેમી

(D) $\sqrt{\frac{12\sqrt{3}}{\pi}}$ સેમી

$$\text{ઉકેલ : સમબાજુ ત્રિકોણનું ક્ષેત્રફળ} = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(4)^2 = 4\sqrt{3}$$

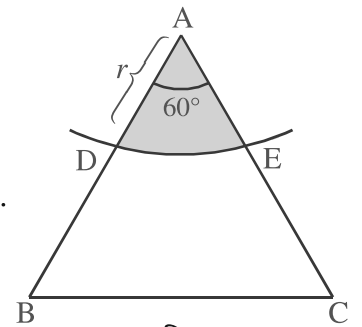
$$\therefore \frac{1}{2} \text{ સમબાજુ ત્રિકોણનું ક્ષેત્રફળ} = 2\sqrt{3}$$

સમબાજુ $\triangle ABC$ માં A ને કેન્દ્ર તરીકે લઈ દોરેલ વર્તુળની કોઈ ચાપ $\widehat{DE}$ એ $\triangle ABC$ ને બે સમાન ભાગમાં વિભાજિત કરે છે. જેથી વૃત્તાંશ ADE શક્ય બને.

$$\text{હવે, વૃત્તાંશ ADE નું ક્ષેત્રફળ} = \frac{1}{2}r^2\theta = 2\sqrt{3}, \text{ જ્યાં } \theta = \frac{\pi}{3}$$

$$\therefore r^2 = \frac{2\sqrt{3} \times 6}{\pi}$$

$$\therefore r = \sqrt{\frac{12\sqrt{3}}{\pi}} \text{ સેમી}$$



જવાબ : (D)

- (3) એક ચતુષ્કોણના ચાર ખૂણાનાં માપ સમાંતર શ્રેણીમાં છે. સૌથી મોટા તથા નાના ખૂણાના માપનો ગુણોત્તર $2 : 1$ છે, તો સૌથી મોટા ખૂણાનું રેડિયન માપ છે.

(A) $\frac{5\pi}{6}$ (B) $\frac{2\pi}{3}$ શ્ર (C) $\frac{5\pi}{9}$ (D) આમાંથી એક પણ નહિ.

ઉકેલ : ધારો કે ચતુષ્કોણના ચાર ખૂણાઓનાં માપ $a - 3d, a - d, a + d, a + 3d$ છે.

$\therefore$ ચારેયનો સરવાળો 2π છે.

$$a - 3d + a - d + a + d + a + 3d = 2\pi$$

$$\therefore 4a = 2\pi$$

$$\therefore a = \frac{\pi}{2}$$

$$\frac{\text{સૌથી મોટા ખૂણાનું માપ}}{\text{સૌથી નાના ખૂણાનું માપ}} = \frac{2}{1}$$

$$\therefore \frac{a + 3d}{a - 3d} = 2$$

$$\therefore 9d = \frac{\pi}{2}$$

$$\therefore \frac{\pi}{2} + 3d = 2\left(\frac{\pi}{2} - 3d\right)$$

$$\therefore d = \frac{\pi}{18}$$

$$\therefore \text{સૌથી મોટા ખૂણાનું રેડિયન માપ} = a + 3d = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$$

જવાબ : (B)

- (4) r ત્રિજ્યાવાળા વર્તુળના કોઈ એક ચાપની લંબાઈ, તે જ વર્તુળના બંધ $\frac{1}{8}$ (વર્તુળ) પરિમિતિ જેટલી છે, તો આ ચાપે કેન્દ્ર આગળ આંતરેલા ખૂણાનું રેડિયન માપ છે.

(A) π (B) $\pi - 2$ (C) $\pi + 2$ (D) $\pi + 3$

ઉકેલ : r ત્રિજ્યાવાળા વર્તુળના ચાપની લંબાઈ l

$$= \text{બંધ} \left(\frac{1}{8} \text{ વર્તુળ}\right) \text{ની પરિમિતિ}$$

$$= \frac{\pi r}{4} + 2r$$

$$= \left(\frac{\pi}{4} + 2\right)r$$

પરંતુ જો r ત્રિજ્યાવાળા વર્તુળ દ્વારા કેન્દ્ર આગળ આંતરેલા, ખૂણાનું રેડિયન માપ θ હોય, તો

$$\text{ચાપની લંબાઈ } l = r\theta$$

$$\therefore r\theta = \left(\frac{\pi}{4} + 2\right)r$$

$$\therefore \theta = \frac{\pi}{4} + 2 \text{ રેડિયન}$$

જવાબ : (C)

- (5) $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$ તો,

$$(A) \tan^2 x = \frac{1}{5}$$

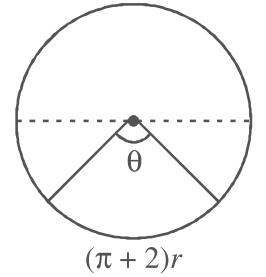
$$(B) \tan^2 x = \frac{1}{3}$$

$$(C) \frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$$

$$(D) \frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$$

ઉકેલ : $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$

$$\therefore \frac{\sin^4 x}{2} + \frac{(1 - \sin^2 x)^2}{3} = \frac{1}{5}$$



આકૃતિ 12.4

$$\text{ધારો કે } \sin^2 x = m$$

$$\therefore \frac{m^2}{2} + \frac{(1-m)^2}{3} = \frac{1}{5}$$

$$\therefore \frac{3m^2 + 2(1-m)^2}{6} = \frac{1}{5}$$

$$\therefore 5[3m^2 + 2(1 - 2m + m^2)] = 6$$

$$\therefore 25m^2 - 20m + 4 = 0$$

$$\therefore (5m - 2)^2 = 0$$

$$\therefore m = \frac{2}{5} = \sin^2 x$$

$$\therefore \sin^2 x = \frac{2}{5} \Rightarrow \cos^2 x = \frac{3}{5}$$

$$\therefore \tan^2 x = \frac{2}{3}$$

$$\therefore (A), (B) \text{ શક્ય નથી.}$$

$$\text{હવે, } \frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{(\sin^2 x)^4}{8} + \frac{(\cos^2 x)^4}{27} = \frac{\left(\frac{2}{5}\right)^4}{8} + \frac{\left(\frac{3}{5}\right)^4}{27} = \frac{2+3}{5^4} = \frac{1}{125} \text{ જવાબ : (C)}$$

$$(6) \quad x = \sec \theta - \tan \theta, y = \operatorname{cosec} \theta + \cot \theta \text{ તો } xy + 1 = \dots$$

$$(A) x + y$$

$$(B) x - y$$

$$(C) y - x$$

$$(D) -x - y$$

$$\text{ઉકેલ : } xy + 1 = (\sec \theta - \tan \theta)(\operatorname{cosec} \theta + \cot \theta) + 1$$

$$= \left(\frac{1 - \sin \theta}{\cos \theta} \right) \left(\frac{1 + \cos \theta}{\sin \theta} \right) + 1$$

$$= \frac{1 + \cos \theta - \sin \theta - \sin \theta \cos \theta + \sin \theta \cos \theta}{\sin \theta \cos \theta}$$

$$= \frac{1 + \cos \theta - \sin \theta}{\sin \theta \cos \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta + \cos \theta - \sin \theta}{\sin \theta \cos \theta}$$

$$= \tan \theta + \cot \theta + \operatorname{cosec} \theta - \sec \theta$$

$$= (\cot \theta + \operatorname{cosec} \theta) - (\sec \theta - \tan \theta)$$

$$= y - x$$

$$\text{જવાબ : (C)}$$

$$(7) \quad \theta \text{ અને } \phi \text{ બંને લઘુકોણ છે. જો } \sin \theta = \frac{1}{2} \text{ તથા } \cos \phi = \frac{1}{3} \text{ હોય, તો } \theta + \phi \in \dots [\text{IIT : 2004}]$$

$$(A) \left[\frac{\pi}{3}, \frac{\pi}{2} \right]$$

$$(B) \left[\frac{\pi}{2}, \frac{2\pi}{3} \right]$$

$$(C) \left[\frac{2\pi}{3}, \frac{5\pi}{6} \right]$$

$$(D) \left[\frac{5\pi}{6}, \pi \right]$$

$$\text{ઉકેલ : } \theta \text{ અને } \phi \text{ બંને લઘુકોણ છે.}$$

$$\sin \theta = \frac{1}{2}. \text{ તેથી } \theta = \frac{\pi}{6} \quad (1)$$

$$\cos \text{ વિધેય પ્રથમ ચરણમાં ઘટતું વિધેય હોવાથી,}$$

$$\cos \phi = \frac{1}{3} < \frac{1}{2} = \cos \frac{\pi}{3}$$

$$\text{હવે, } \cos \phi = \frac{1}{3}$$

$$\text{સ્પષ્ટ છે કે, } \frac{\pi}{3} < \phi < \frac{\pi}{2}$$

$$\therefore \frac{\pi}{3} + \theta < \theta + \phi < \frac{\pi}{2} + \theta$$

$$\therefore \frac{\pi}{3} + \frac{\pi}{6} < \theta + \phi < \frac{\pi}{2} + \frac{\pi}{6} \quad ((1) \text{ પરથી})$$

$$\therefore \frac{\pi}{2} < \theta + \phi < \frac{2\pi}{3}$$

$$\therefore \theta + \phi \in \left[\frac{\pi}{2}, \frac{2\pi}{3} \right] \quad \text{જવાબ : (B)}$$

(8) $\sin^2\theta + 3\cos\theta - 2 = 0$ તો $\cos^3\theta + \sec^3\theta = \dots$, $\theta \neq (2k+1)\frac{\pi}{2}$, $k \in \mathbb{Z}$

(A) 18

(B) 9

(C) 4

(D) $\frac{1}{4}$

ઉકેલ : $\sin^2\theta + 3\cos\theta - 2 = 0$

$\therefore 1 - \cos^2\theta + 3\cos\theta - 2 = 0$

$\therefore \cos^2\theta - 3\cos\theta + 1 = 0$

$\therefore \cos\theta - 3 + \sec\theta = 0$ ($\cos\theta \neq 0$)

$\therefore \cos\theta + \sec\theta = 3$

$\therefore (\cos\theta + \sec\theta)^3 = \cos^3\theta + \sec^3\theta + 3\cos\theta \sec\theta(\cos\theta + \sec\theta)$

$\therefore 27 = \cos^3\theta + \sec^3\theta + 3(3)$

$\therefore \cos^3\theta + \sec^3\theta = 18$

જવાબ : (A)

(9) જો $x = \sin^2\theta \cos\theta$ અને $y = \cos^2\theta \sin\theta$ હોય તો, નીચેનામાંથી કયું સત્ય છે ? $0 < \theta < \frac{\pi}{2}$, $x > y > 0$

(A) $(x^2y)^{\frac{2}{3}} + (xy^2)^{\frac{2}{3}} = 1$

(B) $\left(\frac{x^2}{y}\right)^{\frac{2}{3}} + \left(\frac{y^2}{x}\right)^{\frac{2}{3}} = 1$

(C) $x^2 + y^2 = x^2y^2$

(D) આમાંથી એક પણ નહિ.

ઉકેલ : અહીં, $\frac{x}{y} = \frac{\sin^2\theta \cos\theta}{\sin\theta \cos^2\theta} = \tan\theta$ મળે.

$\therefore \frac{x^2}{y^2} + 1 = \sec^2\theta$

$\therefore \cos^2\theta = \frac{y^2}{x^2 + y^2}, \sin^2\theta = \frac{x^2}{x^2 + y^2}$

આથી, $x = \sin^2\theta \cos\theta$ પરથી, $x = \frac{x^2}{x^2 + y^2} \cdot \frac{y}{\sqrt{x^2 + y^2}}$

$\therefore (x^2 + y^2)^{\frac{3}{2}} = xy$. આથી, $x^2 + y^2 = (xy)^{\frac{2}{3}}$

$\therefore \frac{x^2}{(xy)^{\frac{2}{3}}} + \frac{y^2}{(xy)^{\frac{2}{3}}} = 1$. આથી, $\frac{x^{\frac{4}{3}}}{y^{\frac{2}{3}}} + \frac{y^{\frac{4}{3}}}{x^{\frac{2}{3}}} = 1$

$\therefore \left(\frac{x^2}{y}\right)^{\frac{2}{3}} + \left(\frac{y^2}{x}\right)^{\frac{2}{3}} = 1$

જવાબ : (B)

(10) $\sin\theta \cos\theta$ ની ન્યૂનતમ કિંમત છે.

(A) 1

(B) 0

(C) $-\frac{1}{2}$

(D) $\frac{1}{2}$

ઉકેલ : $(\sin\theta + \cos\theta)^2 \geq 0$

$\therefore \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta \geq 0$

$\therefore 1 + 2\sin\theta\cos\theta \geq 0$

$$\therefore \sin\theta\cos\theta \geq -\frac{1}{2}$$

$$\text{ન્યૂનતમ કિંમત} = -\frac{1}{2} \text{ છે.}$$

બીજી રીત :

$$\sin\theta\cos\theta = \frac{1}{2} \cdot 2\sin\theta\cos\theta = \frac{1}{2} \cdot \sin 2\theta$$

$$\text{હવે } \sin 2\theta \geq -1. \text{ આથી } \frac{1}{2}\sin 2\theta \geq -\frac{1}{2}$$

$$\text{ન્યૂનતમ કિંમત} = -\frac{1}{2}$$

જવાબ : (C)

$$\text{નોંધ : } \theta = \frac{3\pi}{4} \text{ માટે } \sin\theta\cos\theta = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{-1}{\sqrt{2}}\right) = -\frac{1}{2} \text{ છે.}$$

$$(11) \sin x + \sin^2 x = 1 \text{ તો } \cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x + 2\cos^4 x + \cos^2 x - 2 = \dots\dots$$

$$(A) 0$$

$$(B) 1$$

$$(C) 2$$

$$(D) \sin^2 x$$

$$\text{ઉકેલ : } \sin x + \sin^2 x = 1 \Rightarrow \sin x = 1 - \sin^2 x = \cos^2 x$$

$$\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x + 2\cos^4 x + \cos^2 x - 2 = \dots\dots$$

$$= \sin^6 x + 3\sin^5 x + 3\sin^4 x + \sin^3 x + 2\sin^2 x + \sin x - 2$$

$$= \left[(\sin^2 x)^3 + \binom{3}{1} (\sin^2 x)^2 (\sin x) + \binom{3}{2} (\sin^2 x) (\sin x)^2 + \binom{3}{3} (\sin x)^3 \right] + (\sin^2 x + \sin x) - 2 + \sin^2 x$$

$$= [\sin^2 x + \sin x]^3 + [\sin^2 x + \sin x] - 2 + \sin^2 x$$

$$= 1 + 1 - 2 + \sin^2 x$$

$$= \sin^2 x$$

જવાબ : (D)

$$(12) \frac{\sin 3\alpha}{\cos 2\alpha} \text{ નું મૂલ્ય અંતરાલો } (p) \text{ તથા } (q) \text{ માંથી કયા અંતરાલમાં ધન છે ? કયા અંતરાલ માટે ઋણ છે ? તે નક્કી કરો.}$$

$$(p) \left(\frac{13\pi}{48}, \frac{14\pi}{48} \right) \quad (q) \left(\frac{18\pi}{48}, \frac{23\pi}{48} \right)$$

$$(A) (p) \text{ ધન } (q) \text{ ઋણ}$$

$$(B) (p) \text{ ઋણ } (q) \text{ ધન}$$

$$(C) \text{ બંને ધન}$$

$$(D) \text{ બંને ઋણ}$$

$$\text{ઉકેલ : } (p) \frac{13\pi}{48} < \alpha < \frac{14\pi}{48}$$

$$\Rightarrow \frac{13\pi}{16} < 3\alpha < \frac{14\pi}{16}$$

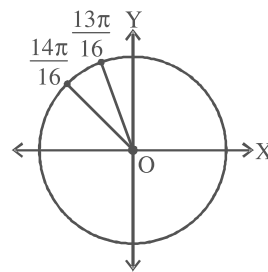
$$\therefore P(3\alpha) \text{ બીજા ચરણમાં છે.}$$

$$\therefore \sin 3\alpha > 0$$

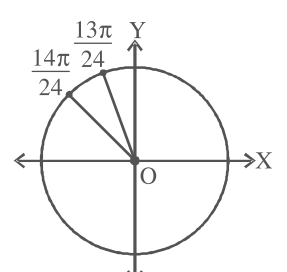
$$\text{હવે, } \frac{13\pi}{48} < \alpha < \frac{14\pi}{48}$$

$$\frac{13\pi}{24} < 2\alpha < \frac{14\pi}{24}$$

$$\therefore P(2\alpha) \text{ બીજા ચરણમાં છે.}$$



આકૃતિ 12.5



આકૃતિ 12.6

$$\cos 2\alpha < 0$$

$$\therefore \frac{\sin 3\alpha}{\cos 2\alpha} < 0$$

$\therefore$ અંતરાલ (p) માટે ઋણ છે.

$$(q) \frac{18\pi}{48} < \alpha < \frac{23\pi}{48}$$

$$\Rightarrow \frac{18\pi}{16} < 3\alpha < \frac{23\pi}{16}$$

$\therefore P(3\alpha)$ ત્રીજા ચરણમાં છે.

$$\therefore \sin 3\alpha < 0$$

$$\frac{18\pi}{48} < \alpha < \frac{23\pi}{48}$$

$$\Rightarrow \frac{18\pi}{24} < 2\alpha < \frac{23\pi}{24}$$

$\therefore P(2\alpha)$ બીજા ચરણમાં છે.

$$\therefore \cos 2\alpha < 0$$

$$\therefore \frac{\sin 3\alpha}{\cos 2\alpha} > 0$$

$\therefore (q)$ માટે ધન છે.

(13) નીચેનામાંથી કોનું મૂલ્ય સૌથી મોટું છે ?

(A) $\tan 1$

(B) $\tan 2$

(C) $\tan 3$

(D) $\tan 4$

ઉકેલ : $\tan 2 < 0$, કારણ કે $\frac{\pi}{2} < 2 < \pi$

$$\tan 3 < 0, \text{ કારણ કે } \frac{\pi}{2} < 3 < \pi$$

$$\tan 4 = \tan(4 - \pi) > 0 \text{ કારણ કે } 0 < 4 - \pi < \frac{\pi}{2}. \text{ વળી } 0 < 4 - \pi < 1 < \frac{\pi}{2}$$

$$\therefore \tan(4 - \pi) = \tan 4 < \tan 1$$

($\tan$ પ્રથમ ચરણમાં વધતું વિધેય છે.)

$$\therefore \tan 1 \text{ સૌથી મોટી સંખ્યા છે.}$$

જવાબ : (A)

(14) જો $y = \frac{\sec^2 \theta - \tan \theta}{\sec^2 \theta + \tan \theta}$ તો $y \in \dots\dots$

(A) $\left(\frac{1}{3}, 3\right)$

(B) $\left(-3, -\frac{1}{3}\right)$

(C) $\left[\frac{1}{3}, 3\right]$

(D) $\left(\frac{1}{3}, 4\right)$

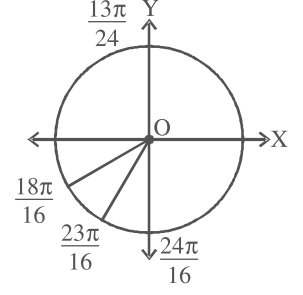
ઉકેલ : ધારો કે $y = \frac{\sec^2 \theta - \tan \theta}{\sec^2 \theta + \tan \theta}$

$$\therefore y = \frac{1 + \tan^2 \theta - \tan \theta}{1 + \tan^2 \theta + \tan \theta}$$

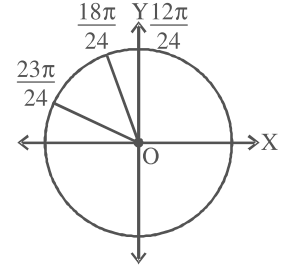
$$\therefore y + y \tan^2 \theta + y \tan \theta = 1 + \tan^2 \theta - \tan \theta$$

$$\therefore (y - 1) \tan^2 \theta + (y + 1) \tan \theta + (y - 1) = 0$$

$\tan \theta$ માં દ્વિઘાત સમીકરણનાં વાસ્તવિક બીજ માટે $\Delta \geq 0$ જરૂરી છે.



આકૃતિ 12.7



આકૃતિ 12.8

જવાબ : (B)

$$\begin{aligned}
&\therefore (y+1)^2 - 4(y-1)^2 \geq 0 \\
&\therefore y^2 + 2y + 1 - 4(y^2 - 2y + 1) \geq 0 \\
&\therefore -3y^2 + 10y - 3 \geq 0 \\
&\therefore 3y^2 - 10y + 3 \leq 0 \\
&\therefore (3y-1)(y-3) \leq 0 \Rightarrow y \in \left[\frac{1}{3}, 3\right]
\end{aligned}$$

$$\therefore y \in \left[\frac{1}{3}, 3\right] \quad \text{જવાબ : (C)}$$

- (15) કોઈ વાસ્તવિક $\theta \in \mathbb{R}$ માટે $\cos\theta = x + \frac{1}{x}$, $x \neq 0$ હોય તો નીચેનામાંથી કયું સત્ય બને ?
 (A) θ લઘુકોણ છે. (B) θ ગુરુકોણ છે. (C) θ કાટકોણ છે. (D) θ શક્ય નથી.

ઉકેલ : $\cos\theta = x + \frac{1}{x}$

$$\therefore \cos^2\theta = x^2 + 2 + \frac{1}{x^2} > 2 \text{ શક્ય નથી.} \quad \text{જવાબ : (D)}$$

- (16) $\sqrt{\cos^2 x - 10\cos x + 25} + 2$ નો વિસ્તાર છે.

- (A) [4, 6] (B) (6, 8] (C) [6, 8] (D) [8, 10]

ઉકેલ : $\sqrt{\cos^2 x - 10\cos x + 25} + 2$

$$\begin{aligned}
&= \sqrt{(\cos x - 5)^2} + 2 \\
&= |\cos x - 5| + 2 \\
&-1 \leq \cos x \leq 1 \Leftrightarrow -6 \leq \cos x - 5 \leq -4 \\
&\Leftrightarrow 4 \leq |\cos x - 5| \leq 6 \\
&\Leftrightarrow 6 \leq |\cos x - 5| + 2 \leq 8 \\
&\Leftrightarrow 6 \leq f(x) \leq 8
\end{aligned}$$

$$\therefore \text{વિસ્તાર } [6, 8] \quad \text{જવાબ : (C)}$$

- (17) $\sum_{i=1}^3 \sin^2\theta_i = 0$ તો $\sum_{i=1}^3 \cos\theta_i$ ની કિંમત નીચેનામાંથી કઈ શક્ય ન બને ?

- (A) 3 (B) -3 (C) -1 (D) -2

ઉકેલ : $\sum_{i=1}^3 \sin^2\theta_i = 0$

$$\therefore \sin^2\theta_1 + \sin^2\theta_2 + \sin^2\theta_3 = 0$$

$$\therefore \sin\theta_1 = \sin\theta_2 = \sin\theta_3 = 0$$

$$\therefore \cos\theta_1 = \cos\theta_2 = \cos\theta_3 = \pm 1 \quad (1)$$

$\cos\theta_1$	$\cos\theta_2$	$\cos\theta_3$	સરવાળો
1	1	1	3
-1	-1	-1	-3
1	1	-1	1
-1	-1	1	-1

આમ, કોષ્ટક પરથી સ્પષ્ટ છે કે સરવાળો -2 શક્ય નથી.

જવાબ : (D)

(18) જો $\sec^2\theta = \frac{4xy}{(x+y)^2}$ સત્ય હોય, તો નીચેનામાંથી કયું સત્ય છે ? ($x \neq -y$) [IIT : 1996]

- (A) $x + y = 0$ (B) $x = y, x \neq 0$ (C) $x = y$ (D) $x \neq y$

ઉકેલ : $\sec^2\theta \geq 1$

$$\frac{4xy}{(x+y)^2} \geq 1 \Leftrightarrow 4xy \geq (x+y)^2 \quad ((x+y)^2 > 0)$$

$$\Leftrightarrow x^2 - 2xy + y^2 \leq 0$$

$$\Leftrightarrow (x - y)^2 \leq 0$$

$(x - y)^2 < 0$ શક્ય નથી.

$$\therefore x - y = 0$$

$$\therefore x = y \text{ અને } x \neq 0$$

જવાબ : (B)

(19) $\sqrt{\frac{1 - \sin A}{1 + \sin A}} = \dots\dots -\frac{\pi}{2} < A < \frac{\pi}{2}$

- (A) $\sec A + \tan A$ (B) $\sec A - \tan A$ (C) $-\sec A + \tan A$ (D) $-(\sec A + \tan A)$

ઉકેલ : $\sqrt{\frac{1 - \sin A}{1 + \sin A}} \times \sqrt{\frac{1 - \sin A}{1 - \sin A}}$

$$= \frac{1 - \sin A}{\sqrt{1 - \sin^2 A}}$$

$$= \frac{1 - \sin A}{|\cos A|}$$

$$= \frac{1 - \sin A}{\cos A} \quad \left[\left(-\frac{\pi}{2} < A < \frac{\pi}{2} \right) \text{ હોવાથી } \cos A > 0 \right]$$

$$= \sec A - \tan A$$

જવાબ : (B)

(20) જો $A = \sin^8\theta + \cos^{14}\theta, \forall \theta \in \mathbb{R}$ હોય તો, નીચેનામાંથી કયું સત્ય બને ?

- (A) $A \geq 1$ (B) $0 < A \leq 1$ (C) $\frac{1}{2} \leq A \leq \frac{3}{2}$ (D) આમાંથી એક પણ નહિ.

ઉકેલ : આપણે જાણીએ છીએ કે,

$$0 \leq \sin^2 A \leq 1 \text{ તથા } 0 \leq \cos^2 A \leq 1$$

$$\Rightarrow \sin^4 A \leq \sin^2 A$$

$$\Rightarrow \sin^6 A \leq \sin^4 A \leq \sin^2 A$$

$$\Rightarrow \sin^8 A \leq \sin^2 A$$

$$\cos^4 A \leq \cos^2 A$$

$$\Rightarrow \cos^6 A \leq \cos^4 A \leq \cos^2 A$$

$$\Rightarrow \cos^{14} A \leq \cos^2 A$$

$$0 < \sin^8 A + \cos^{14} A \leq \sin^2 A + \cos^2 A \quad (\because \sin^8 A + \cos^{14} A \text{ ની કિંમત શૂન્ય શક્ય નથી.})$$

$$\Rightarrow 0 < \sin^8 A + \cos^{14} A \leq 1$$

જવાબ : (B)

નોંધ : $\sin A$ તથા $\cos A$ બંને એક સાથે શૂન્ય હોય, તો જ $\sin^8 A + \cos^{14} A$ ની કિંમત શૂન્ય થાય અને $\sin^2 A + \cos^2 A = 1$ હોવાથી, આ શક્ય નથી.

$$(21) \quad \cos\theta - \sin\theta = \frac{1}{5}, \quad 0 < \theta < \frac{\pi}{2}$$

I		II	
(i) $\frac{(\cos\theta + \sin\theta)}{2}$		(a) $\frac{4}{5}$	(b) $\frac{7}{10}$
(ii) $\cos\theta$		(c) $\frac{24}{25}$	(d) $\frac{7}{25}$
(A) (i) (a) (ii) (c)	(B) (i) (b) (ii) (d)	(C) (i) (a) (ii) (d)	(D) (i) (b) (ii) (a)

$$\text{ઉકેલ : } \cos\theta = \frac{1}{5} + \sin\theta$$

$$\therefore 5\cos\theta = 1 + 5\sin\theta$$

$$\therefore 25\cos^2\theta = 1 + 10\sin\theta + 25\sin^2\theta$$

$$\therefore 25(1 - \sin^2\theta) = 1 + 10\sin\theta + 25\sin^2\theta$$

$$\therefore 50\sin^2\theta + 10\sin\theta - 24 = 0$$

$$\therefore 25\sin^2\theta + 5\sin\theta - 12 = 0$$

$$\therefore (5\sin\theta - 3)(5\sin\theta + 4) = 0$$

$$\therefore \sin\theta = \frac{3}{5} \text{ કારણ કે } 0 < \theta < \frac{\pi}{2}$$

$$0 < \theta < \frac{\pi}{2}. \text{ આથી, } \cos\theta = \frac{4}{5}$$

$$\therefore \frac{\sin\theta + \cos\theta}{2} = \frac{7}{10}$$

$$\therefore (i) \rightarrow (b) \quad (ii) \rightarrow (a)$$

જવાબ : (D)

$$(22) \quad 2 - \cos x + \sin^2 x \text{ ની મહત્તમ તથા ન્યૂનતમ કિંમતનો ગુણોત્તર છે.}$$

$$(A) \frac{7}{4}$$

$$(B) \frac{11}{4}$$

$$(C) \frac{13}{4}$$

(D) આમાંથી એક પણ નહિ.

$$\begin{aligned} \text{ઉકેલ : } f(x) &= 2 - \cos x + \sin^2 x \\ &= 2 - \cos x + 1 - \cos^2 x \\ &= 3 - \cos x - \cos^2 x \\ &= 3 + \frac{1}{4} - \frac{1}{4} - \cos x - \cos^2 x \\ &= \frac{13}{4} - \left(\cos x + \frac{1}{2}\right)^2 \end{aligned}$$

$$f(x) \text{ મહત્તમ મેળવવા } \left(\cos x + \frac{1}{2}\right)^2 \text{ ન્યૂનતમ થવું જોઈએ.}$$

$$\therefore \left(\cos x + \frac{1}{2}\right) = 0 \text{ થવું જોઈએ. આથી, મહત્તમ કિંમત} = \frac{13}{4} \text{ મળે, તે માટે } \cos x = -\frac{1}{2} \text{ જરૂરી છે.}$$

$$f(x)_{\max} = \frac{13}{4}$$

$$f(x) \text{ ન્યૂનતમ મેળવવા } \left(\cos x + \frac{1}{2}\right)^2 \text{ મહત્તમ થવું જોઈએ. આથી, } \cos x = 1 \text{ હોવું જોઈએ.}$$

$$\therefore f(x) = \frac{13}{4} - \left(1 + \frac{1}{2}\right)^2 = \frac{13}{4} - \frac{9}{4} = 1$$

$$\therefore f(x)_{\min} = 1$$

$$\therefore \frac{f(x)_{\max}}{f(x)_{\min}} = \frac{13}{4}$$

જવાબ : (C)

- (23) $\sec^2(a+2)x + a^2 - 1 = 0$, $-\pi < x < \pi$ સમીકરણના ઉકેલ (a, x) હોય, તો આ ઉકેલની સંખ્યા છે.
 (A) 2 (B) 1 (C) 3 (D) અનંત

ઉકેલ : $\sec^2(a+2)x + a^2 - 1 = 0$

$\therefore \sec^2(a+2)x = 1 - a^2$

$\therefore a = 0$ માટે જ શક્ય બને. આથી, $a = 0$

$a = 0 \Rightarrow \sec^2 2x = 1 \Rightarrow \sec 2x = \pm 1$

$-\pi < x < \pi$. આથી, $-2\pi < 2x < 2\pi$

$\sec 2x = 1 \Rightarrow x = 0$ તથા $\sec 2x = -1 \Rightarrow 2x = \pi, -\pi \Rightarrow x = \frac{\pi}{2}, -\frac{\pi}{2}$

$\therefore$ ઉકેલની સંખ્યા 3 છે.

જવાબ : (C)

- (24) $\tan x - \tan^2 x = 1$ તો $\tan^4 x - 2\tan^3 x - \tan^2 x + 2\tan x + 1$ ની કિંમત છે.

(A) 1 (B) 2 (C) 3 (D) 4

ઉકેલ : $\tan^4 x - 2\tan^3 x + \tan^2 x - 2\tan^2 x + 2\tan x + 1$

$= (\tan^2 x - \tan x)^2 - 2(\tan^2 x - \tan x) + 1 = (-1)^2 - 2(-1) + 1 = 1 + 2 + 1 = 4$ જવાબ : (D)

- (25) $2\cos^2 \frac{x}{2} \sin^2 x = x^2 + x^{-2}$, $0 < x \leq \frac{\pi}{2}$ ને

[IIT : 1984]

(A) એક પણ વાસ્તવિક ઉકેલ નથી.

(B) એક જ વાસ્તવિક ઉકેલ મળે.

(C) એક કરતાં વધુ ઉકેલ મળે.

(D) આમાંથી એક પણ નહિ.

ઉકેલ : અહીં, $\left(x - \frac{1}{x}\right)^2 \geq 0$ લેતાં,

$\therefore x^2 - 2 + \frac{1}{x^2} \geq 0$

$\therefore x^2 + \frac{1}{x^2} \geq 2$

$\therefore 2\cos^2 \frac{x}{2} \sin^2 x \geq 2$

$\therefore \cos^2 \frac{x}{2} \sin^2 x \geq 1$

$\cos^2 \frac{x}{2} \sin^2 x > 1$ શક્ય નથી.

વળી, $\cos^2 \frac{x}{2} \sin^2 x = 1$ પણ ના હોઈ શકે, કારણ કે, $\cos^2 \frac{x}{2} \leq 1$, $\sin^2 x \leq 1$

$\therefore \sin^2 x = \cos^2 \frac{x}{2} = 1$ થવા જોઈએ.

પરંતુ $\cos^2 \frac{x}{2} = 1$ તો $\sin^2 x = 4\sin^2 \frac{x}{2} \cos^2 \frac{x}{2} = 4(0)(1) = 0$ થાય.

$\therefore$ વાસ્તવિક ઉકેલની સંખ્યા '0' છે.

જવાબ : (A)

- (26) જો $\tan \alpha = \frac{m}{m+1}$ અને $\tan \beta = \frac{1}{2m+1}$ હોય, તો $(\alpha + \beta)$ ની શક્ય કિંમત છે.

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{6}$

$$\text{ઉકેલ : } \tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$$

$$= \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \left(\frac{m}{m+1}\right)\left(\frac{1}{2m+1}\right)}$$

$$= \frac{m(2m+1) + (m+1)}{(m+1)(2m+1) - m} = \frac{2m^2 + 2m + 1}{2m^2 + 2m + 1} = 1$$

$$\therefore (\alpha + \beta) = \frac{\pi}{4}$$

જવાબ : (B)

$$(27) \quad \frac{2(\sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \dots + \sin 89^\circ)}{2(\cos 1^\circ + \cos 2^\circ + \dots + \cos 44^\circ) + 1} = \dots$$

(A) $\sqrt{2}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{2}$

(D) 1

$$\text{ઉકેલ : } 2(\sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \dots + \sin 89^\circ)$$

$$= 2[(\sin 1^\circ + \sin 89^\circ) + (\sin 2^\circ + \sin 88^\circ) + \dots + (\sin 44^\circ + \sin 46^\circ) + \sin 45^\circ]$$

$$= 2\left[2\sin 45^\circ \cos 44^\circ + 2\sin 45^\circ \cos 43^\circ + \dots + 2\sin 45^\circ \cos 1^\circ + \frac{1}{\sqrt{2}}\right]$$

$$= 2\sqrt{2}(\cos 44^\circ + \cos 43^\circ + \dots + \cos 1^\circ) + \sqrt{2} \quad (1)$$

$$\text{હવે, } \frac{2(\sin 1^\circ + \sin 2^\circ + \dots + \sin 89^\circ)}{2(\cos 1^\circ + \cos 2^\circ + \dots + \cos 44^\circ) + 1}$$

$$= \frac{[2\sqrt{2}(\cos 44^\circ + \cos 43^\circ + \dots + \cos 1^\circ) + \sqrt{2}]}{2(\cos 1^\circ + \cos 2^\circ + \dots + \cos 44^\circ) + 1} \quad ((1) \text{ પરથી})$$

$$= \frac{\sqrt{2}[2(\cos 1^\circ + \cos 2^\circ + \dots + \cos 44^\circ) + 1]}{2(\cos 1^\circ + \cos 2^\circ + \dots + \cos 44^\circ) + 1}$$

$$= \sqrt{2}$$

જવાબ : (A)

$$(28) \quad \sin(y + z - x), \sin(z + x - y), \sin(x + y - z) \text{ સમાંતર શ્રેણીમાં છે. તો } \tan x, \tan y, \tan z \text{ એ } \dots$$

(A) સમાંતર શ્રેણીમાં છે.

(B) સમગુણોત્તર શ્રેણીમાં છે.

(C) સ્વરિત શ્રેણીમાં છે.

(D) આમાંથી એક પણ નહિ.

$$\text{ઉકેલ : } \sin(y + z - x), \sin(z + x - y), \sin(x + y - z) \text{ સમાંતર શ્રેણીમાં છે.}$$

$$\therefore \sin(y + z - x) - \sin(z + x - y) = \sin(z + x - y) - \sin(x + y - z)$$

$$\therefore 2\cos z \sin(y - x) = 2\cos x \sin(z - y)$$

$$\therefore \cos z[\sin y \cos x - \cos y \sin x] = \cos x(\sin z \cos y - \cos z \sin y)$$

$$\therefore \cos z \sin y \cos x - \cos z \cos y \sin x = \cos x \sin z \cos y - \cos x \cos z \sin y$$

$$\therefore \tan y - \tan x = \tan z - \tan y \quad (\cos x \cos y \cos z \neq 0)$$

$$\therefore 2\tan y = \tan x + \tan z$$

$$\therefore \tan x, \tan y, \tan z \text{ સમાંતર શ્રેણીમાં છે.}$$

જવાબ : (A)

(29) $\cos(\alpha + \beta) = \frac{4}{5}$ અને $\sin(\alpha - \beta) = \frac{5}{13}$; $0 < \alpha, \beta < \frac{\pi}{4}$ હોય, તો $\tan 2\alpha = \dots$

(A) $\frac{33}{56}$

(B) $\frac{56}{33}$

(C) $\frac{55}{32}$

(D) $\frac{32}{55}$

ઉકેલ : $0 < \alpha < \frac{\pi}{4}$

$0 < \beta < \frac{\pi}{4}$

$0 < \alpha + \beta < \frac{\pi}{2}$

$0 < \alpha < \frac{\pi}{4}$

$-\frac{\pi}{4} < -\beta < 0$

$-\frac{\pi}{4} < \alpha - \beta < \frac{\pi}{4}$ અને $\sin(\alpha - \beta) = \frac{5}{13} > 0$

$\therefore P(\alpha - \beta)$ પ્રથમ ચરણમાં જ હોય.

$\therefore \tan(\alpha + \beta) = \frac{3}{4}$ તથા $\tan(\alpha - \beta) = \frac{5}{12}$

$2\alpha = (\alpha + \beta) + (\alpha - \beta)$

$\therefore \tan 2\alpha = \tan[(\alpha + \beta) + (\alpha - \beta)]$

$= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta) \tan(\alpha - \beta)}$

$= \frac{\frac{3}{4} + \frac{5}{12}}{1 - \left(\frac{3}{4}\right)\left(\frac{5}{12}\right)} = \frac{56}{33}$

જવાબ : (B)

(30) જો $\sin^{-1}x \leq \cos^{-1}x$ હોય તો, $x \in \dots$

(A) $\left(-1, \frac{1}{\sqrt{2}}\right)$

(B) $\left[-1, \frac{1}{\sqrt{2}}\right]$

(C) $\left[\frac{1}{\sqrt{2}}, 1\right]$

(D) આમાંથી એકપણ નહિ

ઉકેલ : $\sin^{-1}x \leq \cos^{-1}x$

$\sin^{-1}x \leq \frac{\pi}{2} - \sin^{-1}x$

$\therefore 2\sin^{-1}x \leq \frac{\pi}{2}$

$\therefore \sin^{-1}x \leq \frac{\pi}{4}$

અથવા, $-\frac{\pi}{2} \leq \sin^{-1}x \leq \frac{\pi}{4}$

$\therefore \sin\left(-\frac{\pi}{2}\right) \leq x \leq \sin\frac{\pi}{4}$

$\therefore -1 \leq x \leq \frac{1}{\sqrt{2}}$

જવાબ : (B)

(31) $2\cos^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})$ તો ઉકેલ $\dots$

(A) $\left[\frac{1}{\sqrt{2}}, 1\right]$

(B) $\left[\frac{-1}{\sqrt{2}}, 1\right]$

(C) $\left[-1, \frac{-1}{\sqrt{2}}\right]$

(D) $\left(\frac{1}{\sqrt{2}}, 1\right)$

ઉકેલ : $2\cos^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})$

ધારી કે, $\cos^{-1}x = \theta$, $\theta \in [0, \pi]$

$\therefore x = \cos\theta$

$\therefore 2\theta = \sin^{-1}(2\cos\theta\sqrt{1-\cos^2\theta})$

$$\therefore 2\theta = \sin^{-1}(2\cos\theta\sin\theta) \quad (\sin\theta \geq 0 \text{ કારણ કે } 0 \leq \theta \leq \pi)$$

$$\therefore 2\theta = \sin^{-1}(\sin 2\theta)$$

$$\text{હવે, } -\frac{\pi}{2} \leq 2\theta \leq \frac{\pi}{2}$$

$$\therefore -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4} \text{ તથા } \theta \in [0, \pi] \text{ હોવાથી, } \theta \in \left[0, \frac{\pi}{4}\right] \text{ થાય.}$$

વળી, પ્રથમ ચરણમાં $\cos$ ઘટતું વિધેય હોવાથી,

$$0 \leq \theta \leq \frac{\pi}{4} \Rightarrow \cos 0 \geq \cos \theta \geq \cos \frac{\pi}{4}$$

$$\therefore \frac{1}{\sqrt{2}} \leq \cos \theta \leq 1$$

$$\therefore x \in \left[\frac{1}{\sqrt{2}}, 1\right] \quad \text{જવાબ : (A)}$$

$$(32) \quad f(x) = \sin^{-1}\left(\frac{1+x^2}{2|x|}\right) \text{ હોય, તો } x \in \dots \text{ શક્ય છે.}$$

$$(A) \{-1, 1\} \quad (B) [-1, 1] \quad (C) [0, 1] \quad (D) [-1, 0]$$

ઉકેલ : આપણે જાણીએ છીએ કે, $(1 \pm x)^2 \geq 0$

$$\therefore 1 \pm 2x + x^2 \geq 0$$

$$\therefore 1 + x^2 \geq 2|x|$$

$$\therefore \frac{1+x^2}{2|x|} \geq 1$$

$$f(x) = \sin^{-1}\left(\frac{1+x^2}{2|x|}\right) \text{ હોવાથી } \frac{1+x^2}{2|x|} > 1 \text{ શક્ય નથી.}$$

$$\therefore \frac{1+x^2}{2|x|} = 1$$

$$\therefore x = \pm 1$$

$$\therefore x \in \{-1, 1\} \quad \text{જવાબ : (A)}$$

$$(33) \quad \sin^{-1}\left(\cot\left(\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}} + \cos^{-1}\frac{\sqrt{12}}{4} + \sec^{-1}\sqrt{2}\right)\right) = \dots$$

$$(A) 1 \quad (B) 0 \quad (C) -1 \quad (D) 2$$

$$\text{ઉકેલ : } \sin^{-1}\left(\cot\left(\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}} + \cos^{-1}\frac{\sqrt{3}}{2} + \sec^{-1}\sqrt{2}\right)\right)$$

$$= \sin^{-1}\left(\cot\left(\sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) + \cos^{-1}\frac{\sqrt{3}}{2} + \sec^{-1}\sqrt{2}\right)\right) \quad \left(\because \sqrt{\frac{2-\sqrt{3}}{4}} = \sqrt{\frac{4-2\sqrt{3}}{8}} = \frac{\sqrt{3}-1}{2\sqrt{2}}\right)$$

$$= \sin^{-1}\left(\cot\left(\frac{\pi}{12} + \frac{\pi}{6} + \frac{\pi}{4}\right)\right) = \sin^{-1}\left(\cot\frac{\pi}{2}\right) = \sin^{-1}(0) = 0 \quad \text{જવાબ : (B)}$$

(34) $f(x) = x^{11} + x^9 + x^7 + x^3 + 1$ તથા

$f(\sin^{-1}(\sin 8)) = \alpha$ જ્યાં $\alpha = \text{અચળ હોય, તો } f(\tan^{-1}(\tan 8)) = \dots$

(A) α (B) $\alpha - 2$ (C) $\alpha + 2$ (D) $2 - \alpha$

ઉકેલ : $\frac{5\pi}{2} < 8 < 3\pi$

$\therefore -\frac{\pi}{2} < 8 - 3\pi < 0$

$\sin(8 - 3\pi) = -\sin 8$

$\therefore \alpha = f(\sin^{-1}(\sin 8))$
 $= f(\sin^{-1}(-\sin(8 - 3\pi)))$
 $= f(-\sin^{-1}(\sin(8 - 3\pi)))$
 $= f(-8 + 3\pi)$

(1)

હવે, $\tan(8 - 3\pi) = \tan 8$

$f(\tan^{-1}(\tan 8)) = f(\tan^{-1}(\tan(8 - 3\pi)))$
 $= f(8 - 3\pi)$

(2)

$$\begin{array}{l} f(x) = x^{11} + x^9 + x^7 + x^3 + 1 \\ f(-x) = -x^{11} - x^9 - x^7 - x^3 + 1 \\ \hline f(x) + f(-x) = 2 \end{array}$$

$\therefore f(-8 + 3\pi) + f(8 - 3\pi) = 2$

$\therefore \alpha + f(8 - 3\pi) = 2$ ((1) પરથી)

$\therefore f(8 - 3\pi) = 2 - \alpha$

$\therefore f(\tan^{-1}(\tan 8)) = 2 - \alpha$ ((2) પરથી)

જવાબ : (D)

(35) $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$ નો ઉકેલ હોય.

[IIT : 1989]

(A) $n\pi + \frac{\pi}{8}, n \in \mathbb{Z}$

(B) $\frac{n\pi}{2} + \frac{\pi}{8}, n \in \mathbb{Z}$

(C) $(-1)^n \frac{n\pi}{2} + \frac{\pi}{8}, n \in \mathbb{Z}$

(D) $2n\pi + \cos^{-1} \frac{2}{3}, n \in \mathbb{Z}$

ઉકેલ : $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$

$\therefore (\sin 3x + \sin x) - 3\sin 2x = (\cos 3x + \cos x) - 3\cos 2x$

$\therefore 2\sin 2x \cos x - 3\sin 2x = 2\cos 2x \cos x - 3\cos 2x$

$\therefore \sin 2x(2\cos x - 3) = \cos 2x(2\cos x - 3)$

$\therefore (\sin 2x - \cos 2x)(2\cos x - 3) = 0$

પરંતુ, $\cos x = \frac{3}{2}$ શક્ય નથી.

$\therefore \sin 2x - \cos 2x = 0$

$\therefore \tan 2x = 1$ (જો $\cos 2x = 0 \Rightarrow \sin 2x = 0$ જે શક્ય નથી. આથી, $\cos 2x \neq 0$)

$\therefore \tan 2x = \tan \frac{\pi}{4}$

$\therefore 2x = n\pi + \frac{\pi}{4}, n \in \mathbb{Z}$

$\therefore x = \frac{n\pi}{2} + \frac{\pi}{8}, n \in \mathbb{Z}$

જવાબ : (B)

(36) $\tan x + \sec x = 2\cos x$ ની $[0, 2\pi]$ માં ઉકેલની સંખ્યા છે.

(A) 0

(B) 1

(C) 2

(D) 3

ઉકેલ : $\tan x + \sec x = 2\cos x$

$\therefore \frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$

$\therefore \sin x + 1 = 2\cos^2 x$

$\therefore \sin x + 1 = 2(1 - \sin^2 x)$

$$\begin{aligned}\therefore 2\sin^2x + \sin x - 1 &= 0 \\ \therefore (2\sin x - 1)(\sin x + 1) &= 0 \\ \therefore \sin x &= \frac{1}{2} \text{ અથવા } \sin x = -1\end{aligned}$$

$x \in [0, 2\pi]$ હોવાથી,

$$\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6} \text{ અથવા } \frac{5\pi}{6}$$

પરંતુ $\sin x = -1 \Rightarrow \cos x = 0$ જે શક્ય નથી. ($\tan x$ વ્યાખ્યાયિત છે.)

$$\therefore \sin x = -1 \text{ શક્ય નથી.}$$

$\therefore$ ઉકેલની સંખ્યા 2 છે.

જવાબ : (C)

$$(37) \quad 2\sin^2\theta - 3\sin\theta - 2 = 0 \text{ નો વ્યાપક ઉકેલ છે.}$$

$$(A) \quad n\pi + (-1)^n \frac{\pi}{6}; n \in \mathbb{Z}$$

$$(B) \quad n\pi + (-1)^n \frac{\pi}{2}; n \in \mathbb{Z}$$

$$(C) \quad n\pi + (-1)^n \frac{5\pi}{6}; n \in \mathbb{Z}$$

$$(D) \quad n\pi + (-1)^n \frac{7\pi}{6}; n \in \mathbb{Z}$$

$$\text{ઉકેલ : } 2\sin^2\theta - 3\sin\theta - 2 = 0$$

$$\therefore (2\sin\theta + 1)(\sin\theta - 2) = 0$$

$$\therefore \sin\theta = -\frac{1}{2} \text{ અથવા } \sin\theta = 2$$

$\sin\theta = 2 > 1$ નો ઉકેલગણ $\emptyset$ છે.

$$\sin\theta = -\frac{1}{2} \Rightarrow \sin\theta = \sin\left(\frac{7\pi}{6}\right)$$

$$\Rightarrow \theta = n\pi + (-1)^n \left(\frac{7\pi}{6}\right), n \in \mathbb{Z}$$

જવાબ : (D)

$$(38) \quad \begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0 \text{ ના } \left[\frac{-\pi}{4}, \frac{\pi}{4}\right] \text{ માં વિન ઉકેલની સંખ્યા છે.}$$

[IIT : 2001]

$$(A) \quad 0$$

$$(B) \quad 2$$

$$(C) \quad 1$$

$$(D) \quad 3$$

$$\text{ઉકેલ : } \begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} \sin x - \cos x & 0 & \cos x \\ \cos x - \sin x & \sin x - \cos x & \cos x \\ 0 & \cos x - \sin x & \sin x \end{vmatrix} = 0 \quad C_1 - C_2, C_2 - C_3$$

$$\therefore (\sin x - \cos x)^2 \begin{vmatrix} 1 & 0 & \cos x \\ -1 & 1 & \cos x \\ 0 & -1 & \sin x \end{vmatrix} = 0$$

$$\therefore (\sin x - \cos x)^2 [1(\sin x + \cos x) + \cos x(1 - 0)] = 0$$

$$\therefore (\sin x - \cos x)^2 (\sin x + 2\cos x) = 0$$

$$\therefore \sin x = \cos x \text{ અથવા } \sin x = -2\cos x$$

$$\therefore \tan x = 1 \text{ અથવા } \tan x = -2$$

અહીં $x \in \left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$ હોવાથી, $\tan x \in [-1, 1]$

$$\therefore \tan x = -2 \text{ શક્ય નથી.}$$

$$\therefore \tan x = 1 \Rightarrow x = \frac{\pi}{4}. \text{ આથી, ઉકેલની સંખ્યા 1 છે.}$$

જવાબ : (C)

- (39) સમીકરણ $7\cos x + 5\sin x = 2k + 1$ નો શક્ય ઉકેલ, k પૂર્ણાંક હોય, તેવા પૂર્ણાંક k ની સંખ્યા છે.
[IIT : 2002]

(A) 4 (B) 8 (C) 10 (D) 12

ઉકેલ : $7\cos x + 5\sin x = 2k + 1$

$$a = 7, b = 5$$

$$\therefore r^2 = a^2 + b^2 = 49 + 25 = 74. \text{ આથી, } r = \sqrt{74}$$

$$\therefore -\sqrt{74} \leq 2k + 1 \leq \sqrt{74} \text{ જરૂરી છે.}$$

$$\therefore |2k + 1| \leq \sqrt{74}$$

શક્ય પૂર્ણાંક $k = 0, 1, 2, 3, -1, -2, -3, -4$

$\therefore$ શક્ય પૂર્ણાંકની સંખ્યા 8 છે.

જવાબ : (B)

- (40) $\cos(\alpha - \beta) = 1, \cos(\alpha + \beta) = \frac{1}{e}, \alpha, \beta \in [-\pi, \pi]$ એ બંને સમીકરણનું સમાધાન કરતી ક્રમયુક્ત જોડ (α, β) ની સંખ્યા છે.
[IIT : 2005]

(A) 0 (B) 1 (C) 2 (D) 4

ઉકેલ : $-\pi \leq \alpha \leq \pi$

$$-\pi \leq \beta \leq \pi \Rightarrow \pi \geq -\beta \geq -\pi$$

$$\therefore \frac{-\pi \leq \alpha \leq \pi, -\pi \leq -\beta \leq \pi}{-2\pi \leq \alpha - \beta \leq 2\pi}$$

$$\cos(\alpha - \beta) = 1 \text{ તથા } \alpha - \beta \in [-2\pi, 2\pi] \Rightarrow \alpha - \beta = 0, 2\pi, -2\pi$$

(i) $\alpha - \beta = 0$ (ii) $\alpha = \beta + 2\pi$ (iii) $\alpha = \beta - 2\pi$ નો વિચાર કરીએ.

$$\alpha = \beta + 2\pi \text{ લઈએ, તો}$$

$$-\pi \leq \beta \leq \pi \Rightarrow \pi \leq 2\pi + \beta = \alpha \leq 3\pi \text{ જે શક્ય નથી. કારણ કે } -\pi \leq \alpha \leq \pi$$

$$[\alpha = \pi, \beta = -\pi \text{ શક્ય છે, પરંતુ } \cos(\alpha + \beta) = \frac{1}{e} \neq 1]$$

આ જ રીતે, $\alpha = \beta - 2\pi$ પણ શક્ય નથી.

$$\therefore \alpha = \beta \text{ લેતાં,}$$

$$\cos(\alpha + \beta) = \frac{1}{e} \Rightarrow \cos 2\alpha = \frac{1}{e} \text{ જ્યાં } -\pi \leq \alpha \leq \pi \text{ એટલે કે } -2\pi \leq 2\alpha \leq 2\pi$$

$$\therefore -2\pi \leq 2\alpha \leq 2\pi \text{ અને } \cos 2\alpha = \frac{1}{e}$$

$[-2\pi, 0]$ માં બે ઉકેલ અને $[0, 2\pi]$ માં બે ઉકેલ મળે.

$\therefore$ ઉકેલની કુલ સંખ્યા 4 છે.

જવાબ : (D)

- (41) $\theta \in (0, 2\pi)$ માટે $2\sin^2\theta - 5\sin\theta + 2 > 0$ નો ઉકેલ હોઈ શકે.
[IIT : 2006]

(A) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (B) $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$ (C) $\left(0, \frac{\pi}{8}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (D) $\left(\frac{41\pi}{8}, \pi\right)$

ઉકેલ : $2\sin^2\theta - 5\sin\theta + 2 > 0$

$$\therefore 2\left(\sin\theta - \frac{1}{2}\right)(\sin\theta - 2) > 0$$

$$\therefore \sin\theta < \frac{1}{2} \text{ અથવા } \sin\theta > 2$$

$\sin\theta > 2$ શક્ય નથી.

$$\therefore \sin\theta < \frac{1}{2}$$

$\therefore$ ઉકેલગણ $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ છે. જવાબ : (A)

- (42) સમીકરણ $2\sin^2\theta - \cos 2\theta = 0$ અને $2\cos^2\theta - 3\sin\theta = 0$ ની $[0, 2\pi]$ માં ઉકેલની સંખ્યા છે.
 (A) 0 (B) 1 (C) 2 (D) 4

$$\begin{array}{l|l} \text{ઉકેલ : } 2\sin^2\theta - \cos 2\theta = 0 & \therefore 2(1 - \sin^2\theta) - 3\sin\theta = 0 \\ \therefore 2\sin^2\theta - (1 - 2\sin^2\theta) = 0 & \therefore 2\sin^2\theta + 3\sin\theta - 2 = 0 \\ \therefore 4\sin^2\theta = 1 & \therefore (2\sin\theta - 1)(\sin\theta + 2) = 0 \\ \therefore \sin\theta = \pm \frac{1}{2} & \therefore \sin\theta = \frac{1}{2} \text{ અથવા } \sin\theta = -2 \\ \text{હવે, } 2\cos^2\theta - 3\sin\theta = 0 & \sin\theta = -2 \text{ શક્ય નથી.} \end{array}$$

આથી સમીકરણનો સામાન્ય ઉકેલ $\sin\theta = \frac{1}{2}$ છે. $\theta \in [0, 2\pi] \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\therefore$ ઉકેલની સંખ્યા 2 છે.

જવાબ : (C)

- (43) $3\sin^2x - 7\sin x + 2 = 0$ ના $[0, 5\pi]$ માં ઉકેલની સંખ્યા છે.

(A) 0 (B) 5 (C) 6 (D) 10

ઉકેલ : $3\sin^2x - 7\sin x + 2 = 0$

$$\therefore (3\sin x - 1)(\sin x - 2) = 0$$

$$\therefore \sin x = \frac{1}{3} \text{ અથવા } \sin x = 2. \text{ પરંતુ } \sin x = 2 \text{ શક્ય નથી.}$$

$\sin x = \frac{1}{3} > 0$ હોવાથી, $x \in [0, \pi]$ માં બે ઉકેલ, $x \in [2\pi, 3\pi]$ માં બે ઉકેલ તેમજ $x \in [4\pi, 5\pi]$ માં અન્ય બે ઉકેલ મળે. તેથી કુલ 6 ઉકેલ મળે.

જવાબ : (C)

- (44) x એ એવી નાનામાં નાની ધન વાસ્તવિક સંખ્યા છે કે જેથી $\log_{\cos x} \sin x + \log_{\sin x} \cos x = 2$ થાય, તો $x = \dots$

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

ઉકેલ : $\log_{\cos x} \sin x = a$ લેતાં,

$$a + \frac{1}{a} = 2$$

$$\therefore a^2 - 2a + 1 = 0$$

$$\therefore a = 1$$

$$\therefore \log_{\cos x} \sin x = 1$$

$$\therefore \sin x = \cos x$$

$$\therefore \tan x = 1$$

$$\therefore x = \frac{\pi}{4}, \text{ જે નાનામાં નાની ધન સંખ્યા છે.}$$

જવાબ : (C)

- (45) સમીકરણ $\sin x \cos y = 1$ નો ઉકેલ મળે તેવી કમ્યુક્ત જોડ (x, y) ની સંખ્યા છે, $x, y \in [0, 2\pi]$

(A) 1 (B) 2 (C) 3 (D) 4

ઉકેલ : નીચેના બે વિકલ્પો માટે $\sin x \cos y = 1$ સત્ય થાય.

(i) $\sin x = \cos y = 1$ અથવા (ii) $\sin x = \cos y = -1$

(i) જો $\sin x = \cos y = 1$ હોય તો,

$$(x, y) = \left(\frac{\pi}{2}, 0\right) \text{ અથવા } \left(\frac{\pi}{2}, 2\pi\right) \quad (x, y \in [0, 2\pi])$$

(ii) જો $\sin x = \cos y = -1$ હોય, તો $(x, y) = \left(\frac{3\pi}{2}, \pi\right)$

$$\therefore \text{ઉકેલની સંખ્યા 3 છે.}$$

જવાબ : (C)

(46) જો $\sin^3 x \sin 3x = \sum_{m=0}^n c_m \cos mx$ તો $n = \dots$ જ્યાં $c_0, c_1, \dots, c_n$ અચળ છે, $c_n \neq 0$ [IIT : 1981]
 (A) 4 (B) 5 (C) 6 (D) 7

ઉકેલ : $\sin^3 x \sin 3x = \sum_{m=0}^n c_m \cos mx$

$$\begin{aligned} \therefore \sum_{m=0}^n c_m \cos mx &= \frac{1}{4} (3 \sin x - \sin 3x) \sin 3x \\ &= \frac{1}{4} [3 \sin 3x \sin x - \sin^2 3x] \\ &= \frac{1}{8} [3(2 \sin 3x \sin x) - 2 \sin^2 3x] \\ &= \frac{1}{8} [3(-\cos 4x + \cos 2x) - (1 - \cos 6x)] \end{aligned}$$

$$c_0 + c_1 \cos x + c_2 \cos 2x + c_3 \cos 3x + \dots + c_n \cos nx = \frac{1}{8} [-3 \cos 4x + 3 \cos 2x - 1 + \cos 6x]$$

$\therefore n = 6$ જવાબ : (C)

(47) $\sin \frac{\pi}{14} \sin \frac{3\pi}{14} \sin \frac{5\pi}{14} \sin \frac{7\pi}{14} \sin \frac{9\pi}{14} \sin \frac{11\pi}{14} \sin \frac{13\pi}{14} = \dots$
 (A) $\frac{1}{32}$ (B) $\frac{1}{128}$ (C) $\frac{1}{16}$ (D) $\frac{1}{64}$

ઉકેલ : $\sin \frac{\pi}{14} \sin \frac{3\pi}{14} \sin \frac{5\pi}{14} \sin \frac{7\pi}{14} \sin \frac{9\pi}{14} \sin \frac{11\pi}{14} \sin \frac{13\pi}{14}$

$$= \sin \frac{\pi}{14} \sin \frac{3\pi}{14} \sin \frac{5\pi}{14} \sin \frac{\pi}{2} \sin \left(\pi - \frac{5\pi}{14} \right) \cdot \sin \left(\pi - \frac{3\pi}{14} \right) \cdot \sin \left(\pi - \frac{\pi}{14} \right)$$

$$= \left(\sin \frac{\pi}{14} \sin \frac{3\pi}{14} \sin \frac{5\pi}{14} \right)^2$$

$$= \left(\cos \left(\frac{\pi}{2} - \frac{\pi}{14} \right) \cos \left(\frac{\pi}{2} - \frac{3\pi}{14} \right) \cos \left(\frac{\pi}{2} - \frac{5\pi}{14} \right) \right)^2$$

$$= \left[\cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{3\pi}{7} \right]^2$$

$$= \left[\frac{1}{2 \sin \frac{\pi}{7}} \left(2 \sin \frac{\pi}{7} \cos \frac{\pi}{7} \right) \cos \frac{2\pi}{7} \cos \frac{3\pi}{7} \right]^2$$

$$= \left[\frac{1}{2 \sin \frac{\pi}{7}} \left(\sin \frac{2\pi}{7} \cos \frac{2\pi}{7} \right) \cos \left(\pi - \frac{4\pi}{7} \right) \right]^2$$

$$= \left[\frac{1}{2^2 \sin \frac{\pi}{7}} \left(2 \sin \frac{2\pi}{7} \cos \frac{2\pi}{7} \right) \left(-\cos \frac{4\pi}{7} \right) \right]^2$$

$$= \left[\frac{1}{2 \times 4 \sin \frac{\pi}{7}} \left(2 \sin \frac{4\pi}{7} \cos \frac{4\pi}{7} \right) \right]^2$$

$$= \left[\frac{1}{8 \sin \frac{\pi}{7}} \left(\sin \frac{8\pi}{7} \right) \right]^2$$

$$= \frac{1}{64 \sin^2 \frac{\pi}{7}} \sin^2 \left(\pi + \frac{\pi}{7} \right)$$

$$= \frac{1}{64 \sin^2 \frac{\pi}{7}} \cdot \sin^2 \frac{\pi}{7}$$

$$= \frac{1}{64}$$

જવાબ : (D)

(48) સમીકરણ $x + y = \frac{2\pi}{3}$ તથા $\cos x + \cos y = \frac{3}{2}$ નો ઉકેલ છે. [IIT : 1987]

- (A) R (B) $\emptyset$ (C) $[-1, 1]$ (D) $\{-1, 1\}$

ઉકેલ : $\cos x + \cos y = \frac{3}{2}$

$$\therefore 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$$

$$\therefore 2\cos\left(\frac{2\pi}{3}\right)\cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$$

$$\therefore 2\left(\frac{1}{2}\right)\cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$$

$$\therefore \cos\left(\frac{x-y}{2}\right) = \frac{3}{2} > 1 \text{ જે શક્ય નથી.}$$

$\therefore$ ઉકેલગણ $\emptyset$ છે. જવાબ : (B)

(49) $\sin\frac{\pi}{18} \sin\frac{5\pi}{18} \sin\frac{7\pi}{18} = \dots$

- (A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) આમાંથી એકપણ નહિ

ઉકેલ : $\sin\frac{\pi}{18} \sin\frac{5\pi}{18} \sin\frac{7\pi}{18} = \sin 10^\circ \sin 50^\circ \sin 70^\circ$

$$= \frac{1}{2}(2\sin 70^\circ \sin 10^\circ) \sin 50^\circ$$

$$= \frac{1}{2}(-\cos 80^\circ + \cos 60^\circ) \sin 50^\circ$$

$$= \frac{1}{2}\left(-\sin 10^\circ + \frac{1}{2}\right) \sin 50^\circ$$

$$= -\frac{1}{4}[2\sin 50^\circ \sin 10^\circ] + \frac{1}{4} \sin 50^\circ$$

$$= \frac{1}{4}[\cos 60^\circ - \cos 40^\circ] + \frac{1}{4} \sin 50^\circ$$

$$= \frac{1}{4}\left[\frac{1}{2} - \cos 40^\circ + \sin 50^\circ\right]$$

$$= \frac{1}{4}\left[\frac{1}{2} - \cos 40^\circ + \cos 40^\circ\right]$$

$$= \frac{1}{8} \quad \text{જવાબ : (C)}$$

નોંધ : $= \sin 10^\circ \sin(60^\circ - 10^\circ) \sin(60^\circ + 10^\circ) = \frac{1}{4} \sin 3(10^\circ) = \frac{1}{8}$

(50) $\cos(x - y)$, $\cos x$, $\cos(x + y)$ સ્વરિત શ્રેણીમાં છે, તો $\cos x \cdot \sec \frac{y}{2} = \dots$ $\left(\sin \frac{y}{2} \neq 0\right)$

- (A) $\pm\sqrt{2}$ (B) ± 2 (C) $\pm\sqrt{3}$ (D) ± 4

ઉકેલ : $\cos(x - y)$, $\cos x$, $\cos(x + y)$ સ્વરિત શ્રેણીમાં છે.

$\therefore \frac{1}{\cos(x - y)}, \frac{1}{\cos x}, \frac{1}{\cos(x + y)}$ સમાંતર શ્રેણીમાં છે.

$$\therefore \frac{1}{\cos(x - y)} + \frac{1}{\cos(x + y)} = \frac{2}{\cos x}$$

$$\therefore \frac{\cos(x + y) + \cos(x - y)}{\cos(x + y)\cos(x - y)} = \frac{2}{\cos x}$$

$$\therefore \frac{2\cos x \cos y}{\cos^2 x - \sin^2 y} = \frac{2}{\cos x}$$

$$\therefore \cos^2 x \cos y = \cos^2 x - \sin^2 y$$

$$\therefore \cos^2 x - \cos^2 x \cos y = \sin^2 y$$

$$\therefore \cos^2 x (1 - \cos y) = \sin^2 y$$

$$\therefore \cos^2 x \left(2\sin^2 \frac{y}{2}\right) = \left(2\sin \frac{y}{2} \cos \frac{y}{2}\right)^2$$

$$\therefore \cos^2 x \left(2\sin^2 \frac{y}{2}\right) = 4\sin^2 \frac{y}{2} \cos^2 \frac{y}{2}$$

$$\therefore \cos^2 x = 2\cos^2 \frac{y}{2}$$

$$\therefore \frac{\cos^2 x}{\cos^2 \frac{y}{2}} = 2$$

$$\therefore \cos^2 x \sec^2 \frac{y}{2} = 2$$

$$\therefore \cos x \sec \frac{y}{2} = \pm\sqrt{2}$$

જવાબ : (A)

(51) $A > 0, B > 0, A + B = \frac{\pi}{3}$, તો $\tan A \tan B$ નું મહત્તમ મૂલ્ય છે.

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{1}{4}$

(D) $\frac{1}{\sqrt{3}}$

ઉકેલ : ધારો કે, $\tan A \tan B = y$

હવે, $A + B = \frac{\pi}{3}$

$\therefore \tan(A + B) = \tan \frac{\pi}{3}$

$\therefore \frac{\tan A + \tan B}{1 - \tan A \tan B} = \sqrt{3}$

$\therefore \tan A + \tan B = \sqrt{3}(1 - \tan A \tan B)$

$\therefore \frac{y}{\tan B} + \tan B = \sqrt{3}(1 - y)$

$\therefore y + \tan^2 B = \sqrt{3} \tan B(1 - y)$

$\therefore \tan^2 B - \sqrt{3} \tan B(1 - y) + y = 0$

આ $\tan B$ માં દ્વિઘાત સમીકરણ છે.

વાસ્તવિક ઉકેલ માટે $\Delta \geq 0$ થવું જોઈએ.

$\therefore 3(1 - y)^2 - 4y \geq 0$

$\therefore 3(1 - 2y + y^2) - 4y \geq 0$

$\therefore 3y^2 - 10y + 3 \geq 0$

$\therefore (y - 3)(3y - 1) \geq 0$

$A > 0, B > 0$ અને $A + B = \frac{\pi}{3}$

$\therefore 0 < A < \frac{\pi}{3}$ અને $0 < B < \frac{\pi}{3}$

$\therefore 0 < \tan A < \sqrt{3}$ તથા $0 < \tan B < \sqrt{3}$

$\therefore 0 < \tan A \tan B < 3$

$\therefore \tan A \tan B \geq 3$ શક્ય નથી.

$\therefore \tan A \tan B \leq \frac{1}{3}$

$\therefore \tan A \tan B$ ની મહત્તમ સીમા $\frac{1}{3}$ છે.

એ જોઈ શકાય કે $\tan A = \tan B = \frac{1}{\sqrt{3}}$ શક્ય છે.

$\therefore$ મહત્તમ મૂલ્ય $\frac{1}{3}$ છે.

જવાબ : (A)

(52) $\tan A = \frac{(1 - \cos B)}{\sin B}$ તો $\tan 2A = \dots\dots$

(A) $\tan B$

(B) $\sin B$

(C) $\tan 2B$

(D) $\tan 4B$

ઉકેલ : $\tan A = \frac{(1 - \cos B)}{\sin B} = \frac{2 \sin^2 \frac{B}{2}}{2 \sin \frac{B}{2} \cos \frac{B}{2}} = \tan \frac{B}{2}$

$\therefore \tan A = \tan \frac{B}{2}$

$\therefore A = \frac{B}{2} + n\pi$

$\therefore 2A = B + 2n\pi$

$\therefore \tan 2A = \tan B$

જવાબ : (A)

(53) $\sin^4 \theta - 2 \sin^2 \theta - 1 = 0$ ની $[0, 2\pi]$ માં શક્ય ઉકેલની સંખ્યા છે.

[IIT : 1984]

(A) 0

(B) 1

(C) 2

(D) 3

ઉકેલ : $\sin^4 \theta - 2 \sin^2 \theta - 1 = 0$

$\therefore \sin^4 \theta - 2 \sin^2 \theta + 1 - 2 = 0$

$\therefore (1 - \sin^2 \theta)^2 = 2$

$\therefore \cos^4 \theta = 2$, જે શક્ય નથી.

$\therefore$ ઉકેલ સંખ્યા 0 છે.

જવાબ : (A)

- (54) કોઈ ધન પૂર્ણાંક n માટે, $\sin \frac{\pi}{2n} + \cos \frac{\pi}{2n} = \frac{\sqrt{n}}{2}$ હોય તો n ની કિંમતો શક્ય બને. [IIT : 1994]
- (A) 1 (B) 2 (C) 3 (D) 4

ઉકેલ : $\sin \frac{\pi}{2n} + \cos \frac{\pi}{2n} = \frac{\sqrt{n}}{2}$

$$\therefore \left(\sin \frac{\pi}{2n} + \cos \frac{\pi}{2n} \right)^2 = \frac{n}{4}$$

$$\therefore 1 + \sin \frac{2\pi}{2n} = \frac{n}{4}$$

$$\therefore \sin \frac{\pi}{n} = \frac{n}{4} - 1$$

$$\therefore \sin \frac{\pi}{n} = \frac{n-4}{4} \quad (1)$$

હવે, n ધન પૂર્ણાંક છે.

$$0 \leq \sin \frac{\pi}{n} \leq 1$$

$$\sin \frac{\pi}{n} = 0 \text{ તો } n = 4. \text{ પરંતુ } \sin \frac{\pi}{n} \neq 0 \text{ ((1) પરથી)}$$

$$\sin \frac{\pi}{n} = 1 \text{ તો } n = 8 \text{ પરંતુ } \sin \frac{\pi}{8} \neq 1$$

$$\therefore 0 < \sin \frac{\pi}{n} < 1 \Rightarrow 0 < \frac{n-4}{4} < 1$$

$$\therefore 0 < n - 4 < 4$$

$$\therefore 4 < n < 8$$

$$\therefore n = 5, 6, 7 \text{ શક્ય બને.}$$

$$\therefore n \text{ ની 3 કિંમતો શક્ય બને. જવાબ : (C)}$$

- (55) સંકર સંખ્યા ω એ 1 નું ધનમૂળ હોય તો
- $$\sin \left((\omega^{10} + \omega^{23})\pi - \frac{\pi}{4} \right) = \dots \quad (\omega \neq 1)$$

- (A) $-\frac{\sqrt{3}}{2}$ (B) $-\frac{1}{\sqrt{2}}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{\sqrt{3}}{2}$

ઉકેલ : $\sin \left((\omega^{10} + \omega^{23})\pi - \frac{\pi}{4} \right)$

$$\omega^{10} = \omega^9 \cdot \omega = \omega$$

$$\omega^{23} = \omega^{21} \cdot \omega^2 = \omega^2$$

$$\sin \left((\omega + \omega^2)\pi - \frac{\pi}{4} \right)$$

$$= \sin \left(-\pi - \frac{\pi}{4} \right)$$

$$1 + \omega + \omega^2 = 0. \text{ આથી, } \omega + \omega^2 = -1$$

$$= -\sin \left(\pi + \frac{\pi}{4} \right) = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

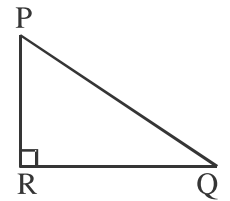
$$\text{જવાબ : (C)}$$

- (56) ΔPQR માટે $R = \frac{\pi}{2}$ હોય તથા $\tan \frac{P}{2}$ અને $\tan \frac{Q}{2}$ એ દ્વિઘાત સમીકરણ $ax^2 + bx + c = 0$ ($a \neq 0$) ની બે બીજ હોય, તો સત્ય બને.

- (A) $a + b = c$ (B) $a + c = b$ (C) $b + c = a$ (D) $b = c$

ઉકેલ : સમીકરણ $ax^2 + bx + c = 0$ ની બે બીજ $\tan \frac{P}{2}$ અને $\tan \frac{Q}{2}$ છે.

$$\therefore \tan \frac{P}{2} + \tan \frac{Q}{2} = -\frac{b}{a} \text{ અને } \tan \frac{P}{2} \tan \frac{Q}{2} = \frac{c}{a}$$



આકૃતિ 12.9

વળી, ΔPQR માટે, $R = \frac{\pi}{2}$ હોવાથી, $P + Q = \frac{\pi}{2}$

$$\therefore \frac{P}{2} + \frac{Q}{2} = \frac{\pi}{4}$$

$$\therefore \tan\left(\frac{P}{2} + \frac{Q}{2}\right) = 1$$

$$\therefore \frac{\tan \frac{P}{2} + \tan \frac{Q}{2}}{1 - \tan \frac{P}{2} \tan \frac{Q}{2}} = 1$$

$$\therefore \frac{\frac{-b}{a}}{1 - \frac{c}{a}} = 1$$

$$\therefore \frac{-b}{a - c} = 1$$

$$\therefore -b = a - c$$

$$\therefore c = a + b$$

જવાબ : (A)

(57) $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$ માટે, નીચેનામાંથી કયું સત્ય બને ?

(A) $f(\theta) < 0, \forall \theta \geq 0$

(B) $f(\theta) < 0, \forall \theta < 0$

(C) $f(\theta) \geq 0, \forall \theta \in \mathbb{R}$

(D) $f(\theta) < 0, \forall \theta \in \mathbb{R}$

ઉકેલ : $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$

$$= \sin\theta(2\sin 2\theta \cos\theta)$$

$$= (2\sin\theta \cos\theta)\sin 2\theta$$

$$= \sin^2 2\theta \geq 0, \forall \theta \in \mathbb{R}$$

જવાબ : (C)

(58) $\alpha + \beta = \frac{\pi}{2}$ અને $\beta + \gamma = \alpha$ તો $\tan\alpha = \dots\dots$

(A) $2(\tan\beta + \tan\gamma)$

(B) $\tan\beta + 2\tan\gamma$

(C) $\tan\beta + \tan\gamma$

(D) $2\tan\beta + \tan\gamma$

ઉકેલ : અહીં, $\alpha + \beta = \frac{\pi}{2}$ છે.

$$\therefore \alpha = \frac{\pi}{2} - \beta$$

$$\tan\alpha = \tan\left(\frac{\pi}{2} - \beta\right) = \cot\beta = \frac{1}{\tan\beta}$$

$$\therefore \tan\alpha \tan\beta = 1$$

(1)

$$\text{હવે, } \beta + \gamma = \alpha$$

$$\therefore \gamma = \alpha - \beta$$

$$\therefore \tan\gamma = \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$$

((1) પરથી)

$$\therefore \tan\gamma = \frac{\tan\alpha - \tan\beta}{2}$$

$$\therefore \tan\alpha = \tan\beta + 2\tan\gamma$$

જવાબ : (B)

(59) $(\cos\alpha_1)(\cos\alpha_2) \dots (\cos\alpha_n)$ ની મહત્તમ કિંમત છે,

જ્યાં $0 \leq \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n \leq \frac{\pi}{2}$ તથા $\cot\alpha_1 \cdot \cot\alpha_2 \dots \cot\alpha_n = 1$

(A) $\frac{1}{2^2}$

(B) $\frac{1}{2^n}$

(C) $\frac{1}{2^n}$

(D) 1

ઉકેલ : ધારો કે $y = (\cos\alpha_1)(\cos\alpha_2) \dots (\cos\alpha_n)$

$$\text{હવે, } \cot\alpha_1 \cot\alpha_2 \dots \cot\alpha_n = 1$$

$$\therefore \frac{\cos\alpha_1}{\sin\alpha_1} \cdot \frac{\cos\alpha_2}{\sin\alpha_2} \dots \frac{\cos\alpha_n}{\sin\alpha_n} = 1$$

$$\therefore \cos\alpha_1 \cdot \cos\alpha_2 \dots \cos\alpha_n = \sin\alpha_1 \cdot \sin\alpha_2 \dots \sin\alpha_n = y$$

$$y^2 = (\cos\alpha_1 \cos\alpha_2 \cos\alpha_3 \dots \cos\alpha_n)(\sin\alpha_1 \sin\alpha_2 \dots \sin\alpha_n)$$

$$\therefore y^2 = \frac{(2\sin\alpha_1 \cos\alpha_1) (2\sin\alpha_2 \cos\alpha_2) \dots (2\sin\alpha_n \cos\alpha_n)}{2^n}$$

$$\therefore y^2 = \frac{\sin 2\alpha_1 \sin 2\alpha_2 \dots \sin 2\alpha_n}{2^n}$$

$$0 \leq \alpha_1, \alpha_2, \dots, \alpha_n \leq \frac{\pi}{2}$$

$$\therefore 0 \leq 2\alpha_1, 2\alpha_2, \dots, 2\alpha_n \leq \pi$$

$$\therefore 0 \leq \sin 2\alpha_1 \sin 2\alpha_2, \dots, \sin 2\alpha_n \leq 1$$

$$\therefore 0 \leq \frac{\sin 2\alpha_1 \sin 2\alpha_2 \dots \sin 2\alpha_n}{2^n} \leq \frac{1}{2^n}$$

$$\therefore 0 \leq y^2 \leq \frac{1}{2^n}$$

$$\therefore 0 \leq y \leq \frac{1}{2^{\frac{n}{2}}}$$

$$\therefore y \text{ ની મહત્તમ કિંમત } \frac{1}{2^{\frac{n}{2}}} \text{ છે.}$$

$$\text{વળી } \alpha_1, \alpha_2 \dots \alpha_n = \frac{\pi}{4} \text{ લેતાં,}$$

$$\cot\alpha_1 \cot\alpha_2 \dots \cot\alpha_n = 1$$

$$\text{તથા } \cos\alpha_1 \cos\alpha_2 \dots \cos\alpha_n = \left(\frac{1}{\sqrt{2}}\right)^n = \frac{1}{2^{\frac{n}{2}}}$$

જવાબ : (A)

(60) α, β, γ એવી વાસ્તવિક સંખ્યાઓ છે કે જેથી $\alpha + \beta + \gamma = \pi$ હોય, તો $\sin\alpha + \sin\beta + \sin\gamma$ ની ન્યૂનતમ કિંમત

(A) હંમેશા ધન હોય. (B) ઋણ હોઈ શકે. (C) 0 થાય. (D) -3 થાય.

ઉકેલ : $\alpha + \beta + \gamma = \pi$

$$\alpha = -\frac{\pi}{2}, \beta = -\frac{\pi}{2}, \gamma = 2\pi \text{ માટે આ સત્ય છે.}$$

$$\sin\alpha + \sin\beta + \sin\gamma = \sin\left(-\frac{\pi}{2}\right) + \sin\left(-\frac{\pi}{2}\right) + \sin 2\pi$$

$$= (-1) + (-1) + 0 = -2$$

$$\therefore \sin\alpha + \sin\beta + \sin\gamma \text{ ની કિંમત } -2 \text{ છે.}$$

(A), (C), (D) શક્ય નથી.

$$\therefore \text{કિંમત ઋણ હોઈ શકે.}$$

જવાબ : (B)

[(A) તો સ્પષ્ટ અસત્ય છે. (D) સત્ય માટે $\alpha = \beta = \gamma = \frac{3\pi}{2}, \frac{7\pi}{2}$ વગેરે આ શક્ય નથી, કારણ કે $\alpha + \beta + \gamma = \pi$. (C) પણ સ્પષ્ટ રીતે અસત્ય છે.]

(61) $5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$ નો વિસ્તાર છે.

(A) [4, 10] (B) [-4, 10] (C) [3, 10] (D) [-3, 10]

$$\text{ઉકેલ : } f(\theta) = 5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right)$$

$$= 5\cos\theta + 3\left(\cos\theta \cos\frac{\pi}{3} - \sin\theta \sin\frac{\pi}{3}\right)$$

$$= 5\cos\theta + \left(\frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta\right)$$

$$= \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta. \text{ અથી, } a = \frac{13}{2}, b = -\frac{3\sqrt{3}}{2}$$

$$r^2 = a^2 + b^2 = \frac{169+27}{4} = 49. \text{ અથી, } r = 7$$

$\therefore f$ નો વિસ્તાર $[-7, 7]$ છે.

$$\therefore -7 \leq 5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) \leq 7$$

$$\Rightarrow -4 \leq 5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3 \leq 10$$

માંગેલ વિસ્તાર $[-4, 10]$ છે.

જવાબ : (B)

(62) $f(\theta) = \sin^2\theta$ નું મુખ્ય આવર્તમાન છે.

[AIEEE : 2002]

(A) π^2

(B) π

(C) 2π

(D) $\frac{\pi}{2}$

ઉકેલ : $f(\theta) = \sin^2\theta = \sin^2(\pi + \theta) = f(\pi + \theta)$

આવી નાનામાં નાની ધન વાસ્તવિક સંખ્યા π છે. અથી, મુખ્ય આવર્તમાન = π

જવાબ : (B)

(63) α, β એવી સંખ્યા છે કે જેથી $\pi < \alpha - \beta < 3\pi$

$$\sin\alpha + \sin\beta = \frac{-21}{65} \text{ અને } \cos\alpha + \cos\beta = \frac{-27}{65}, \text{ તો } \cos\left(\frac{\alpha - \beta}{2}\right) = \dots \quad [\text{AIEEE : 2004}]$$

(A) $\frac{-6}{65}$

(B) $\frac{3}{\sqrt{130}}$

(C) $\frac{6}{65}$

(D) $\frac{-3}{\sqrt{130}}$

$$\text{ઉકેલ : } \sin\alpha + \sin\beta = \frac{-21}{65}. \text{ અથી, } 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right) = \frac{-21}{65} \quad (1)$$

$$\cos\alpha + \cos\beta = \frac{-27}{65}. \text{ અથી } 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right) = \frac{-27}{65} \quad (2)$$

(1) અને (2) વર્ગીકરણ કરતાં,

$$\therefore 4\sin^2\left(\frac{\alpha + \beta}{2}\right)\cos^2\left(\frac{\alpha - \beta}{2}\right) + 4\cos^2\left(\frac{\alpha + \beta}{2}\right)\cos^2\left(\frac{\alpha - \beta}{2}\right) = \frac{441 + 729}{65^2}$$

$$\therefore 4\cos^2\left(\frac{\alpha - \beta}{2}\right) = \frac{18}{65}$$

$$\therefore \cos^2\left(\frac{\alpha - \beta}{2}\right) = \frac{9}{130}$$

$$\therefore \cos\left(\frac{\alpha - \beta}{2}\right) = \pm\frac{3}{\sqrt{130}}$$

$$\therefore \cos\left(\frac{\alpha - \beta}{2}\right) = \frac{-3}{\sqrt{130}} \text{ કારણ કે } \frac{\pi}{2} < \frac{\alpha - \beta}{2} < \frac{3\pi}{2}$$

જવાબ : (D)

(64) સમીકરણ $2\sin^2x + 5\sin x - 3 = 0$ ની $[0, 3\pi]$ માં શક્ય ઉકેલોની સંખ્યા છે. [AIEEE : 2006]

- (A) 4 (B) 6 (C) 1 (D) 2

ઉકેલ : $2\sin^2x + 5\sin x - 3 = 0$

$$\therefore \sin x = \frac{1}{2} \text{ અથવા } \sin x = -3$$

હવે, $\sin x = -3$ શક્ય નથી.

$$\therefore \sin x = \frac{1}{2} \text{ અને } x \in [0, 3\pi]$$

$\therefore [0, \pi]$ માં બે ઉકેલ અને $[2\pi, 3\pi]$ માં અન્ય બે ઉકેલ મળે.

$$\therefore \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6} \text{ ચાર ઉકેલો શક્ય બને.}$$

ઉકેલોની કુલ સંખ્યા 4 છે.

જવાબ : (A)

(65) $0 < x < \pi$ અને $\cos x + \sin x = \frac{1}{2}$ હોય, તો $\tan x = \dots$

- (A) $\frac{1-\sqrt{7}}{4}$ (B) $\frac{4-\sqrt{7}}{3}$ (C) $\frac{-4 \pm \sqrt{7}}{3}$ (D) $\frac{1+\sqrt{7}}{4}$

ઉકેલ : $\cos x + \sin x = \frac{1}{2}$ આપેલ છે.

$$\therefore 1 + \tan x = \frac{1}{2} \sec x \quad (\cos x \neq 0)$$

$$\therefore 2 + 2\tan x = \sec x$$

$$\therefore 4 + 8\tan x + 4\tan^2 x = \tan^2 x + 1$$

$$\therefore 3\tan^2 x + 8\tan x + 3 = 0$$

$$\therefore \tan x = \frac{-8 \pm \sqrt{28}}{6} = \frac{-8 \pm 2\sqrt{7}}{6} = \frac{-4 \pm \sqrt{7}}{3}$$

જવાબ : (C)

(66) જો $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$ હોય, તો P : $\cos\alpha + \cos\beta + \cos\gamma = 0$

અને Q : $\sin\alpha + \sin\beta + \sin\gamma = 0$ માટે

[AIEEE 2009]

(A) P અસત્ય છે અને Q સત્ય છે.

(B) P અને Q બંને સત્ય છે.

(C) P અને Q બંને અસત્ય છે.

(D) P સત્ય છે અને Q અસત્ય છે.

ઉકેલ : $(\cos\alpha + \cos\beta + \cos\gamma)^2 + (\sin\alpha + \sin\beta + \sin\gamma)^2$

$$= 3 + 2[(\cos\alpha \cos\beta + \sin\alpha \sin\beta) + (\cos\beta \cos\gamma + \sin\beta \sin\gamma) + (\cos\alpha \cos\gamma + \sin\alpha \sin\gamma)]$$

$$= 3 + 2[\cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\alpha - \gamma)]$$

$$= 3 + 2\left(-\frac{3}{2}\right) = 0$$

$$\therefore \cos\alpha + \cos\beta + \cos\gamma = 0 \text{ તથા } \sin\alpha + \sin\beta + \sin\gamma = 0 \text{ બંને સત્ય છે.}$$

જવાબ : (B)

(67) $a_1, a_2 \dots a_{n+1}$ પદો સમાંતર શ્રેણીમાં હોય, તો $\tan^{-1} \frac{d}{1+a_1a_2} + \tan^{-1} \frac{d}{1+a_2a_3} + \dots + n$ પદ સુધી =

(જ્યાં $d > 0, a_i > 0, \forall i$) (સામાન્ય તફાવત છે.)

$$(A) \tan^{-1} \frac{nd}{1+a_1a_{n+1}}$$

$$(B) \tan^{-1} \left(\frac{nd}{1+a_1a_n} \right)$$

$$(C) \tan^{-1} \left(\frac{(n-1)d}{1+a_1a_n} \right)$$

(D) આમાંથી એકપણ નહિ

ઉકેલ : $a_1, a_2, a_3, \dots, a_{n+1}$ સમાંતર શ્રેણીમાં છે.

$$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \dots a_{n+1} - a_n = d \quad (1)$$

$$\begin{aligned} & \tan^{-1} \frac{d}{1+a_1a_2} + \tan^{-1} \frac{d}{1+a_1a_3} + \dots (n \text{ પદ સુધી}) \\ &= \tan^{-1} \left(\frac{a_2 - a_1}{1+a_1a_2} \right) + \tan^{-1} \left(\frac{a_3 - a_2}{1+a_2a_3} \right) + \dots + (n \text{ પદ સુધી}) \\ &= (\tan^{-1} a_2 - \tan^{-1} a_1) + (\tan^{-1} a_3 - \tan^{-1} a_2) + \dots + [\tan^{-1}(a_{n+1}) - \tan^{-1}(a_n)] \\ &= \tan^{-1}(a_{n+1}) - \tan^{-1} a_1 \\ &= \tan^{-1} \left(\frac{a_{n+1} - a_1}{1+a_{n+1}a_1} \right) \end{aligned} \quad (2)$$

હવે, (1) પરથી

$$a_2 - a_1 + a_3 - a_2 + a_4 - a_3 + \dots + a_{n+1} - a_n = nd$$

$$\therefore a_{n+1} - a_1 = nd$$

$$\therefore \tan^{-1} \left(\frac{a_{n+1} - a_1}{1+a_{n+1}a_1} \right) = \tan^{-1} \left(\frac{nd}{1+a_{n+1}a_1} \right)$$

જવાબ : (A)

(68) $\sin^2 x + \cos^4 x, \forall x \in \mathbb{R}$ નો વિસ્તાર છે. [AIEEE : 2011]

(A) $\left[\frac{13}{16}, 1 \right]$

(B) $[1, 2]$

(C) $\left[\frac{3}{4}, \frac{13}{16} \right]$

(D) $\left[\frac{3}{4}, 1 \right]$

$$\begin{aligned} \text{ઉકેલ : } \sin^2 x + \cos^4 x &= \left(\frac{1 - \cos 2x}{2} \right) + \left(\frac{1 + \cos 2x}{2} \right)^2 \\ &= \frac{1 - \cos 2x}{2} + \frac{1 + 2\cos 2x + \cos^2 2x}{4} \\ &= \frac{2 - 2\cos 2x + 1 + 2\cos 2x + \cos^2 2x}{4} \\ &= \frac{3 + \cos^2 2x}{4} \end{aligned}$$

$$= \frac{3}{4} + \frac{1}{4} \left(\frac{1 + \cos 4x}{2} \right)$$

$$= \frac{7}{8} + \frac{1}{8} \cos 4x$$

$\cos 4x$ નો વિસ્તાર $[-1, 1]$ છે.

$$\therefore \frac{7}{8} + \frac{1}{8} \cos 4x \text{ નો વિસ્તાર } \left[\frac{7}{8} - \frac{1}{8}, \frac{7}{8} + \frac{1}{8} \right] = \left[\frac{3}{4}, 1 \right] \text{ છે.}$$

જવાબ : (D)

(69) $\sin \theta + \sin 4\theta + \sin 7\theta = 0$ નું $(0, \pi)$ માં સમાધાન કરતી θ ની કિંમતો છે. [AIEEE : 2011]

(A) $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

(B) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$

(C) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

(D) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$

ઉકેલ : $\sin \theta + \sin 4\theta + \sin 7\theta = 0$

$$\therefore (\sin 7\theta + \sin \theta) + \sin 4\theta = 0$$

$$\therefore 2\sin 4\theta \cos 3\theta + \sin 4\theta = 0$$

$$\therefore \sin 4\theta (2\cos 3\theta + 1) = 0$$

$$\therefore \sin 4\theta = 0 \text{ અથવા } \cos 3\theta = -\frac{1}{2}$$

$$(i) \sin 4\theta = 0 \text{ લેતાં,}$$

$$0 < \theta < \pi \Rightarrow 0 < 4\theta < 4\pi$$

$$\therefore 4\theta = \pi, 2\pi, 3\pi. \text{ આથી, } \theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4} \quad (1)$$

$$(ii) \cos 3\theta = -\frac{1}{2} \text{ લેતાં,}$$

$$0 < \theta < \pi \Rightarrow 0 < 3\theta < 3\pi$$

$$\therefore 3\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}. \text{ આથી, } \theta = \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9} \quad (2)$$

$$\therefore \theta = \frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$$

જવાબ : (D)

(70) કોઈ એક સમલંબ ચતુષ્કોણ ABCD માટે $\overline{AB} \parallel \overline{CD}$ તથા $\overline{BC} \perp \overline{CD}$ હોય તથા

જો $m\angle ADB = \theta$, $BC = p$ તથા $CD = q$ હોય, તો $AB = \dots\dots$ [JEE Main : 2013]

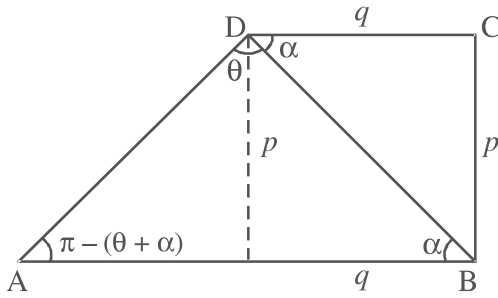
$$(A) \frac{(p^2 + q^2) \sin \theta}{p \cos \theta + q \sin \theta}$$

$$(B) \frac{p^2 + q^2 \cos \theta}{p \cos \theta + q \sin \theta}$$

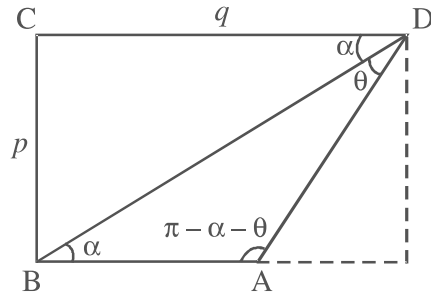
$$(C) \frac{p^2 + q^2}{p^2 \cos \theta + q^2 \sin \theta}$$

$$(D) \frac{(p^2 + q^2) \sin \theta}{(p \cos \theta + q \sin \theta)^2}$$

ઉકેલ :



આકૃતિ 12.10



આકૃતિ 12.11

સમલંબ ચતુષ્કોણ ABCD માટે, $\overline{AB} \parallel \overline{CD}$ તેમજ $\overline{BC} \perp \overline{CD}$ આપેલ હોવાથી, આકૃતિમાં દર્શાવ્યા મુજબ કાટકોણ $\triangle BDC$ રચાશે. જ્યાં $m\angle BDC = \alpha$

$$\therefore BD^2 = BC^2 + CD^2 = p^2 + q^2 \quad (1)$$

$$\text{અને } \cos \alpha = \frac{CD}{BD} = \frac{q}{\sqrt{p^2 + q^2}} \text{ તેમજ } \sin \alpha = \frac{BC}{BD} = \frac{p}{\sqrt{p^2 + q^2}} \quad (2), (3)$$

વળી, $\triangle ABD$ માટે, sine સૂત્રનો ઉપયોગ કરતાં,

$$\frac{AB}{\sin \theta} = \frac{BD}{\sin A} \text{ મળે.}$$

$$\therefore \frac{AB}{\sin \theta} = \frac{\sqrt{p^2 + q^2}}{\sin(\pi - (\theta + \alpha))} \quad ((1) \text{ પરથી})$$

$$\frac{AB}{\sin \theta} = \frac{\sqrt{p^2 + q^2}}{\sin \theta \cos \alpha + \cos \theta \sin \alpha}$$

$$\therefore AB = \frac{\sqrt{p^2 + q^2} \sin \theta}{\sin \theta \left(\frac{q}{\sqrt{p^2 + q^2}} \right) + \cos \theta \left(\frac{p}{\sqrt{p^2 + q^2}} \right)} \quad ((2), (3) \text{ પરથી})$$

$$\therefore AB = \frac{(p^2 + q^2) \sin \theta}{p \cos \theta + q \sin \theta} \quad \text{જવાબ : (A)}$$

$$(71) \quad \tan^{-1} y = \tan^{-1} x + \tan^{-1} \left(\frac{2x}{1-x^2} \right), \quad 0 < x < \frac{1}{\sqrt{3}} \quad \text{હોય, તો } y = \dots$$

$$(A) \frac{3x-x^3}{1-3x^2} \quad (B) \frac{3x+x^3}{1-3x^2} \quad (C) \frac{3x-x^3}{1+3x^2} \quad (D) \frac{3x+x^3}{1+3x^2}$$

$$\text{ઉકેલ : } \tan^{-1} y = \tan^{-1} \left(\frac{x + \frac{2x}{1-x^2}}{1 - \frac{2x^2}{1-x^2}} \right) \quad \left(0 < x < \frac{1}{\sqrt{3}} \right)$$

$$= \tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right)$$

$$\therefore y = \left(\frac{3x-x^3}{1-3x^2} \right) \quad \text{જવાબ : (A)}$$

$$(72) \quad \sin^{-1}(\sin 10) = \dots$$

$$(A) 10 \quad (B) 10-3\pi \quad (C) 3\pi-10 \quad (D) \text{ આમાંથી એક પણ નહિ.}$$

$$\text{ઉકેલ : } \sin^{-1}(\sin 10) = \sin^{-1}[\sin(3\pi-10)] = 3\pi-10 \quad \left(-\frac{\pi}{2} < 3\pi-10 < 0 \right)$$

$$\text{જવાબ : (C)}$$

$$(73) \quad \sin^{-1} \left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2} \right] = \dots$$

$$(A) \sin^{-1} \sqrt{x} + \sin^{-1} x \quad (B) \sin^{-1} x - \sin^{-1} \sqrt{x} \quad (C) \sin^{-1} \sqrt{x} - \sin^{-1} x \quad (D) \text{ આમાંથી એક પણ નહિ.}$$

$$\text{ઉકેલ : } \text{ધારો કે } x = \sin \theta \text{ અને } \sqrt{x} = \sin \alpha,$$

$$\text{અહીં } x \in [0, 1] \Rightarrow \theta, \alpha \in \left[0, \frac{\pi}{2} \right]$$

$$\therefore \theta - \alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$

$$\sin^{-1} \left(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2} \right)$$

$$= \sin^{-1} \left(\sin \theta \sqrt{1 - \sin^2 \alpha} - \sin \alpha \sqrt{1 - \sin^2 \theta} \right)$$

$$= \sin^{-1} (\sin \theta \cos \alpha - \cos \theta \sin \alpha)$$

$$(\cos \alpha > 0, \cos \theta > 0)$$

$$= \sin^{-1} (\sin(\theta - \alpha))$$

$$= \theta - \alpha$$

$$= \sin^{-1} x - \sin^{-1} \sqrt{x}$$

$$\left(0 \leq \theta, \alpha \leq \frac{\pi}{2} \right) \quad \text{જવાબ : (B)}$$

$$(74) \quad \cot^{-1} \frac{n}{\pi} > \frac{\pi}{6}, \quad n \in \mathbb{N} \text{ તો } n \text{ ની મહત્તમ કિંમત } \dots\dots$$

$$(A) \quad 6$$

$$(B) \quad 7$$

$$(C) \quad 5$$

$$(D) \quad \text{આમાંથી એક પણ નહિ.}$$

$$\text{ઉકેલ : } \cot^{-1} \frac{n}{\pi} > \frac{\pi}{6}$$

$$\therefore \frac{n}{\pi} < \cot \frac{\pi}{6} \quad (\cot \text{ ઘટતું વિધેય છે.})$$

$$\therefore \frac{n}{\pi} < \sqrt{3}$$

$$\therefore n < \sqrt{3} \pi \Rightarrow n < 6$$

$$\therefore n \text{ ની મહત્તમ કિંમત } = 5.$$

$$\text{જવાબ : (C)}$$

$$(75) \quad \sum_{r=1}^n \sin^{-1} \left(\frac{\sqrt{r} - \sqrt{r-1}}{\sqrt{r(r+1)}} \right) = \dots\dots$$

$$(A) \quad \tan^{-1} \sqrt{n} - \frac{\pi}{4}$$

$$(B) \quad \tan^{-1} (\sqrt{n+1}) - \frac{\pi}{4}$$

$$(C) \quad \tan^{-1} \sqrt{n}$$

$$(D) \quad \tan^{-1} (\sqrt{n+1})$$

$$\text{ઉકેલ : } \sin^{-1} \left(\frac{\sqrt{r} - \sqrt{r-1}}{\sqrt{r(r+1)}} \right) = \tan^{-1} \left(\frac{\sqrt{r} - \sqrt{r-1}}{1 + \sqrt{r}\sqrt{r-1}} \right)$$

$$= \tan^{-1} \sqrt{r} - \tan^{-1} (\sqrt{r-1})$$

$$\therefore \sum_{r=1}^n \sin^{-1} \left(\frac{\sqrt{r} - \sqrt{r-1}}{1 + \sqrt{r}\sqrt{r-1}} \right) = \sum_{r=1}^n \left(\tan^{-1} \sqrt{r} - \tan^{-1} \sqrt{r-1} \right) = \tan^{-1} \sqrt{n} \quad \text{જવાબ : (C)}$$

$$(76) \quad \sum_{r=1}^n \tan^{-1} \left(\frac{2^r - 1}{1 + 2^{2r-1}} \right) = \dots\dots$$

$$(A) \quad \tan^{-1} 2^n$$

$$(B) \quad \tan^{-1} 2^n - \frac{\pi}{4}$$

$$(C) \quad \tan^{-1} 2^{n+1}$$

$$(D) \quad \tan^{-1} 2^{n+1} - \frac{\pi}{4}$$

$$\text{ઉકેલ : } \sum_{r=1}^n \tan^{-1} \left(\frac{2^r - 1}{1 + 2^{2r-1}} \right) = \sum_{r=1}^n \tan^{-1} \left(\frac{2^r - 2^{r-1}}{1 + 2^r \cdot 2^{r-1}} \right)$$

$$= \sum_{r=1}^n \left[\tan^{-1} (2^r) - \tan^{-1} (2^{r-1}) \right]$$

$$= \tan^{-1} 2^n - \tan^{-1} 1 = \tan^{-1} 2^n - \frac{\pi}{4}$$

$$\text{જવાબ : (B)}$$

$$(77) \quad \sum_{m=1}^n \tan^{-1} \left(\frac{2m}{m^4 + m^2 + 2} \right) = \dots$$

(A) $\tan^{-1} \left(\frac{n^2 + n}{n^2 + n + 2} \right)$ (B) $\tan^{-1} \left(\frac{n^2 - n}{n^2 - n + 2} \right)$ (C) $\tan^{-1} \left(\frac{n^2 + n + 2}{n^2 + n} \right)$ (D) આમાંથી એક પણ નહિ.

ઉકેલ :
$$\begin{aligned} \sum_{m=1}^n \tan^{-1} \left(\frac{2m}{m^4 + m^2 + 2} \right) &= \sum_{m=1}^n \tan^{-1} \left(\frac{(m^2 + m + 1) - (m^2 - m + 1)}{1 + (m^2 + m + 1)(m^2 - m + 1)} \right) \\ &= \sum_{m=1}^n \left[\tan^{-1}(m^2 + m + 1) - \tan^{-1}(m^2 - m + 1) \right] \\ &= (\tan^{-1}3 - \tan^{-1}1) + (\tan^{-1}7 - \tan^{-1}3) + (\tan^{-1}13 - \tan^{-1}7) + \dots \\ &\quad + (\tan^{-1}(n^2 + n + 1) - \tan^{-1}(n^2 - n + 1)) \\ &= \tan^{-1}(n^2 + n + 1) - \tan^{-1}1 \\ &= \tan^{-1} \left(\frac{n^2 + n + 1 - 1}{1 + (n^2 + n + 1)(1)} \right) \quad (n^2 + n + 1 > 0) \\ &= \tan^{-1} \left(\frac{n^2 + n}{n^2 + n + 2} \right) \end{aligned}$$

જવાબ : (A)

(78) a, b, c ધન વાસ્તવિક સંખ્યા માટે

$$\theta = \tan^{-1} \sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1} \sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1} \sqrt{\frac{c(a+b+c)}{ab}} \text{ હોય, તો } \tan \theta = \dots \text{ [IIT : 1981]}$$

(A) 0 (B) 2 (C) 1 (D) $\sqrt{3}$

ઉકેલ : ધારો કે $\tan \alpha = \sqrt{\frac{a(a+b+c)}{bc}}, \tan \beta = \sqrt{\frac{b(a+b+c)}{ca}}, \tan \gamma = \sqrt{\frac{c(a+b+c)}{ab}}$

$$0 < \alpha, \beta, \gamma < \frac{\pi}{2}$$

$$\therefore \theta = \alpha + \beta + \gamma \text{ મળે.}$$

$$\tan \alpha + \tan \beta + \tan \gamma = \frac{\sqrt{a+b+c}(a+b+c)}{\sqrt{abc}} = \frac{(a+b+c)^{\frac{3}{2}}}{\sqrt{abc}}$$

$$\tan \alpha \tan \beta \tan \gamma = \frac{(a+b+c)^{\frac{3}{2}}}{\sqrt{abc}}$$

$$\therefore \tan \alpha + \tan \beta + \tan \gamma = \tan \alpha \tan \beta \tan \gamma$$

$$\therefore \alpha + \beta + \gamma = (2k + 1)\pi; k \in \mathbb{Z}$$

પરંતુ $0 < \alpha + \beta + \gamma < \frac{3\pi}{2}$. આથી, $\alpha + \beta + \gamma = \pi$

$$\therefore \tan \theta = \tan(\alpha + \beta + \gamma) = \tan \pi = 0$$

જવાબ : (A)

બીજી રીત :

ધારો કે $a + b + c = u$

$$\therefore \theta = \tan^{-1} \sqrt{\frac{au}{bc}} + \tan^{-1} \sqrt{\frac{bu}{ca}} + \tan^{-1} \sqrt{\frac{cu}{ab}}$$

$$\text{હવે, } \sqrt{\frac{au}{bc}} \cdot \sqrt{\frac{bu}{ca}} = \frac{u}{c} = \frac{a+b+c}{c} > 1$$

$$\therefore \theta = \pi + \tan^{-1} \left(\frac{\sqrt{\frac{au}{bc}} + \sqrt{\frac{bu}{ca}}}{1 - \left(\sqrt{\frac{au}{bc}} \right) \left(\sqrt{\frac{bu}{ca}} \right)} \right) + \tan^{-1} \sqrt{\frac{cu}{ab}}$$

$$= \pi + \tan^{-1} \left(\frac{\frac{(a+b)\sqrt{u}}{\sqrt{abc}}}{1 - \frac{u}{c}} \right) + \tan^{-1} \sqrt{\frac{cu}{ab}}$$

$$= \pi + \tan^{-1} \left(\frac{(u-c)\sqrt{u}}{(c-u)} \cdot \frac{c}{\sqrt{abc}} \right) + \tan^{-1} \sqrt{\frac{cu}{ab}}$$

$$= \pi - \tan^{-1} \sqrt{\frac{cu}{ab}} + \tan^{-1} \sqrt{\frac{cu}{ab}}$$

$$= \pi$$

$$\therefore \tan \theta = \tan \pi = 0$$

જવાબ : (A)

$$(79) \quad \tan \left[2 \tan^{-1} \left(\frac{1}{5} \right) - \frac{\pi}{4} \right] = \dots\dots$$

$$(A) \quad \frac{7}{17}$$

$$(B) \quad \frac{-7}{17}$$

$$(C) \quad \frac{17}{7}$$

$$(D) \quad \frac{15}{7}$$

$$\text{ઉકેલ : } \tan \left[2 \tan^{-1} \left(\frac{1}{5} \right) - \frac{\pi}{4} \right]$$

$$= \tan \left[\tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{5} - \tan^{-1} 1 \right]$$

$$= \tan \left[\tan^{-1} \frac{\frac{2}{5}}{1 - \frac{1}{25}} - \tan^{-1} 1 \right]$$

$$= \tan \left[\tan^{-1} \frac{5}{12} - \tan^{-1} 1 \right]$$

$$= \tan \left[\tan^{-1} \left(\frac{\frac{5}{12} - 1}{1 + \frac{5}{12}} \right) \right]$$

$$= \tan \left[\tan^{-1} \left(\frac{-7}{17} \right) \right] = \frac{-7}{17}$$

જવાબ : (B)

$$(80) \quad \lim_{n \rightarrow \infty} \sum_{r=1}^{\infty} \tan^{-1} \left(\frac{1}{2r^2} \right) = \dots\dots$$

$$(A) \quad \frac{\pi}{6}$$

$$(B) \quad \frac{\pi}{4}$$

$$(C) \quad \frac{\pi}{2}$$

$$(D) \quad \frac{\pi}{3}$$

$$\text{ଓଡ଼ିଶା : } \sum_{r=1}^{\infty} \tan^{-1}\left(\frac{1}{2r^2}\right) = \sum_{r=1}^{\infty} \tan^{-1}\left(\frac{2}{4r^2}\right)$$

$$= \sum_{r=1}^{\infty} \tan^{-1}\left[\frac{2}{1+4r^2-1}\right]$$

$$= \sum_{r=1}^{\infty} \tan^{-1}\left[\frac{(2r+1)-(2r-1)}{1+(2r-1)(2r+1)}\right]$$

$$= \sum_{r=1}^{\infty} \left[\tan^{-1}(2r+1) - \tan^{-1}(2r-1) \right]$$

$$\therefore \lim_{n \rightarrow \infty} \sum_{r=1}^{\infty} \tan^{-1}\left(\frac{1}{2r^2}\right) = \lim_{n \rightarrow \infty} \sum_{r=1}^n \left[\tan^{-1}(2r+1) - \tan^{-1}(2r-1) \right]$$

$$= \lim_{n \rightarrow \infty} \left[\tan^{-1}(2n+1) - \frac{\pi}{4} \right] = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4} \quad \text{ଞ୍ଜାଲିଆ : (B)}$$

$$(81) \quad \cot^{-1}(\sqrt{\cos \alpha}) - \tan^{-1}(\sqrt{\cos \alpha}) = x \quad \text{ଓଲ} \quad \sin x = \dots \quad [\text{AIEEE : 2002}]$$

$$(A) \tan^2\left(\frac{\alpha}{2}\right)$$

$$(B) \cot^2\left(\frac{\alpha}{2}\right)$$

$$(C) \tan \alpha$$

$$(D) \cot \frac{\alpha}{2}$$

$$\text{ଓଡ଼ିଶା : } x = \cot^{-1}(\sqrt{\cos \alpha}) - \tan^{-1}(\sqrt{\cos \alpha})$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{\cos \alpha}}\right) - \tan^{-1}(\sqrt{\cos \alpha})$$

$$= \tan^{-1}\left(\frac{\frac{1}{\sqrt{\cos \alpha}} - \sqrt{\cos \alpha}}{1+1}\right)$$

$$= \tan^{-1}\left(\frac{1 - \cos \alpha}{2\sqrt{\cos \alpha}}\right)$$

$$\therefore \sin x = \sin \left[\tan^{-1}\left(\frac{1 - \cos \alpha}{2\sqrt{\cos \alpha}}\right) \right]$$

$$\sin x = \sin \left[\sin^{-1}\left(\frac{1 - \cos \alpha}{1 + \cos \alpha}\right) \right]$$

$$\left(\tan^{-1} x = \sin^{-1} \frac{x}{\sqrt{1+x^2}} \right)$$

$$= \sin \left(\sin^{-1}\left(\tan^2 \frac{\alpha}{2}\right) \right)$$

$$= \tan^2 \frac{\alpha}{2}$$

$$\text{ଞ୍ଜାଲିଆ : (A)}$$

(82) $\sin^{-1}x = 2\sin^{-1}a$ સમીકરણનો ઉકેલ મળે તે માટે નીચેનામાંથી કઈ શરત જરૂરી છે ? [AIEEE : 2003]

(A) $|a| \geq \frac{1}{\sqrt{2}}$ (B) $|a| \leq \frac{1}{\sqrt{2}}$ (C) $\forall a \in \mathbb{R}$ (D) $|a| < \frac{1}{2}$

ઉકેલ : $\sin^{-1}x = 2\sin^{-1}a$

આપણે જાણીએ છીએ કે, $-\frac{\pi}{2} \leq \sin^{-1}x \leq \frac{\pi}{2}$

$$\begin{array}{l} \therefore -\frac{\pi}{2} \leq 2\sin^{-1}a \leq \frac{\pi}{2} \\ \therefore -\frac{\pi}{4} \leq \sin^{-1}a \leq \frac{\pi}{4} \end{array} \quad \left| \quad \begin{array}{l} \therefore -\frac{1}{\sqrt{2}} \leq a \leq \frac{1}{\sqrt{2}} \\ \therefore |a| \leq \frac{1}{\sqrt{2}} \end{array} \right.$$

જવાબ : (B)

(83) $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, તો $4x^2 - 4xy \cos\alpha + y^2 = \dots\dots$ [AIEEE : 2005]

(A) $2\sin 2\alpha$ (B) 4 (C) $4\sin^2\alpha$ (D) $-4\sin^2\alpha$

ઉકેલ : ધારો કે $\cos^{-1}x = p$, $\cos^{-1}\frac{y}{2} = q$, $p \in [0, \pi]$, $q \in [0, 2\pi]$

$\therefore \cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$

હવે, $p - q = \alpha$

$\therefore \cos(p - q) = \cos\alpha$

$\therefore \cos p \cos q + \sin p \sin q = \cos\alpha$

$\frac{xy}{2} + \sqrt{1-x^2} \sqrt{1-\frac{y^2}{4}} = \cos\alpha$ ($\sin p > 0$, $\sin q > 0$)

$\left(\frac{xy}{2} - \cos\alpha\right)^2 = (1-x^2)\left(1-\frac{y^2}{4}\right)$

$\frac{x^2y^2}{4} - xycos\alpha + \cos^2\alpha = 1 - x^2 - \frac{y^2}{4} + \frac{x^2y^2}{4}$

$x^2 + \frac{y^2}{4} = 1 - \cos^2\alpha + xycos\alpha$

$\therefore 4x^2 - 4xycos\alpha + y^2 = 4\sin^2\alpha$

જવાબ : (C)

(84) $\sin^{-1}\left(\frac{x}{5}\right) + \csc^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$ હોય, તો $x = \dots\dots$ [AIEEE : 2007]

(A) 4 (B) 5 (C) 1 (D) 3

ઉકેલ : $\sin^{-1}\left(\frac{x}{5}\right) + \csc^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$

$\therefore \sin^{-1}\left(\frac{x}{5}\right) + \sin^{-1}\frac{4}{5} = \frac{\pi}{2}$

$\therefore \sin^{-1}\left(\frac{x}{5}\right) + \cos^{-1}\frac{3}{5} = \frac{\pi}{2}$

$\therefore x = 3$

જવાબ : (D)

$$(85) \quad \cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right) = \dots\dots$$

$$(A) \quad \frac{6}{17}$$

$$(B) \quad \frac{3}{17}$$

$$(C) \quad \frac{4}{17}$$

$$(D) \quad \frac{5}{17}$$

$$\text{ઉકેલ : } \operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3} = \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3} = \tan^{-1}\left(\frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{6}{12}}\right) = \tan^{-1}\left(\frac{17}{6}\right)$$

$$\therefore \cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right) = \cot\left(\tan^{-1}\frac{17}{6}\right) = \frac{6}{17}$$

જવાબ : (A)

$$(86) \quad 0 < \theta < \frac{\pi}{2}$$

$$\sum_{m=1}^6 \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right) \operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2} \quad \text{સમીકરણનું ઉકેલ હોય.} \quad [\text{IIT : 2009}]$$

$$(A) \quad \frac{\pi}{4}$$

$$(B) \quad \frac{\pi}{6}$$

$$(C) \quad \frac{\pi}{12}$$

$$(D) \quad \frac{5\pi}{12}$$

$$\text{ઉકેલ : } 0 < \theta < \frac{\pi}{2}$$

$$\sum_{m=1}^6 \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right) \operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$$

$$\therefore \sum_{m=1}^6 \frac{1}{\sin\left(\theta + \frac{(m-1)\pi}{4}\right) \sin\left(\theta + \frac{m\pi}{4}\right)} = 4\sqrt{2}$$

$$\therefore \sum_{m=1}^6 \frac{\sin\frac{\pi}{4}}{\sin\frac{\pi}{4} \sin\left(\theta + \frac{(m-1)\pi}{4}\right) \sin\left(\theta + \frac{m\pi}{4}\right)} = 4\sqrt{2}$$

$$\therefore \sum_{m=1}^6 \frac{\sin\left[\left(\theta + \frac{m\pi}{4}\right) - \left(\theta + \frac{(m-1)\pi}{4}\right)\right]}{\sin\frac{\pi}{4} \left[\sin\left(\theta + \frac{(m-1)\pi}{4}\right) \sin\left(\theta + \frac{m\pi}{4}\right)\right]} = 4\sqrt{2}$$

$$\therefore \sum_{m=1}^6 \left[\cot\left(\theta + \frac{(m-1)\pi}{4}\right) - \cot\left(\theta + \frac{m\pi}{4}\right) \right] = 4$$

$$\therefore \cot\theta - \cot\left(\theta + \frac{\pi}{4}\right) + \cot\left(\theta + \frac{\pi}{4}\right) - \cot\left(\theta + \frac{2\pi}{4}\right) + \dots + \cot\left(\theta + \frac{5\pi}{4}\right) - \cot\left(\theta + \frac{6\pi}{4}\right) = 4$$

$$\therefore \cot\theta - \cot\left(\theta + \frac{3\pi}{2}\right) = 4$$

$$\therefore \cot\theta + \tan\theta = 4$$

$$\therefore \frac{1}{\tan\theta} + \tan\theta = 4$$

$$\therefore 1 + \tan^2 \theta = 4 \tan \theta$$

$$\therefore \tan^2 \theta - 4 \tan \theta + 1 = 0$$

$$\therefore \tan \theta = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

$$\tan \theta = 2 + \sqrt{3} \Rightarrow \theta = \frac{5\pi}{12} \text{ તથા } \tan \theta = 2 - \sqrt{3} \Rightarrow \theta = \frac{\pi}{12} \quad \text{જવાબ : (C), (D)}$$

(87) x, y, z સમાંતર શ્રેણીમાં છે તથા $\tan^{-1}x, \tan^{-1}y, \tan^{-1}z$ પણ સમાંતર શ્રેણીમાં છે. તો [JEE : 2013]
($0 < x, y, z < 1$)

$$(A) x = y = z \quad (B) 2x = 3y = 6z \quad (C) 6x = 3y = 6z \quad (D) 6x = 4y = 3z$$

ઉકેલ : x, y, z સમાંતર શ્રેણીમાં છે.

$$\therefore x + z = 2y \quad (1)$$

હવે, $\tan^{-1}x, \tan^{-1}y, \tan^{-1}z$ પણ સમાંતર શ્રેણીમાં છે.

$$\therefore \tan^{-1}x + \tan^{-1}z = 2\tan^{-1}y$$

$$\therefore \tan^{-1}\left(\frac{x+z}{1-xz}\right) = 2\tan^{-1}y = \tan^{-1}\left(\frac{y+y}{1-y^2}\right)$$

$$\therefore \tan^{-1}\left(\frac{2y}{1-xz}\right) = \tan^{-1}\left(\frac{2y}{1-y^2}\right) \quad ((1) \text{ પરથી})$$

$$\therefore 1 - xz = 1 - y^2$$

$$\therefore y^2 = xz$$

$\therefore x, y, z$ સમગુણોત્તર શ્રેણીમાં છે તથા x, y, z સમાંતર શ્રેણીમાં છે.

$$\therefore x = y = z \quad \text{જવાબ : (A)}$$

(88) $\tan\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\frac{2}{3}\right] = \dots\dots$ [IIT : 1993]

$$(A) \frac{6}{17} \quad (B) \frac{7}{16} \quad (C) \frac{16}{7} \quad (D) \text{ આમાંથી એક પણ નહિ.}$$

$$\text{ઉકેલ : હવે, } \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3} = \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3} = \tan^{-1}\left(\frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{6}{12}}\right) = \tan^{-1}\left(\frac{17}{6}\right)$$

$$\therefore \tan\left[\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right] = \tan\left(\tan^{-1}\frac{17}{6}\right) = \frac{17}{6} \quad \text{જવાબ : (D)}$$

(89) $\tan\left[\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right] = \dots\dots$ [IIT : 1994]

$$(A) \frac{\sqrt{29}}{3} \quad (B) \frac{29}{3} \quad (C) \frac{\sqrt{3}}{29} \quad (D) \frac{3}{29}$$

$$\text{ઉકેલ : } \tan\left[\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right]$$

$$= \tan[\tan^{-1}7 - \tan^{-1}4]$$

$$= \tan\left(\tan^{-1}\left(\frac{7-4}{1+28}\right)\right) = \frac{3}{29} \quad \text{જવાબ : (D)}$$

(90) સમીકરણ $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ ની વાસ્તવિક બીજની સંખ્યા [IIT : 1999]

(A) 0

(B) 1

(C) 2

(D) અનંત

ઉકેલ : $\tan^{-1}\sqrt{x(x+1)} = \frac{\pi}{2} - \sin^{-1}\sqrt{x^2+x+1}$

$$\therefore \tan^{-1}\sqrt{x(x+1)} = \cos^{-1}\sqrt{x^2+x+1}$$

$$\therefore \cos^{-1}\frac{1}{\sqrt{x^2+x+1}} = \cos^{-1}\sqrt{x^2+x+1}$$

$$\therefore x^2 + x + 1 = 1$$

$$\therefore x^2 + x = 0$$

$$\therefore x(x+1) = 0$$

$$\therefore x = 0 \text{ અથવા } x = -1$$

$\therefore$ વાસ્તવિક બીજની સંખ્યા 2 છે. ચકાસણી કરતા બંને બીજ, આપેલ સમીકરણનું સમાધાન કરે છે.

જવાબ : (C)

(91) $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \dots\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^8}{4} - \dots\right) = \frac{\pi}{2}$, $0 < |x| < \sqrt{2}$ હોય, તો $x = \dots$

(A) $\frac{1}{2}$

(B) 1

(C) $-\frac{1}{2}$

(D) -1 [IIT : 2015]

ઉકેલ : $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \dots\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^8}{4} - \dots\right) = \frac{\pi}{2}$

$$\therefore \left(x - \frac{x^2}{2} + \frac{x^3}{4} - \dots\right) = \left(x^2 - \frac{x^4}{2} + \frac{x^8}{4} - \dots\right)$$

$$\therefore \frac{x}{1 - \left(\frac{-x}{2}\right)} = \frac{x^2}{1 - \left(\frac{-x^2}{2}\right)} \quad (0 < x^2 < 2, 0 < |x| < \sqrt{2})$$

$$\therefore \frac{2x}{2+x} = \frac{2x^2}{2+x^2}$$

$$\therefore x(2+x^2) = x^2(2+x)$$

$$\therefore 2x + x^3 = 2x^2 + x^3. \text{ અર્થાત્, } x^2 - x = 0. \text{ અર્થાત્, } x(x-1) = 0$$

$$\therefore x = 0 \text{ અથવા } x = 1 \text{ પરંતુ } 0 < |x| < \sqrt{2}$$

$$\therefore x = 1$$

જવાબ : (B)

(92) $\sin(\cot^{-1}(1+x)) = \cos(\tan^{-1}x)$ હોય, તો $x = \dots$

(A) $\frac{1}{2}$

(B) 1

(C) 0

(D) $-\frac{1}{2}$

ઉકેલ : $\sin(\cot^{-1}(1+x)) = \cos(\tan^{-1}x)$

$$\therefore \sin\left(\sin^{-1}\frac{1}{\sqrt{x^2+2x+2}}\right) = \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)$$

$$\therefore \frac{1}{\sqrt{x^2+2x+2}} = \frac{1}{\sqrt{1+x^2}} \text{ . અર્થથી, } 1+x^2 = x^2+2x+2$$

$$\therefore 2x = -1. \text{ અર્થથી, } x = -\frac{1}{2} \left(x = -\frac{1}{2} \text{ મૂકી ચકાસણી કરો.} \right)$$

જવાબ : (D)

(93) $\sqrt{1+x^2} \left[\left\{ x \cos(\cot^{-1}x) + \sin(\cot^{-1}x) \right\}^2 - 1 \right]^{\frac{1}{2}} = \dots\dots (0 < x < 1)$ [IIT : 2008]

(A) $\frac{x}{\sqrt{1+x^2}}$

(B) x

(C) $x\sqrt{1+x^2}$

(D) $\sqrt{1+x^2}$

ઉકેલ : $\sqrt{1+x^2} \left[\left\{ x \cos(\cot^{-1}x) + \sin(\cot^{-1}x) \right\}^2 - 1 \right]^{\frac{1}{2}}$

$$= \sqrt{1+x^2} \left[\left\{ x \left(\cos \left(\cos^{-1} \frac{x}{\sqrt{1+x^2}} \right) \right) + \sin \left(\sin^{-1} \frac{1}{\sqrt{1+x^2}} \right) \right\}^2 - 1 \right]^{\frac{1}{2}}$$

$$= \sqrt{1+x^2} \left[\left\{ \frac{x^2}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}} \right\}^2 - 1 \right]^{\frac{1}{2}}$$

$$= \sqrt{1+x^2} [1+x^2-1]^{\frac{1}{2}}$$

$$= x\sqrt{1+x^2}$$

($x > 0$)

જવાબ : (C)

(94) કોઈ ત્રિકોણની ત્રણ બાજુનાં માપ $3x+4y$, $4x+3y$, $5x+5y$ હોય, તો તે ત્રિકોણ પ્રકારનો હોય $x, y > 0$

(A) કાટકોણ

(B) ગુરુકોણ

(C) સમબાજુ

(D) આમાંથી એક પણ નહિ.

ઉકેલ : ધારો કે ΔABC માં $BC = a = 3x+4y$, $AC = b = 4x+3y$, $AB = c = 5x+5y$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

$$= \frac{(3x+4y)^2 + (4x+3y)^2 - (5x+5y)^2}{2(3x+4y)(4x+3y)} = \frac{-2xy}{2(3x+4y)(4x+3y)} < 0$$

$\therefore C > \frac{\pi}{2}$. અર્થથી, ΔABC ગુરુકોણ ત્રિકોણ છે.

જવાબ : (B)

(95) a માપવાળા અને n બાજુવાળા નિયમિત બહુકોણના અંતર્ગત અને બહિર્ગત વર્તુળની ત્રિજ્યાઓનો સરવાળો

- (A) $\frac{a}{4} \cot \frac{\pi}{2n}$ (B) $a \cot \frac{\pi}{n}$ (C) $\frac{a}{2} \cot \frac{\pi}{2n}$ (D) $a \cot \frac{\pi}{2n}$

ઉકેલ : $\tan \frac{\pi}{n} = \frac{a}{2r}$

$\sin \frac{\pi}{n} = \frac{a}{2R}$

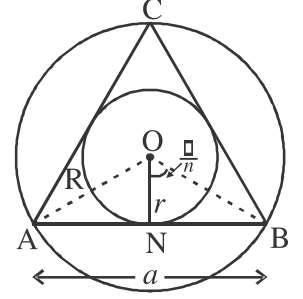
$\therefore r = \frac{a}{2} \cot \frac{\pi}{n}$

$R = \frac{a}{2} \operatorname{cosec} \frac{\pi}{n}$

$\therefore r + R = \frac{a}{2} \left[\cot \frac{\pi}{n} + \operatorname{cosec} \frac{\pi}{n} \right]$

$= \frac{a}{2} \left[\frac{1 + \cos \frac{\pi}{n}}{\sin \frac{\pi}{n}} \right]$

$= \frac{a}{2} \left[\frac{2 \cos^2 \frac{\pi}{2n}}{2 \sin \frac{\pi}{2n} \cos \frac{\pi}{2n}} \right] = \frac{a}{2} \cot \frac{\pi}{2n}$



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જવાબ : (C)

(96) $(2\cos\theta - 1)(2\cos2\theta - 1)(2\cos4\theta - 1) = \dots$

- (A) $\frac{2\cos8\theta + 1}{2\cos\theta + 1}$ (B) $\frac{2\cos8\theta - 1}{2\cos\theta + 1}$ (C) $\frac{2\cos8\theta + 1}{2\cos\theta - 1}$ (D) આમાંથી એક પણ નહિ.

ઉકેલ : $(2\cos\theta - 1)(2\cos2\theta - 1)(2\cos4\theta - 1)$

$= \frac{(2\cos\theta + 1)(2\cos\theta - 1)(2\cos2\theta - 1)(2\cos4\theta - 1)}{(2\cos\theta + 1)}$

$= \frac{(4\cos^2\theta - 1)(2\cos2\theta - 1)(2\cos4\theta - 1)}{2\cos\theta + 1}$

$= \frac{\left[4 \left(\frac{1 + \cos2\theta}{2} \right) - 1 \right] (2\cos2\theta - 1)(2\cos4\theta - 1)}{2\cos\theta + 1}$

$= \frac{(2\cos2\theta + 1)(2\cos2\theta - 1)(2\cos4\theta - 1)}{2\cos\theta + 1}$

$= \frac{4\cos^2 4\theta - 1}{2\cos\theta + 1}$

$= \frac{2(1 + \cos8\theta) - 1}{2\cos\theta + 1}$

$= \frac{2\cos8\theta + 1}{2\cos\theta + 1}$

જવાબ : (A)

(97) $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$ હોય, તો $x = \dots$

- (A) -1 (B) 1 (C) 0 (D) આમાંથી એક પણ નહિ.

ઉકેલ : $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$

$\therefore (\tan^{-1}x)^2 + \left(\frac{\pi}{2} - \tan^{-1}x \right)^2 = \frac{5\pi^2}{8}$

$$\therefore (\tan^{-1}x)^2 + (\tan^{-1}x)^2 - \pi(\tan^{-1}x) + \frac{\pi^2}{4} = \frac{5\pi^2}{8}$$

$$\therefore 2(\tan^{-1}x)^2 - \pi(\tan^{-1}x) - \frac{3\pi^2}{8} = 0$$

$$\text{ધારી કે } y = \tan^{-1}x$$

$$\therefore 2y^2 - \pi y - \frac{3\pi^2}{8} = 0$$

$$\therefore 16y^2 - 8\pi y - 3\pi^2 = 0$$

$$\therefore (4y - 3\pi)(4y + \pi) = 0$$

$$\therefore y = -\frac{\pi}{4} \text{ અથવા } y = \frac{3\pi}{4}$$

$$\therefore \tan^{-1}x = -\frac{\pi}{4} \text{ અથવા } \tan^{-1}x = \frac{3\pi}{4}$$

$$\therefore \tan^{-1}x = -\frac{\pi}{4} \quad \left(\tan^{-1}x \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \right)$$

$$\therefore x = \tan\left(-\frac{\pi}{4}\right)$$

$$\therefore x = -1$$

$x = -1$ ની ચકાસણી કરતા તે સમીકરણનું સમાધાન કરે છે. આથી $x = -1$ ઉકેલ છે. જવાબ : (A)

$$(98) \quad e^{-\frac{\pi}{2}} < \theta < \frac{\pi}{2} \text{ હોય, તો નીચેનામાંથી કયું સત્ય બને ? } (0 < \cos\theta < 1)$$

$$(A) \cos(\log\theta) > \log(\cos\theta)$$

$$(B) \cos(\log\theta) < \log(\cos\theta)$$

$$(C) \cos(\log\theta) = \log(\cos\theta)$$

$$(D) \cos(\log\theta) = \frac{2}{3}\log(\cos\theta)$$

$$\text{ઉકેલ : } e^{-\frac{\pi}{2}} < \theta < \frac{\pi}{2}$$

$$\log e^{-\frac{\pi}{2}} < \log\theta < \log\frac{\pi}{2}$$

$$\therefore -\frac{\pi}{2} < \log\theta < \frac{\pi}{2} \quad \left(\frac{\pi}{2} < e \Rightarrow \log\frac{\pi}{2} < 1 < \frac{\pi}{2} \right)$$

$$\therefore \cos(\log\theta) > 0 \quad (1)$$

$$\text{હવે, } 0 < \cos\theta < 1$$

$$\therefore \log(\cos\theta) < \log 1$$

$$\therefore \log(\cos\theta) < 0 \quad (2)$$

(1) અને (2) પરથી

$$\cos(\log\theta) > \log(\cos\theta)$$

જવાબ : (A)