

AJ-1544

M. A. / M.Sc. (Previous)
Term End Examination, 2021-22
Mathematics (Paper-V)
(Advanced Discrete Mathematics)

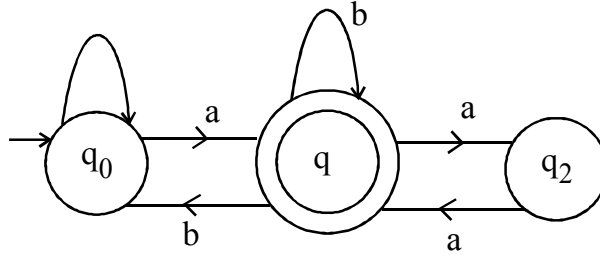
Time : 3 hrs.]**[Maximum Marks : 100**

Note – Answer any **five** questions. Answer to each question should begin on a fresh page. All questions carry equal marks.

1. (a) Define and explain each of the following :
 - (i) Logical Equivalence
 - (ii) Validity
 - (iii) Demorgan's Law
 - (iv) Conditional and Negation
 - (v) Idempotent.
- (b) Show that :

$$P \Rightarrow (q \Rightarrow r) \equiv (p \wedge q) \Rightarrow r$$
2. (a) Define Quantifier, Universal Quantifier and Existential Quantifier.
 Prove that the following argument
 $P, P \rightarrow q, q \rightarrow r \vdash$ is valid.
- (b) Prove the validity of the following argument :
 - (i) “If I get the admission and work hard, then I will get first division.
 If I get first division, then I will be happy. I will not be happy.”
 - (ii)
$$\frac{[(p \Rightarrow q) \Rightarrow r] \Rightarrow s}{r}$$
3. (a) Define Homomorphism of semigroup and monoids.
 Let $\langle M, *, e \rangle$ and $\langle T, \Delta, e' \rangle$ be two monoids with identities e and e' .
 If f is an onto mapping from M to T i.e.,
 $f : M \rightarrow T$ is an isomorphism.
 Then $f(e) = e'$.
- (b) Define congruence relation and quotient semigroups.
 Let $(S, *)$ and $(T, 0)$ be semigroups prove that $S \times T$ and $T \times S$ are isomorphic semigroup.
4. (a) Define Lattice and Direct product and show that dual of a Lattice is a Lattice.
- (b) Define complete, complemented and distributive lattice, and show that two bounded lattices L_1 and L_2 are complemented iff $L_1 \times L_2$ is complemented.
5. (a) Define order relation in a Boolean algebra and show that the order relation “ \leq ” is a partial order relation in a Boolean algebra.
- (b) (i) Define Boolean function and minimal Boolean function.
 - (ii) Change the following functions to disjunctive normal form,
 $f(u, v, w) = (u + v + w) (u \cdot v + u' \cdot w)'$.
6. (a) What is minimization of Boolean functions ? Use a Karnaugh map to find a minimal sum for
 $E = x'yz + x'yz't + y'zt' + xyzt' + xy'z't'$
- (b) Define equivalent circuits.
 Then show that the algebra of Boolean switching circuits is a Boolean algebra.

7. (a) Define context free Grammar and Regular grammar. Construct a grammar for the language $L = \{a^i b^{2i} : i \geq 1\}$
 (b) Explain Regular expressions. Find the regular expression for the following df a.



8. (a) Define equivalence of finite state machine then show that the following machines M_1 and M_2 are equivalent.

State	Input		Output
	1	2	
$\Rightarrow A$	B	C	0
B	F	D	0
C	G	E	0
D	H	B	0
E	B	F	1
F	D	H	0
G	E	B	0
H	B	C	1

State	Input		Output
	1	2	
$\Rightarrow A$	B	C	0
B	C	D	0
C	D	E	0
D	E	B	0
E	B	C	1

- (b) Describe Moore Mealy machines with examples.
9. (a) Define the following and with examples :
- (i) Complete graph
 - (ii) Bipartite graph
 - (iii) Eulerian graph
 - (iv) Cutsets
 - (v) Planar graph
- (b) The maximum number of edges in a simple graph with n-vertices is $\frac{n(n-1)}{2}$.
10. (a) Define the following with examples :
- (i) Hamiltonian Graph
 - (ii) Spanning tree
 - (iii) Indegree and Outdegree
 - (iv) Adjacency matrix
 - (v) Incidence matrix,
- (b) The necessary and sufficient condition for a connected graph G to be an Euler graph is that “all vertices of G are of even degree.