

AJ-1542**M. A./M. Sc. (Previous)****Term End Examination, 2021-22****Mathematics (Paper-IV)****(Complex Analysis)****Time : 3 hrs.]****[Maximum Marks : 100**

Note – Answer any **five** questions. Answer to each question should begin on a fresh page. All questions carry equal marks.

1. (a) State and prove Marera's theorem.

(b) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent series valid for the regions.

(i) $|z| < 1$

(ii) $1 < |z| < 3$

(iii) $|z| > 3$

(iv) $0 < |z+1| < 2$

2. (a) State and prove Liouville's theorem.

(b) State and prove the fundamental theorem of Algebra.

3. (a) State and prove Cauchy's Residue theorem.

(b) Evaluate $\int_0^\pi \frac{a d\theta}{a^2 + \sin^2 \theta}$, $a > 0$ by the theory of residues.

4. (a) Prove that cross-ratios are invariant under a bilinear transformation.

(b) Find the bilinear transformation which maps the point $z_1 = 2$, $z_2 = i$, $z_3 = -2$ into the points $\omega_1 = 1$, $\omega_2 = i$ and $\omega_3 = -1$

5. (a) State and prove Schwarz lemma.

(b) State and prove maximum modulus principle.

6. (a) If $|z| \leq 1$ and $p \geq 0$ then prove that :

$$|1 - E_p(z)| \leq |z|^{p+1}$$

(b) If $|z| < \frac{1}{2}$, then prove that :

$$(i) \quad |E_p(z)| \leq \exp(2|z|^{p+1})$$

$$(ii) \quad |E_p(z)| \geq \exp\left(\frac{-2}{|z|^{p+1}}\right)$$

7. State and prove Mittag-Leffler's theorem.

8. State and prove Riemann-mapping theorem.

9. State and prove Bloch's theorem.

10. (a) State and prove Poisson-Jensen formula.

(b) State and prove Jensen's theorem.