

2019
MATHEMATICS

Full marks: 100

Time: 3 hours

General instructions:

- i) Approximately 15 minutes is allotted to read the question paper and revise the answers.
 - ii) The question paper consists of 26 questions. All questions are compulsory.
 - iii) Marks are indicated against each question.
 - iv) Internal choice has been provided in some questions.
 - v) Use of simple calculators (non-scientific and non-programmable) only is permitted.
- N.B:** Check that all pages of the question paper is complete as indicated on the top left side.

Section – A**1. Choose the correct answer from the given alternatives:**

- (a) If $f(x)=|x|$ and $g(x)=[x]$, then $g \circ f(-3.7)$ is equal to 1
 (i) -3.7 (ii) 3 (iii) 3.7 (iv) 4
- (b) Consider the set Q with the binary operation $*$ as $a * b = \frac{ab}{4}$. Then the identity element is 1
 (i) $\frac{1}{4}$ (ii) 1 (iii) 4 (iv) 16
- (c) If a matrix A is both symmetric and skew-symmetric matrix, then 1
 (i) A is a diagonal matrix (ii) A is a zero matrix
 (iii) A is a square matrix (iv) none of these
- (d) If $y = a^x x^a$ then $\frac{dy}{dx}$ is equal to 1
 (i) $a^x x^{a-1} (a - x \log a)$ (ii) $a^x x^{a-1} (a + x \log a)$
 (iii) $a^x x^a (a + x \log a)$ (iv) $a^x x^{a-1} (x + a \log a)$
- (e) The point on the curve $y = 2x^2$, where the slope of the tangent is 8, is 1
 (i) (0, 2) (ii) (0, 8) (iii) (2, 8) (iv) (8, 2)
- (f) The value of $\int \tan^2 x dx$ is 1
 (i) $x - \tan x + C$ (ii) $\tan x + x + C$ (iii) $\tan x - x + C$ (iv) $x \tan x + C$

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(g) The value of $\int_{-1}^1 \log\left(\frac{2-x}{2+x}\right) dx$ is 1

- (i) -1 (ii) 0 (iii) 1 (iv) 2

(h) If $p\hat{i} + 3\hat{j}$ is a vector of magnitude 5, then the value of p is 1

- (i) 0 (ii) 1 (iii) ± 3 (iv) ± 4

(i) Let A and B be events such that $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, then $P(A|B)$ is equal to 1

- (i) $\frac{4}{9}$ (ii) $\frac{7}{13}$ (iii) $\frac{2}{3}$ (iv) $\frac{9}{4}$

(j) If A & B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{3}$, $P(A \cup B) = \frac{1}{2}$, then the events A and B are 1

- (i) independent (ii) dependent
(iii) mutually exclusive (iv) none of these

Section - B

2. Consider the set of real numbers \mathbf{R} . Define the relation R on \mathbf{R} as " aRb if and only if $a^2 + b^2 = 1$ ". Write the domain of R. Also, prove that R is not transitive. 2

3. Find $f \circ g$ and $g \circ f$ if $f(x) = |x|$ and $g(x) = |4x + 3|$. Are they equal? 2

4. Find the value of $\tan\left(\tan^{-1}\sqrt{3} + \sin^{-1}\frac{1}{\sqrt{2}} - \cot^{-1}1\right)$ 2

5. Solve the following equation for x : $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$ 2

6. If $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$, show that $A'A = I_2$ 2

7. Differentiate $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x . 2

8. If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$ 2

9. Evaluate $\int \sin^4 x dx$ 2

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10. Form a differential equation representing the given curve, $y = ae^{bx}$, where a & b are arbitrary constants. 2

11. Find the value of λ for which \vec{a} and \vec{b} are perpendicular if $\vec{a} = 7\hat{i} - \lambda\hat{j} - 7\hat{k}$ and $\vec{b} = 4\hat{i} + 5\hat{j} - \hat{k}$ 2

Section - C

12. a. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, show that $2A^{-1} = 9I - A$. 4

Or

b. Using properties of determinants, prove that:

$$\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

13. a. If $y = a \cos(\log x) + b \sin(\log x)$, prove that $x^2 y_2 + x y_1 + y = 0$ 4

Or

b. Find the coordinates of the point at which the tangent to the curve $f(x) = x^2 - 6x + 1$ is parallel to the chord joining the points (1, -4) and (3, -8)

14. a. If $x = a \sin 2t (1 + \cos 2t)$, $y = b \cos 2t (1 - \cos 2t)$, show that $\frac{dy}{dx} = \frac{b}{a}$ at $t = \frac{\pi}{4}$ 4

Or

b. If $f(x) = \left(\frac{3+x}{1+x}\right)^{2+3x}$, find $f'(0)$.

15. Evaluate $\int \frac{\cos^5 x}{\sin x} dx$ 4

16. a. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos x}{(1 + \sin x)(2 + \sin x)} dx$ 4

Or

b. Evaluate $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1 + \sin x} dx = \pi(\sqrt{2} - 1)$

17. Solve the differential equation $x \sin \frac{y}{x} \frac{dy}{dx} + x - y \sin \frac{y}{x} = 0$, given that $y(1) = \frac{\pi}{2}$ 4

18. a. If $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = 3\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - \hat{k}$, find the vector \vec{d} which is perpendicular to both \vec{a} and \vec{b} and $\vec{c} \cdot \vec{d} = 4$

Or

b. Show that the points A, B, C, D with position vectors $4\hat{i} + 8\hat{j} + 12\hat{k}$, $2\hat{i} + 4\hat{j} + 6\hat{k}$, $3\hat{i} + 5\hat{j} + 4\hat{k}$ and $5\hat{i} + 8\hat{j} + 5\hat{k}$ respectively are coplanar. 4

19. Find the foot and the length of the perpendicular drawn from the point (3, 4, 5) to the plane $2x - 5y + 3z = 39$ 4

20. In a bulb factory, machines A, B and C manufacture 60%, 30% and 10% bulbs respectively. Out of these bulbs, 1%, 2% and 3% of the bulbs produced respectively by A, B and C are found to be defective. A bulb is picked up at random from the total production and found to be defective. Find the probability that this bulb was produced by the machine A. 4

21. A die is tossed once. If the random variable X is defined as:

$$X = \begin{cases} 1, & \text{if the die results in an even number} \\ 0, & \text{if the die results in an odd number} \end{cases}$$

Then, find the mean and variance of X. 4

Section - D

22. a. Using elementary row transformations, find the inverse of the matrix $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$ 6

Or

b. Solve the following system of linear equations using matrix method:

$$\begin{aligned} 2x + 3y + 3z &= 5 \\ x - 2y + z &= -4 \\ 3x - y - 2z &= 3 \end{aligned}$$

23. a. Show that the semi-vertical angle of a cone maximum volume and given slant height is $\tan^{-1} \sqrt{2}$

Or

b. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

24. a. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the

$$\text{straight line } \frac{x}{a} + \frac{y}{b} = 1$$

Or

b. Using the method of integration, find the area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$ 6

25. a. Find the image of the point (1, 3, 4) in the plane $2x - y + z + 3 = 0$. Also, find the distance of the point from its image. 6

Or

b. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Also, find their point of intersection.

26. a. A housewife wishes to mix two types of food X and Y in such a way that the mixture contains at least 8 units of vitamin A and 10 units of vitamin B. X contains 2 units/kg of vitamin A and 1 unit/kg of vitamin B. While Y contains 1 unit/kg of vitamin A and 2 units/kg of vitamin B. It costs Rs 60/kg of X and Rs 80/kg of Y. Formulate this problem as a linear programming problem to minimize the cost of such a mixture and solve it.

Or

b. A shopkeeper wants to invest Rs 5400 on two types of pens. Type A costs Rs 180 per packet and type B costs Rs 60 per packet. He can get a profit of Rs 15 on type A and Rs 10 on type B. He has a space for 50 packets only. Formulate this as an LPP so as to get the number of each type of packets and the maximum profit. Also, find the maximum profit.
